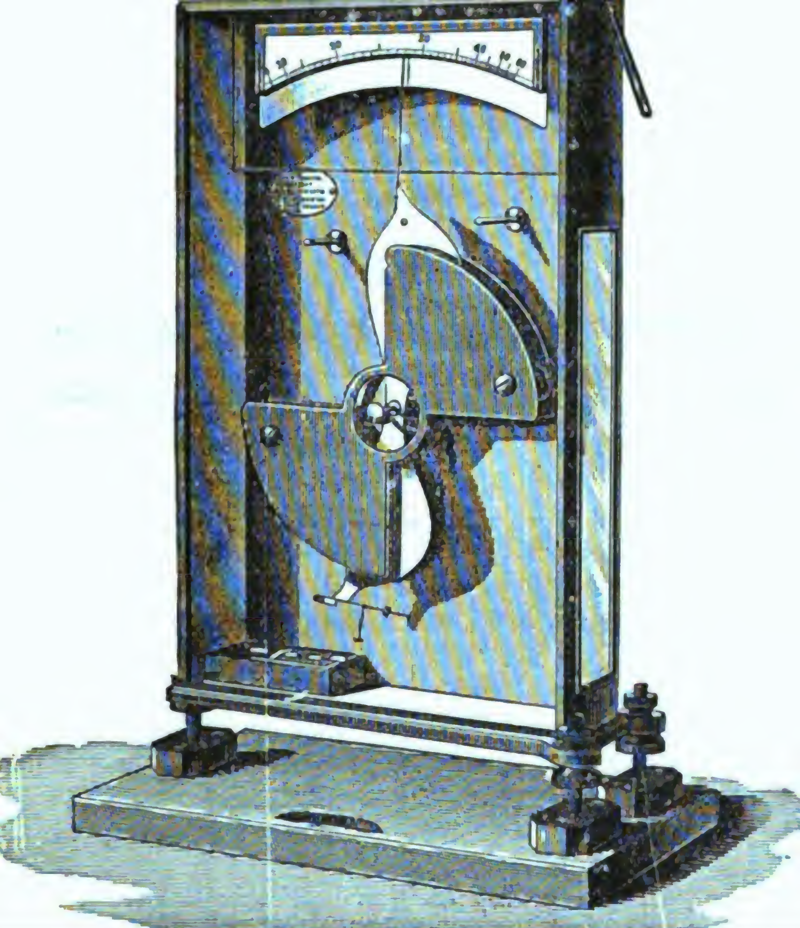

Magnets and Electric Currents

John Ambrose Fleming

T. N. Whitehead.

MAGNETS

AND

ELECTRIC CURRENTS

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MAGNETS

AND

ELECTRIC CURRENTS

AN ELEMENTARY TREATISE
FOR THE USE OF ELECTRICAL ARTISANS
AND SCIENCE TEACHERS

BY

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PREFACE.

TWELVE years ago the Author published a reprint of a course of lectures given to the pupils and workmen at an electrical engineering factory under the title 'Short Lectures to Electrical Artisans.' After the issue of several editions of this little book it became necessary to re-write it entirely, and the present volume may be considered therefore as taking the place of the former one. In recasting the information in such a manner as to conform more nearly to the present state of knowledge, the Author still desired to fulfil the original aim of supplying electrical artisans and engineering students with a brief and elementary account of the scientific principles underlying modern applications of electricity in engineering. With this object in view, the use of mathematical symbols has as far as possible been avoided, but at the same time an endeavour has been made to give the reader clear notions on the nature of the quantitative measurements which lie at the root of all applications of electrical facts in the arts. In spite of the attention which has been paid to the subject of electrical and magnetic terminology and units, it can hardly be considered that these are yet reduced to their final and most satisfactory form. The difficulties which students generally experience in grasping the real physical significance of magnetic and electric definitions and state-

ments, arise to some extent from the irrational basis on which the British Association system of units has been built up. It was felt therefore essential to give the reader a brief account of the rational system of units proposed by Mr. Oliver Heaviside. The general facts of the subject have, however, been explained and stated in the text in terms of the usual definitions and language, because in a short elementary treatise intended for the use of beginners, it is not desirable to depart too far from the use of expressions which are current in more advanced text-books, and to which such a volume as the present one is intended to be introductory. The Author has also followed the ordinary custom in using such terms as electromotive *force*, magnetomotive *force*, and magnetic *force*. There can be but little doubt, however, that students would sooner acquire clear notions on the subject of quantitative electromagnetism, if the multiplicity of equivalent terms, and especially the customary extended use of the word *force*, were discontinued by common consent. A clear and unambiguous magnetic phraseology could be constructed on the following basis:—

Let the idea now conveyed by the term magnetomotive force be expressed, as Mr. Heaviside and other writers have already done, by the word *gaussage* (pronounce "gowsage"), and let the gaussage between any two points or around any magnetic circuits be measured in a unit termed a *gauss*, such that 80 ampere-turns are equal to one microgauss.

Let electromotive force similarly be called as usual, *voltage*, and measured in *volts*.

Let the gaussage per centimetre along any direction at any place, be called the *gaussivity* at that point

Gaussivity is therefore the name for that quantity which is commonly called the magnetic force or the magnetic intensity at any point.

Similarly, let *voltivity* be the name for the voltage per centimetre in any direction, or for the so-called electric force at any point.

The term *magnetic flux*, or simply *the flux* or *fluxage*, may then very well be used to denote what is commonly called the total induction, and be stated to be measured in units called webers.

The flux density might then consistently be called the *fluxivity* at any point.

The essential qualities of a ferromagnetic body are then defined, as explained in the text, by the terms *retentivity*, *coercivity*, *reluctivity*, and *permeability*, and the qualities of a magnetic circuit, by the terms, *reluctance* and *permeance*, as usually is the case.

The *fluxivity* is therefore the name for the magnetic state produced by *gaussivity* at any point in empty or matter-occupied space. A distribution of gaussivity exists in the regions near all electric currents or magnetic poles.

Retentivity is the property possessed by ferromagnetic bodies, of maintaining fluxivity when the impressed gaussivity is withdrawn.

The reluctivity of the material is the measure of the gaussivity required to produce unit fluxivity at any point. The energy stored up per unit of volume of the magnetic circuit is measured by half the product of the gaussivity and fluxivity at that point. By the use of these or similar terms the student is led to recognise clearly the relations of the magnetic measurable quantities. He is not confused by the employment of the

term *force* in other than its strict dynamical sense, and this latter word is reserved entirely as the name for the cause, whatever its nature, which effects a change in momentum in material substances. The analogies as far as they exist, between mechanical stress and strain and magnetic gaussivity and fluxivity are easily seen and pointed out. If similarly the total current through any area is called the *amperage* (pronounce "ampeer-age"), and the rational system of units adopted, at least in theory, we should have a simple and easily remembered series of statements concerning the fundamental relations and methods of measuring the various magnetic and electric quantities concerned in electromagnetism. The two principal relations would then admit of the simplest possible statement as follows.

In a co-linked electric and magnetic circuit, such as exists in a dynamo or transformer, the *gaussage round* the magnetic circuit is numerically equal to the *amperage through* the magnetic circuit. Any change in the *fluxage through* the electric circuit produces a *voltage round* the electric circuit, numerically equal to the time rate of change of the fluxage.

The methods of marking out spaces occupied by magnetic flux by "lines" and "tubes" are useful from a mathematical point of view, but the practical student learns more readily to submit the facts to arithmetical calculation when he discards these notions, and thinks of magnetic flux or fluxage merely as a physical state existing in a magnetic circuit which is measured in webers or microwebers, just as he thinks of electric current or amperage as another kind of physical state in an electric conducting circuit, and which is measured in amperes or milliamperes, as the case may be. In the

first case the active cause producing flux or *fluxage* is called *gaussage*, and in the second case the active cause of current or *amperage* is called *voltage*. Electric and magnetic terminology is undoubtedly in a transition condition. Terms and phrases based upon "action-at-a-distance" notions and "fluid" theories still survive, and are intermixed with others of later coinage.

The objection which is generally raised to the introduction of new words and terms may be met by the contention that whilst scientific progress both in research and teaching is assisted up to a certain stage by the use of language which involves some hypothesis, or by words descriptive of some imagined state or process, yet beyond that point it is desirable to divest terminology of unnecessary hypotheses, and that therefore new terms are needed to replace the older ones when these last are found to suggest erroneous ideas, or to involve undesirable ambiguity of language.

In conclusion, the Author desires to express his thanks for the loan of blocks for some diagrams to the Society of Arts, the *Electrician* Publishing Company, Messrs. Elliott Bros., Mr. James Pitkin, and the General Electric Company, and also in particular to Mr. W. C. Clinton for kind assistance in revising the proof sheets.

J. A. F.

UNIVERSITY COLLEGE, LONDON:
December 1897.

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MAGNETS

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CHAPTER I.

MAGNETS AND MAGNETISM.

§ 1. **Magnets.**—In order that the student may properly understand the fundamental facts concerning magnets and electric currents, he is recommended to try, or to witness, the simple experiments described in the succeeding pages, and as far as possible to verify for himself the truth of the statements made in the chapters of this short treatise.

Let him procure in the first place a sample of *lodestone*, the scientific name for which is *Magnetite*, some iron filings, a bundle of steel knitting needles, a skein of floss silk, some fine copper binding wire, a few iron nails, and also small screws of iron, steel, and brass. In addition include, if possible, one or two small pieces of metallic nickel and cobalt.

Lodestone or *magnetite* can be obtained from nearly all mineralogists or dealers in physical and chemical apparatus. Good specimens are sold generally for about 2*s*. 6*d*. or 5*s*. a pound. In choosing a sample, select a piece of an elongated shape, like a plum or large olive, and which picks up iron tacks or brads well at both ends. Two such pieces, at least, should be procured. They should be tested on purchasing, to see if they each will attract and pick up well a small iron screw or three or four iron tacks at each extremity.

Magnetite or lodestone is an oxide of iron, or compound of iron and oxygen, having a chemical composition represented by the formula Fe_3O_4. It is very widely distributed in Nature in the earth's crust, and is found in masses or veins as a lustrous black or dull brown stone, in Sweden, Nova Scotia, Canada, Arkansas in the United States of America, and in many other places. It is found sometimes in enormous masses, forming beds or even entire mountains, and as an ore of iron, furnishes a source of very pure iron. It occurs also in New Zealand, Scotland, India and other localities, in a finely granular state known as *iron sand,* and this is also a source from which an excellent quality of iron may be extracted. It may be found frequently in crystals having a black lustre and an octagonal form, and it is the principal constituent in a non-magnetic state of the scale or oxide which is detached from iron when it is heated red-hot and forged under the hammer.

This ore of iron has been known for long ages past to be possessed in varying degrees of two remarkable properties which are called its *Magnetic Properties.* In the first place, if laid in iron filings, or in contact with small pieces of iron or magnetite, these last cling to it chiefly at certain points or edges of the mass which are spoken of as the *poles* of the lodestone.

The earliest mention, in English, of the attractive proper ties of magnetite, or the natural magnet, occurs in the writings of Alexander Neckam, in 1200 A.D. In 1269 a very full account of the chief facts known about it was given in a celebrated letter by Peter Peregrinus, or Peter the Pilgrim, a soldier monk serving in the army of Charles of Anjou, brother of Louis IX. of France. This letter, written in the trenches at the siege of Lucera, is remarkable for its complete statement of the facts of magnetic attraction. On the other hand, long before this date much knowledge of the properties of lodestone had been obtained by other nations. Early Phrygian iron miners, settled in Samothrace, an island in the Ægean Sea, were well acquainted with the attractive power of magnetite for iron. Priests of Samothrace, in 514 B.C., even sold magnetised iron

rings as a remedy for gout in the fingers, just as magnetic quacks retail various so-called magnetic remedial appliances to-day.

The second important fact with regard to this particular iron ore is, that if an elongated mass of it, having two well-defined poles at the extremities, is suspended by a thread or floated upon water in a little wooden cup, it always tends to place itself with the line joining the poles in a certain position with regard to the meridian of the place at which it is suspended, and if disturbed from that position it returns to it again. In the northern hemisphere of the earth, one pole of the lodestone sets itself approximately towards the north, and the other towards the south.

Let the reader take one of his pieces of magnetite, and having determined the position of the poles, suspend it by a few fibres of floss silk, so that it hangs with the line joining the poles in a horizontal position. It may conveniently be upheld by attaching the silk support to a little wooden stand. Verify then the following facts. If the suspended loadstone is dipped into iron filings, these cling on to it, chiefly at the poles, arranging themselves in little tufts. If another piece of lodestone is held near to the suspended one it will be found that each end or pole of the movable lodestone is differently acted upon by one selected pole or end of the fixed lodestone. Each pole of the fixed lodestone attracts one end of the movable one, and repels the other end of the movable one. It is clear, therefore, that the two poles of the lodestone are not alike in properties, and their behaviour is summed up in the following law :—

Similar poles Repel, Opposite poles Attract, understanding by the term *similar* poles, those that are found to be magnetically alike, and by *opposite* poles those that are not alike in a magnetic sense, when tested against the same pole of a third lodestone.

Dr. Gilbert, of Colchester, in his great book 'De Magnete ('On the Magnet') published in 1600 A.D., first suggested that

the earth as a whole is a great magnet, or acts as if it were one large lodestone, magnetised in an irregular manner. The tendency of the suspended lodestone at any place on the earth's surface to set its axis in a certain direction is merely due to the effort of the small lodestone to place its poles towards the dissimilar magnetic poles of the earth acting as a great lodestone. Hence we ought properly to call that end of the suspended lodestone which directs itself towards the north pole of the earth the *south pole* of the lodestone, because that north-pointing end is a pole magnetically similar to the southern hemisphere of the earth.

To distinguish the lodestone poles they are generally called the *North pole* or *North-seeking pole* and the *South pole* or *South-seeking pole*, and they are also called sometimes the *Red pole* and the *Blue pole*, and distinguished by red and blue paint or paper placed on the poles. The student can recollect that *R*ed corresponds to No*R*th, and Bl*U*e to So*U*th, by noting the way in which the letters R and U occur in the words Red and North, and Blue and South.

Hence the law of magnet attraction and repulsion is otherwise stated :—

$\left\{\begin{array}{l}\text{Red}\\ \text{North pole}\end{array}\right\}$	attracts	$\left\{\begin{array}{l}\text{Blue}\\ \text{South pole}\end{array}\right\}$	and
North pole	repels	North pole.	
South pole	repels	South pole, and	
South pole	attracts	North pole.	

The next fact of importance with regard to the magnetic properties of magnetite is that it can bestow them upon iron and steel when placed in contact with it.

Take a steel knitting needle, and having ascertained, by dipping the end of it in iron filings, that it is free from magnetism, stroke the knitting needle, say twenty times, over the pole of the lodestone with an action like a violin bow over the strings ; but drawing the needle always one way, and not backwards and forwards. After so doing, test the knitting needle by dipping its ends in

iron filings. It will be found that they have now acquired the property of picking up iron particles, and also if the knitting needle is suspended by its centre by a fibre of floss silk, it will be seen that the knitting needle has acquired a *directive* property like the lodestone, and that it has a north and a south pole. The knitting needle is then said to be magnetised by the

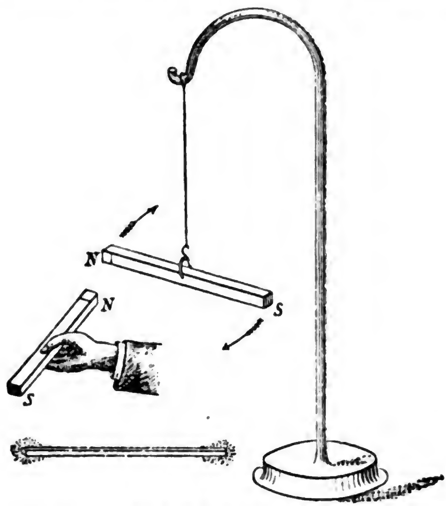

Fig. 1.—Repulsion of similar Magnetic Poles.

lodestone. Pieces of steel so treated are called *Permanent Magnets*, and are spoken of as bar magnets or horseshoe magnets, from their shape. Each permanent magnet has two opposite magnetic poles ; one of these is attracted and the other repelled by one selected pole of another permanent magnet. See Fig. 1.

The student or teacher should procure a pair of steel bar magnets, not less than 8 or 10 inches in length. These are

sold in pairs, and generally placed parallel to one another in a wooden tray and furnished with short pieces of soft iron called *keepers* placed across the ends. In order to prevent the steel from rusting when continually handled by damp hands, the magnets should be wiped over occasionally with a cloth just sprinkled with a drop or two of paraffin oil. This will keep them bright. Also round pieces of red and blue paper should be glued on to the poles to indicate clearly the north and south seeking poles. In putting them into the tray, lay the magnets parallel to each other, but not touching, and with their poles in opposite directions. Then place the keepers so as to complete the magnetic circuit. The action of the keepers in preserving the magnets from loss of power is due to the fact that on thus forming a *closed magnetic circuit* the free poles are practically abolished. These free poles exercise a demagnetising action on the rest of the bar. After using the magnets in iron filings, be careful to wipe off all stray particles of iron adhering, so that the keepers may make close contact with the magnet poles. Also a steel horse-shoe magnet should be procured, and kept when not in use with the keeper on. Contrary to usual statements, it does no harm to the magnet to pull *off* the keeper rapidly. In magnetising a knitting needle or other piece of steel lay the steel on the table and draw one pole of a bar magnet uniformly along it from one end to the other. Repeat the process many times, always drawing the bar magnet one way, and not backwards and forwards. The last end of the steel touched becomes a pole of opposite name to that of the end of the bar magnet with which the steel is stroked.

Magnetise a steel sewing needle by stroking it with a bar magnet, and then suspend it by a short length of a fibre of floss silk. A small suspended magnetic needle of this kind or a small compass needle is called an *Exploring Needle.* Hold the exploring needle anywhere near to a piece of lodestone, or to one of the bar magnets, and move it about. It will be seen that at every point in the neighbourhood of the magnetic mass, the small exploring needle takes up a certain direction when its vibrations have ceased. This shows that magnets, whether natural or steel permanent magnets,

affect all the space round them, and the region within which their influence is felt is called their *Magnetic Field.*

Near to the poles of the magnet the field is said to be *strong*, and far away it is said to be *weak*. The magnetic field has therefore two qualities at every point, viz. *strength* and *direction*. Its direction is shown by the direction in which the small exploring needle sets itself, and later on we shall show how its strength may be measured at any point.

A convenient form of exploring needle or *Magnetometer* is made by fitting a glass test-tube, $\frac{5}{8}$ of an inch in diameter, with a cork through which passes a pin bent into a hook at the lower end. Attach to this hook a fibre of floss silk, and to the end of the silk a fragment of magnetised sewing needle half an inch in length. Let the needle hang as near as possible to the closed end of the tube. Mark the north-seeking end with a touch of red paint.

§ 2. **Magnetic Classification of Substances.**—There are only three pure metals which can be attracted or influenced by a not very strong magnet, and these are *Iron, Nickel* and *Cobalt.* Small pieces of metallic nickel and cobalt may be bought at any chemical warehouse, and it will be found that an ordinary steel horse-shoe magnet picks them up quite easily. Certain alloys or mixtures of these three metals are also magnetic. In addition to these metals, the compounds of iron, with carbon, known as *Steel;* with oxygen, in the form of *Magnetite* (Fe_3O_4); and with sulphur, in the compound called *Pyrrhotite* (Fe_7S_8), are substances which are distinguished by similar magnetic properties. These bodies and any alloys or mixtures of them exhibiting the same powers are called the *Ferromagnetic* or *Strongly Magnetic* bodies.

In the next place, there are a number of bodies such as the metals *Manganese, Palladium, Platinum,* and also *Oxygen* (both gaseous and liquid), and numerous salts or compounds of iron, manganese, cobalt and nickel, which, though not visibly attracted by a feeble magnet

or lodestone, can yet be attracted by the pole of a very powerful magnet. These bodies are called *Paramagnetic* or *Feebly Magnetic* substances. All the ferromagnetic bodies become changed into *feebly magnetic* ones by heating to a certain temperature. Thus iron at and beyond a temperature called its *Critical Temperature*, which lies between 690° C. and 870° C., or a bright red heat, loses all its strong magnetic qualities. In the same way nickel loses them at about 300° C.

Then there is a third class of bodies including *Bismuth*, *Antimony*, *Phosphorus*, *Copper* and many other metals and substances which are called *Diamagnetic* bodies, and these are feebly *repelled* by a powerful magnetic pole. There is, however, no substance which possesses diamagnetic properties to anything like the extent to which iron, for instance, possesses magnetic properties. It is a remarkable thing that the ferromagnetic or strongly magnetic substances should be so few in number, and it points to the fact that there is something unique or special in the structure of iron, nickel and cobalt in the metallic state, and when at temperatures lower than their critical temperatures.

It is also remarkable that certain alloys of iron with other strongly or feebly magnetic metals may be almost non-magnetic or at least very feebly magnetic. Thus, for instance, an alloy of iron with 12 per cent. of manganese in it and some small percentage of carbon, forming what is known as *Hadfield's Manganese-steel*, is a very feebly magnetic body.* It can have its magnetic properties increased very considerably by prolonged heating out of contact with air, and it then passes into a strongly magnetic condition.

Amongst other interesting alloys are the steels containing nickel. A steel having in it 20 per cent. of nickel, for example, can exist in two magnetic condi-

* The fact that there may be 88 per cent. of iron in a practically non-magnetic substance shows that ferromagnetic qualities depend on the *molecule* and not on the *atom*.

tions. If heated to a dull red heat it passes into a feebly magnetic state, and retains this state even when cooled again to ordinary temperatures. If it is cooled to a very low temperature, as by plunging it for a moment into liquid air at a temperature of $-185°$ C., it passes into a strongly magnetic state, and retains this state when heated again up to the ordinary temperature. In the two states, after having been strongly heated or considerably cooled, and then brought back to the ordinary temperature, the physical qualities of the material are quite different. In the feebly magnetic state it is fairly ductile and plastic, in the strongly magnetic state it is very hard and brittle, and in many other respects the physical qualities are quite different, and yet the chemical composition remains the same.

Another fact of some interest is that although iron is the most strongly magnetic metal, and manganese is a feebly magnetic metal, yet chemical compounds of manganese are more strongly magnetic than the corresponding compounds of iron. Thus manganous sulphate is more magnetic than ferrous sulphate, and similarly in many other cases. We are, however, far from understanding the ultimate cause of strong magnetism in certain substances, and why it is that pure iron, nickel and cobalt stand out distinguished especially by this unique property from all other metals and bodies whatever.

§ 3. **Electro-Magnetism.**—Although a piece of copper wire in its ordinary condition possesses no magnetic properties, it is possible to produce in and around a copper wire a magnetic field by means of an electric current.

Let the student construct the following simple piece of apparatus with which to study this effect.

On a pasteboard tube 1 inch in diameter and about 6 or 7 inches long, wind four layers of double cotton-covered copper wire of the size known as No. 20. Procure a piece of zinc 4 inches long and 3 inches wide

and about $\frac{1}{16}$ of an inch in thickness. Bend the zinc into a tube 1 inch in diameter, and fix a cork well soaked in paraffin wax at one end. Through a hole in this cork pass a rod of hard carbon of the kind sold for arc lamp carbons, the carbon being 5 inches long and $\frac{1}{2}$ an inch in diameter. Bind a piece of bare copper wire tightly round the end of the carbon, and solder another piece of copper wire to the zinc tube, and suspend this carbon-zinc couple to the bobbin of wire as shown in Fig. 2, joining one end of the helix of wire to the carbon and the other end to the zinc. Hang up the whole arrangement by a few fibres of floss silk so that the zinc tube is vertical, and let it hang in a glass beaker, gallipot or tumbler (see Fig. 2). Fill the vessel with a solution made by mixing in a pint of water, 2 oz. by weight of strong sulphuric acid, 3 oz. by weight of crushed bichromate of potash, and a quarter of an ounce of mercurous chloride or calomel.

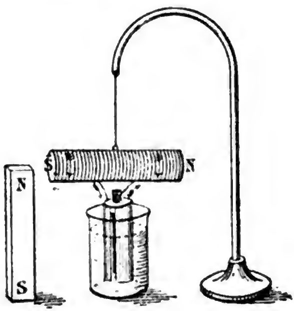

Fig. 2.—Spiral Conductor conveying an Electric Current exhibiting Magnetic Polarity.

In making the above battery solution it is best to pour a pint of boiling water on the finely crushed bichromate of potash, which is a red crystalline salt, to be bought of any chemist. Stir it up well, and when it is dissolved and the solution *quite* cold pour into it *gently* and *slowly* the strong sulphuric acid or oil of vitriol. This last is very corrosive and should be carefully handled. Then stir in the quarter of an ounce of mercurous chloride or *calomel*. The mercurous chloride being poisonous should be used with great care. The solution when made should be kept in a bottle and labelled "Bichromate Solution for Battery." Be careful not to spill the solution about

in using it, as it will destroy and stain all table-cloths and carpets. This solution costs about 1*s.* 6*d.* a gallon to make. When the solution has been used once it can be used again by adding 2 oz. more of strong sulphuric acid to the pint. Before placing the zinc in this battery solution it should be rubbed over with a little mercury to amalgamate it.

When the above described arrangement consisting of a carbon rod and a zinc plate, each attached to the opposite ends of a long spiral of copper wire, is hung freely, so that the carbon and the zinc are immersed about four-fifths of their length in the above mentioned battery solution, but not touching the containing vessel, it will be found that the suspended helix of copper wire possesses magnetic properties. If one end of a strong steel bar magnet is brought near each end of the helix in turn it will be found that the helix or spiral has a north pole at one end and a south pole at the other. It will direct itself if left alone like the suspended lodestone, and in all respects it behaves just as does a permanent steel bar magnet or elongated lodestone. If after some time the hand is placed on the wire spiral it will be felt to be slightly warm. If a small bundle of fine steel knitting needles are placed in the interior of the paper tube it will be found that they become strongly magnetised, and the whole arrangement will possess more marked magnetic properties than before. It is clear, therefore, that the carbon fastened at one end of the wire and the zinc at the other, when both are dipped in an acid solution, enable the copper wire joining them, and formed into a helix or spiral, to magnetise the space all round it and affect it just as does a bar magnet or lodestone. The copper wire under these conditions is said to be traversed by an *Electric Current*, and the term electric current is applied to denote the sum total of all the new and peculiar powers possessed by the wire under these conditions.

The carbon, zinc and acid constitute what is called a *Voltaic Cell* or *Couple*; the carbon being termed the

Positive pole of the cell, and the zinc the *Negative pole.* For the sake of distinction, the current is said to *flow* from the positive pole of the cell through the wire to the negative pole of the cell. This, however, is merely a convention, and we know little or nothing about the actual motions which constitute or accompany the electric current.

If, however, we think for the time being of the current as flowing in the wire like water in a pipe, and as starting out from the carbon, then if looking at the helix end-on, we view it in such a direction that the current would appear to be rotating in the opposite direction to the hands of a watch; that end of the helix nearest the eye will be a *North pole.* (See Fig. 3.) By constructing two

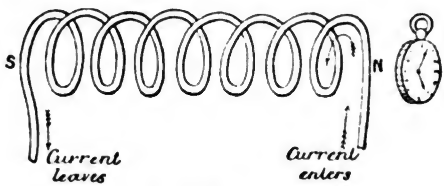

Fig. 3.—Poles of a Solenoid.

helices of this kind, it is possible to show that they behave to each other exactly like two bar magnets, as regards the mutual attraction and repulsion exercised by their poles.

We shall defer until a later chapter a discussion of the various methods of generating electric currents, but henceforth assume that the student or teacher wishing to follow out the experimental demonstrations here given has the means of obtaining electric currents either from an electric lighting service main or by batteries. A small portable secondary battery of about six cells, and which can be charged when required off an electric supply main, affords a convenient source of current for the experiments described.

If a helix of wire is formed by winding cotton-covered wire closely and evenly in several layers on a pasteboard tube, we have what is termed a *Solenoid.* It is convenient to construct several such solenoids as follows:—

Pasteboard tubes can be bought or prepared similar to those used in sending papers or drawings by post. Take such a tube about 1 foot long and 2 inches outside diameter. Provide it with a pair of cheeks or ends by glueing on to the ends of the tube square pieces of hard wood, about 4 inches square, and which have had holes 2 inches in diameter made in them. Then wind on the tube six or eight layers of double cotton-covered copper wire, No. 18 size, and bring out the ends to brass terminals screwed into one end piece or cheek of the coil.

Two such solenoids should be constructed; one made as above described, and a second one made without cheeks and wound on a tube of pasteboard about 1 inch in diameter. This smaller solenoid should be wound with six or eight layers of No. 20 cotton-covered wire, and made so as to slip easily inside the larger solenoid. If the student has access to a lathe he will find it saves much labour in winding on the wire. In both cases the exact number of turns of wire put on the tube should be noted, and also the length of the bobbin between the cheeks. The number of turns and length should be marked on each bobbin for reference. The wire must be wound on very evenly, and when finished the outer layer should be varnished with shellac varnish and wound over with tape to protect it. It is convenient to attach the ends of the copper wire to brass binding screws.

If a bar of soft iron is placed in a helix or spiral of wire, and a current sent through the wire in such a direction that when looked at end-on the current, if it could be seen, would appear to revolve round right-handedly, or like the hands of a watch, then that end nearest the eye is a *South* or *Blue pole.* The direction of the acquired magnetism of the iron is in the same

direction as that of the field of the empty solenoid or helix. (See Fig. 4.)

An arrangement consisting of a bar of iron or bundle of iron wires surrounded by a helix or bobbin of

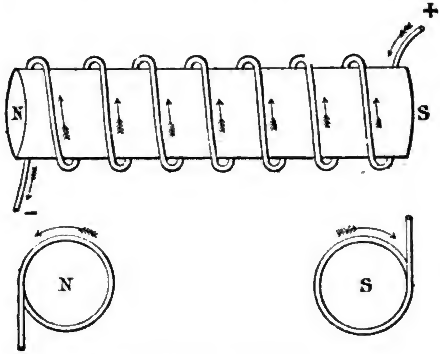

Fig. 4.—Poles of an Electromagnet.

covered wire, which causes the iron to become powerfully magnetic when an electric current is passed through the coil, is called an *Electromagnet.*

For all purposes for which a powerful magnet is practically required, electromagnets are used. In most cases these consist of a large bar of iron bent into the shape of a horse-shoe, or of two round vertical bars of iron united at the bottom by a yoke. Coils of wire are wound on these iron bars, and on passing a current through the coils a very strong magnetism is given to the iron. Generally speaking, removable blocks of iron, formed with blunt points at one end, are fitted on to the poles of the magnet, and these are called *Pole Pieces.* They serve to create a powerful magnetic field when placed on the magnet poles and nearly touching.

In electrical workshops a strong electromagnet is often required to magnetise steel bars or needles. A suitable form of electromagnet for such purpose may thus be made. A strip of

very soft Swedish iron is taken, about 2 feet long, 3 inches wide and 1 inch thick. This is bent twice at right angles about 6 inches from either end, so as to give it a horse-shoe shape. The ends may be planed flat. This horse-shoe is then wound over from end to end with half a dozen layers of No. 10 S.W.G. double cotton-covered wire, and the whole mounted on a convenient wooden stand. (See Fig. 5.) If, through such a magnet, a current of 10 amperes is passed from a few secondary cells, very powerful magnetism is excited in the iron core. A steel bar A B intended to be magnetised can then be placed across the poles and hammered with a wooden mallet whilst the current is passing. A few minutes suffice to magnetise the steel to saturation. If it is desired to magnetise small needles, such as compass-needles, then soft iron plates may be used as pole pieces to enable a small needle, such as a compass-needle, to bridge across the interval and so be magnetised.

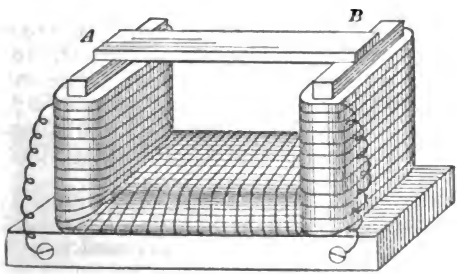

Fig. 5.—Workshop Electromagnet.

§ 4. **Magnetic Retentivity and Coercivity.**—Taking a helix of copper wire as described above, place in it a small bundle of soft or pure iron wire, the wires being cut about the length of the solenoid. These wires should be first carefully heated red-hot to deprive them of all magnetism. On passing an electric current through the copper wire helix, the iron wires will be found to have become powerfully magnetic, and will, whilst in the helix,

attract other pieces of iron and exhibit all the properties of permanent magnets.

Let the iron wires be then removed carefully after interrupting the current, and let them be tested with a small suspended exploring needle. It will be found that they are still strongly magnetic. They may even retain, when removed, as much as 90 per cent. of the magnetism they possess whilst in the helix. If, however, a smart knock or twist is given to these iron wires, and they are then tested again, it will be found that nearly all their magnetic polarity has vanished. The property of retaining the magnetic state after the magnetising force is withdrawn is called *Magnetic Retentivity*, and the ability to retain the magnetic state with varying degrees of power against mechanical shocks or reversed magnetic force is called the *Coercive Power* of the body, or its *Coercivity.* The coercivity of the iron or steel in any magnetic state is measured by the magnetising force which must be applied to the metal to exactly deprive it of all magnetism. This force is called the *Coercive Force.* The retentivity corresponding to any state of magnetisation is measured by the percentage of magnetisation that remains when the magnetising force is merely withdrawn.

If the same experiment is tried with a bundle of steel knitting needles, it will be found that, although magnetised when placed in the helix, the steel is not nearly so easily demagnetised after removal by shocks or knocks as the iron. The difference in the behaviour of iron and steel when magnetised in a helix is expressed by saying that the steel has greater *magnetic coercivity* than the iron, although the iron may have as great or even greater *magnetic retentivity* than the steel, as shown by the percentage of magnetism it can retain. The same fact is most easily shown in another way. Take a thin iron wire of very pure iron, usually called soft iron, and bend up the ends at right angles the better to hold it by. (See Fig. 6.) Draw along this wire lightly the pole of a

steel permanent magnet once or twice in the same direction, and holding it carefully, test this iron wire by means of a small pocket compass, or the exploring needle already mentioned. It will be found that the iron wire is magnetised, and possesses two magnetic poles, a north and south pole. Give the wire a sharp twist or two, and it will be found on testing it again with the compass needle that all magnetic polarity has vanished. Try the same experiment with a steel wire, and it will be discovered that it is not at all easy to deprive it of all the magnetic polarity once imparted to it by a similar procedure.

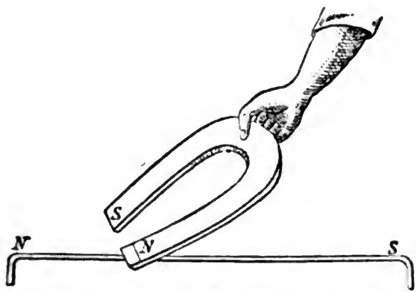

Fig. 6.—Experiment to show Retentivity of Soft Iron.

We shall later on describe more in detail how this magnetic retentivity and magnetic coercivity are measured, but meanwhile the fact to be noticed is that magnetisable bodies, such as steel and iron, have very different powers of retaining their magnetic state against mechanical blows or shocks, or reversed magnetic force. Steel once subjected to magnetic force requires a larger reversed magnetising force than iron to annul the effect of the original magnetising force, but nevertheless a steel magnet *is* deprived to some extent of its magnetism by blows, and should never be allowed to fall on the ground or be roughly used.

If a wire of very soft iron is magnetised, and then handled very gently, so as not to shake out of it the slightly held magnetism, such a wire is instantly demagnetised by sending along it a sudden discharge of electricity.

When a steel bar is magnetised strongly, either by stroking it with another bar or by placing it in a solenoid traversed by a current, part of the magnetism it acquires appears to be held less strongly than the remainder. This loosely held magnetism is called the *Sub-permanent Magnetism.* For many purposes it is necessary to get rid of the sub-permanent magnetism and leave the bar in a more stable magnetic condition, in which its magnetic polarity is not altered permanently by time or small shocks. This process is called *ageing* the magnet. It can be done by boiling the magnet for twenty-four hours in water and also subjecting it to some rough usage. A small piece of magnetised steel can be *aged* immediately by dipping it into liquid air, which exposes it to a very low temperature of 185° below zero Centigrade.

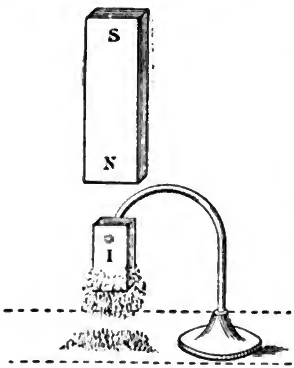

Fig. 7.—Magnetic Induction created in a Soft Iron Bar I, indicated by the attraction of Iron Filings.

When a bar of iron or steel has been magnetised in a solenoid, or in any other way, that part of the magnetism which is retained even against some mechanical shocks is called the *permanent magnetism.*

The difference between the coercivity of iron and steel accounts for the dissimilarity in behaviour in the case of the two metals when placed in a magnetic field and then removed from it. Take a short piece of very pure iron, say an inch long and a quarter of an inch square, and support it by a wire as in Fig. 7. Bring up

above it the pole of a bar magnet, placed near to, but not touching the iron. Then test the iron by applying to it a few iron filings held on a slip of card. It will be found that as long as the magnet remains near the short piece of iron, it is magnetised, and will hold up the filings. On withdrawing the magnet, the short piece of iron loses all its apparent magnetism, and the iron filings fall away. Try the same experiment, but using a piece of steel instead of iron, and it will be seen that the steel retains its magnetised state even after the magnet acting on it is withdrawn. It is customary to say that the iron or steel is in this case magnetised *by induction.* The proper way, however, to regard the effect is to state that the magnet exerts a *magnetic force* at all points in its field. If a piece of iron is placed anywhere in this field, this magnetic force creates *magnetisation* in the iron. If the magnetic force is withdrawn, this magnetisation may disappear almost entirely if the metal is soft or pure iron, because of its small coercivity, but does not disappear if the metal is steel. We shall return to this question in another chapter. It may be well, however, to mention here that particular qualities of steel called *magnet steel* are prepared which are characterised by great coercivity and retentivity. The presence in the steel of the metal tungsten is found to bestow this quality in a large degree, and hence bars of tungsten-steel, tempered glass-hard, are used in making magnets which are required to be very permanent.

In making permanent steel magnets the result to be attained is to prepare a quality of steel with both high retentivity and coercivity. Such steel is called magnetically hard steel. The steel has to be heated and then quenched to give it mechanical hardness, and this process bestows magnetic hardness on it as well. The temperature at which steel becomes magnetically hardened by quenching varies according to the amount of carbon in the steel. It is considerably lower in high-carbon steel than in low-carbon steel, and corresponds closely to the temperature at which *recalescence* occurs. The higher above the

point of recalescence the steel is heated before being quenched, the harder it is magnetically. The magnetic hardening produced by quenching is the larger the higher the percentage of carbon. To produce strong magnets the steel should, after hardening by quenching, be re-heated to 450° C.

§ 5. **Structure of a Magnet.**—In considering the structure or nature of a magnet, we are assisted to some extent by the discovery that it is impossible to separate in a magnetised bar that portion of it showing north polarity, from that portion of it showing south polarity. Let the student magnetise a steel knitting needle, and having tested it with the compass needle, and marked the polarity of the ends, immerse it in iron filings. It will be seen that whilst the filings adhere in bunches to the two poles they do not stick on at all in the centre.

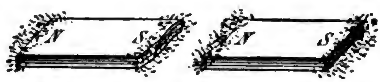

Fig. 8.—Magnet broken in two parts; each part exhibiting complete Polarity.

This alone would seem to indicate that the magnetic power resides in the ends of the bar. Let, however, the needle be broken in two parts at the centre, and two new magnetic poles immediately will make their appearance in the place which was a moment ago an apparently non-magnetic region (see Fig. 8). The two half needles will then on testing be found to be each a complete magnet having two opposite magnetic poles; the original north half gaining a south pole at the central break, and the original south half gaining a north pole as its complement. This experiment may then be repeated again with each half needle, and as long as a piece of needle remains large enough to break in half, so long will each rupture create two complete little magnets each provided with a north and a south pole.

The inference from this experiment is that every

particle or molecule of a magnetised steel bar is a perfect little magnet, and a further deduction from the facts is that the molecules or perhaps groups of molecules of iron or steel are in themselves permanent magnets, and that magnetisation consists in arranging these molecular magnets in an orderly manner. Hence we must think of the molecular magnets in a piece of iron or steel, which is not magnetised, as arranged in a manner so that they exercise no external action. This could arise in the following way :—

Let a group of six or eight or more small magnets be arranged in a circuit with opposite poles in contact as in Fig. 9. Such a ring or group of magnets will

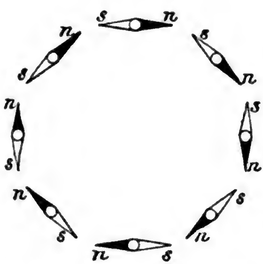

Fig. 9.—Magnets forming self-closed Chain.

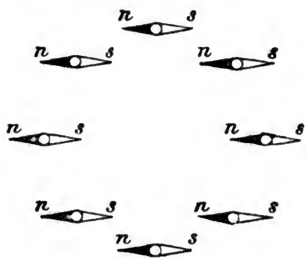

Fig. 10.
Magnets Colineated.

exercise no magnetic action upon an external compass needle or exploring needle. If then this chain or ring of magnets is rearranged so that the constituent magnets are made to point all one way as in Fig. 10, the group will immediately exhibit magnetic polarity as a whole. Hence it has been supposed with some good reason, that iron, nickel or other magnetisable substances are to be thought of as composed of molecules or groups of molecules, each of which is a perfect little magnet with its two opposite magnetic poles. Under ordinary circum-

stances these molecular magnets of which the mass is built up, arrange themselves in closed groups or chains so as to satisfy the mutual attraction of their opposite poles. The process of magnetisation consists then, not in creating a new power which did not previously exist, but simply in opening up some or all of these molecular chains, and compelling the magnetic molecules to face more or less in one and the same direction like soldiers at drill. This supposition has been found to explain quite simply many observed magnetic facts, and we shall revert to it again in connection with some of them.

Before we can advantageously further discuss magnetic phenomena, it is necessary to describe briefly the methods of measurement, and the units in use in scientific investigations, in order that we may study the subject with reference to the exact measurement of the quantities concerned in electric and magnetic work. This we proceed to do in the next chapter.

CHAPTER II.

MEASUREMENT AND UNITS.

§ 1. **Substance and Energy.** — In the external world of Nature there are two permanent causes of all the impressions conveyed to us by our organs of sense. These are called respectively *Substance* or *Matter*, and *Energy*. These agents exist and exhibit themselves to us in many different forms; and are both characterised by one distinctive quality, viz. *permanence;* that is to say, we cannot create or destroy them; but our relations to them are limited entirely to noticing and causing changes or transformations in them.

By carefully analysing many different substances, chemists have come to the conclusion that there are sixty-three or sixty-four different kinds of simple substances which up to the present they have been unable to change into one another or anything simpler. These are called *Elementary Substances.* The fact that the total quantity of these elementary substances can neither be increased nor diminished by any chemical process known to us, is called the law of *Conservation of Matter.*

By attentively considering the physical and chemical changes that can take place in substances, we have been led to see that these changes involve transformations in something else called *Energy*, associated with Matter. We are quite unable to separate *Matter* from *Energy*, or to say what they are apart from one another, even if they are capable of so existing; and no definition of one of them can be given which does not involve, at least implicitly, a mention of the other. It is clear, however, that apart altogether from changes in the nature or

amount of the *substance* present in any place, there is something else which may be associated with it which may be more or less, and which bestows upon it its power of affecting both our senses and other substances. Thus for instance, one and the same piece of iron can be more or less hot, more or less electrified, and set in motion with more or less speed. These states of heat, electrification, or movement, are exhibitions of energy associated with matter, and energy itself is not only a measurable quantity, but, as we shall see, is subject to a law of conservation like matter, and cannot either be created or destroyed. The progress of knowledge has led us to recognise many different *forms of energy*, or manifestations of energy associated with matter in some way, such as :—kinetic energy ; gravitational energy ; electromagnetic energy ; electrostatic energy ; chemical energy ; thermal energy ; radiant energy ; elastic or molecular energy, or energy of configuration.

The above is not by any means an exhaustive or complete list of the various forms of energy, but is merely illustrative. A body is said to possess *Kinetic Energy* when it is in motion, as, for instance, a cannon-ball flying through the air or a railway train moving along a line. All material bodies possess *Gravitational Energy* merely when placed in the presence of one another ; and this energy is increased the further apart they are removed. Thus, if the earth and a stone are separated by lifting up the stone, the gravitational energy of the two taken together is increased. In any inclosed space containing portions of matter, the total gravitational energy in that space is increased by separating the bodies as far apart as possible. We have not been able yet to obtain any insight into the real reason for this existence of energy due to the mere relative position of two or more masses of matter.

By *Elastic Energy* or *Energy of Configuration* is meant the energy possessed by bodies in virtue of the endeavour they make to recover their original shape or

size when deformed. Thus, a bent watch-spring has energy of configuration associated with it in virtue of its power to unbend, and resistance to bending. In some cases changing the *size* of a body is accompanied by an increase in its energy: as when we compress gas or air; but mere change of shape in a mass of air apart from change of size or bulk is not accompanied by any change in its energy. Heat also is a form of energy, and when we heat a body we increase its thermal energy.

It is one of the most remarkable achievements of scientific thought to have arrived at the notion that under all these various forms we may have to do with merely one and the same physical agency which we entitle *Energy*, and this generalisation is based upon the experimental fact that energy in one form can only be increased by making an equivalent amount in some other form less.

Energy not only exists in many forms, but it can be changed from one form to another. When we look a little more closely into the processes going on when an energy transformation is taking place, we find that in every case there are two factors concerned. There is always something of the nature of a *resistance* or opposition which has to be overcome, and there is also a change, generally called a *displacement*, which is made against that resistance. When a displacement is made against a resistance, *work* is said to be done. In the act of doing work a change or transformation of energy from one form to another is effected, and work is then said to be done by one agent or system of bodies on another. Thus, for instance, certain bodies resist change of form, or shape, or configuration, such as a watch-spring. If we make a change in form in opposition to this resistance, we do *work* on the spring and increase its elastic energy or energy of configuration. This is found to necessitate the expenditure of an equivalent amount of some other form of energy. Lifting up a body against the force of gravity, or compressing air against the elastic forces resisting

change of bulk, are also examples of displacements made against resistances, and hence of work done. These acts, therefore, are operations in which the energy associated with certain forms of substance is increased, and careful examination shows that they can only be performed by diminishing energy associated in some other form with matter. When we increase the speed or velocity of a body, we do work against its *inertia* or that quality of it in virtue of which it resists any change in its state of rest or motion ; and to do this we have to expend an equivalent of energy in some other form. In every case the amount of energy transformed is measured by the work done, or by the product of the resistance opposing the displacement, and the distance through which it is overcome. By measuring and comparing the energy-changes which take place during physical actions, three great generalisations have been established which are the fundamental principles of modern physics. They are as follows:—

1. Energy exists in various forms, and these different forms may be transformed into one another. For instance, heat may be changed into kinetic energy, or electrical energy into heat. This is called the principle of *Energy-Transformation.*

2. When such an energy transformation or change takes place, a definite quantity of one kind of energy disappears, and an equivalent quantity of another kind makes its appearance. No human power, however, and no physical process suffices to create or destroy any form of energy. Hence energy is subject to a law of *Conservation of Energy.* On account of its transformability, energy is recognised as a measurable quantity, and we can evaluate any quantity of any form of energy by stating its equivalent in one form, say gravitational or kinetic energy. Thus the increase in thermal energy represented by a rise in temperature of a known mass of some standard substance through 1° C. can be reckoned in foot-pounds or in ergs.

3. Although we can, generally speaking, transform energy from one form to another, we cannot by the machinery at our disposal always effect the complete transformation of any one form into another. We can, for instance, transform the whole of any quantity of kinetic energy into heat, as when a railway train is brought to rest by the brakes, but we cannot transform the whole of any quantity of heat into kinetic energy. Every transformation of one kind of energy into another involves the change of some part of that energy into heat, and in that particular form, as heat, it becomes lessened in *transformability* or *utility* as far as we are concerned. All energy-changes are thus not wholly reversible, but are subject to a condition called the principle of the *Dissipation of Energy*. There is a spontaneous tendency for some energy-changes to take place. Thus all the forms of energy called potential energy, such as a raised weight, bent spring, charged Leyden jar, &c., tend to pass spontaneously, and merely by the withdrawal of some small restraint, into kinetic energy, and then into thermal energy or heat. There is, however, no such natural tendency for thermal energy to change into mechanical or potential energy. This involves the consideration of what is called the Second Law of Thermodynamics.

§ 2. **Space and Time.**—In addition to the two great external permanent causes of sensation, viz. *Substance* and *Energy*, which we can neither create nor destroy, there are two conceptions in our own minds which are equally incapable of being annihilated. These are the ideas of *Space* and *Time*. No resolution of these notions into any simpler ones can be given which is satisfactory; but apart from any definitions, we are quite unable to divest our minds of these fundamental concepts. We may, for instance, think of a room, however large, without any material substances in it; but we cannot think of the *space* in the room as non-existent, nor can we think of *space* as limited to that room. In the same way we are able to consider any series of events as not having happened,

but we are quite unable to destroy the idea of a *duration*, or *time* occupied by events, whether the events have happened or not. Similarly, we are unable to think of time or duration as limited.

These physical and permanent causes of sensations, viz. matter and energy, and these mental and innate ideas, viz. space and time, constitute our fundamental experiences and concepts; and scientific knowledge consists in learning by experiment and trial the laws of their relations and their possible transformations.

The only way in which we can do this is by experience and deductive reasoning based upon it.

§ 3. **Physical Measurement.**—In nearly all the applications of scientific knowledge in the arts we find that an exact system of measurement of the agents or things with which we are concerned is necessary. Any quality of a natural object or agent which can be more or less, and which can have its magnitude or degree defined by reference to the same quality of a selected amount of something of the same kind taken as a standard, is called a *physical magnitude* or *physical quantity*. The act of making a comparison between the physical magnitude in question and another of the same nature made in respect of the particular quality which we are then considering, is called a *physical measurement*.

Thus, for instance, the volume, or (as it is generally called) the size of a body can be more or less. We can have much or little water in a vessel, and much or little space in a room. A volume is selected, say the gallon or cubic foot, and with this standard volume we compare the volume in question, and state the multiple or fraction which it is of the selected volume of comparison. This last is called the *Unit of Volume*, and the number which states the multiple or fraction which the volume we are considering is of the unit of volume, is called *the numerical magnitude* of the volume being measured.

§ 4. **Absolute and Arbitrary Units.**—In selecting our various units, we may do so quite arbitrarily, and

without connecting them together in any systematic way. Thus, our English unit of length is the yard; our unit of volume the gallon; our unit of mass the pound; and the connection between the yard, gallon and pound is not a simple one, but involves numerical factors not easily remembered. Moreover, in selecting units of other kinds, such as those of force, electric current and magnetic field, we are not obliged to choose them with reference to the already selected units of length, mass and time.

The progress of physical science has shown, however, that an unconnected and arbitrary selection of physical units is a great hindrance to the advance of knowledge, and that considerable advantages ensue from the construction of a system of units in which all are related in a simple and logical manner to certain fundamental units of length, mass and time.

A system of physical units of this latter kind is called an *absolute system*, as contrasted wlth an arbitrary and disconnected system of units of measurement.

In the construction of an absolute system of units, the first question which arises is the selection of *Fundamental Units*, to which all the other units are related.

§ 5. **Fundamental Units.**—It is necessary to consider briefly the nature of the fundamental units. It has been found convenient to select, as the starting point, a unit of space called the *Unit of Length*, a unit of duration called the *Unit of Time*, and a unit of matter called the *Unit of Mass*. There is no difficulty in understanding the meaning of the first two terms. A straight line is the shortest or least distance between two points. Two marks may be made on the surface of a body which is sufficiently permanent, and the shortest distance between these points taken as the unit of length. As a practical matter, most civilised nations possess certain bars of very unchangeable metal, such as platinum. On these are made two marks, and the unit of length is defined by law to be the shortest distance between these marks.

With regard to time, the most universally important event is the completion of one rotation of the earth round its polar axis. The duration of this event is called a sidereal day. Most nations take as the unit of time some known fraction of this duration.

The term *Mass*, and the idea connected with it, is more difficult to define; in fact it cannot be defined without reference to the notion of *Force*. The fundamental property of matter is that it cannot set itself in motion or change its own state of rest or motion.

We know perfectly well that substances or bodies can, however, be made to change their position with reference to certain datum points or lines. Thus, for instance, a body can move forward in such a way that an assigned line in the body preserves a constant direction in space: in this case it is said to move without rotation; or it may move so that the body rotates round an axis or some other point: in this case it is said to move with rotation. Whenever material substances are thus set in motion from a state of rest—that is, when their space-relations are changing with time—this *motion* must have been originally due to the action of mechanical force, or simply to *Force*. We recognise force, therefore, as the cause, whatever its nature, which creates changes in a body's state of rest or motion; and we may define *equal forces* as those which, if successively but independently applied to the same substance would produce the same or like changes of motion.

The term *Dynamics* is used to denote that part of natural science which is concerned with the study of the laws of mechanical force.

There are many ways in which we can classify material substances. A chemist, for instance, classifies bodies according to the chemical changes they can produce, and a biologist according to their condition as regards life. *In dynamics we consider substances solely under the aspect of the changes of motion produced in them by Force.* However different a piece of iron may be from a piece of bread, yet, nevertheless, it is found pos-

sible to select certain sized pieces of iron and bread, such that whilst different in every other particular which can be named, they are yet alike in one respect—that is, they are *dynamically identical*, and when acted upon by the same or equal forces for the same time, so that they are set in motion without rotation, will under the action of these equal forces acquire the same velocity. These pieces of iron and bread are then said to have the same *mass*.*

All civilised nations select some piece or lump of very unchangeable substance, such as a block of platinum, which they define by law as their *Unit of Mass*, and with which they compare the mass of other bodies.

§ 6. **The Metric System.**—In selecting the fundamental unit of space, viz. that of length, it has been considered that it would be an advantage to have this standard of length connected with some natural unchangeable length, such as that of a simple pendulum beating seconds, or the length of the earth's polar axis, or the length of a meridional arc of the earth's surface. It was this last dimension which was selected at the end of the eighteenth century as the basis of the metric system of weights and measures as first established in France. The founders of this system took as their unit of length, one ten-millionth part of the length of a quadrant of a meridian of the earth's surface as then known, and called this length a *metre*. Subsequent progress in geodesy showed, however, that the measurement of the earth's meridional quadrant completed in 1799 by Delambre and Mechain was in error by defect by about 1 part in 4000; and that, if the subsequent deductions of Bessel and Airy be correct, the metre as settled by the French Government in 1801 is less than one ten-millionth part of the earth's meridional quadrant. For

* To define *mass* as the *quantity of matter* in a body—a definition sometimes met with in mechanical text-books—is merely a play upon words, and simply confuses the student. The notion of *mass* as a physical concept can only be based on that of force. The student is earnestly recommended to procure, and read, the little treatise on 'Matter and Motion,' by Clerk Maxwell, in which he will find the fundamental notions of dynamics set out by the hand of a master.

all practical purposes it might just as well have been any convenient arbitrary length. The value of the metrical system does not rest in the natural magnitude of the unit of length, but in the simple relations of the various units derived from it to one another.*

On the 2nd November, 1801, the French Government defined the unit of length to be the shortest distance between two marks on a platinum bar constructed by Borda, this measurement to be made with the bar at 0° C. The bar at present recognised as the standard (1897) is preserved at Sèvres, near Paris, and is called the *Mètre International.* It is guarded with jealous care, and only used very occasionally for comparison with other standards. The length so defined is the legal metre or unit of length, and is equal to 39·37011 English standard inches, or 1·09361426 British standard yards.

To obtain larger or smaller units of length, multiples by 10, 100 or 1000, or fractions of ·1, ·01 or ·001 of the metre are taken, and these are denoted by prefixing to the word *metre* a Greek or Latin numerical term. The student should note that—

The prefix	*mega-*	denotes a	multiple of	1,000,000	times.
"	*myria-*	"	"	10,000	"
"	*kilo-*	"	"	1,000	"
"	*hecto-*	"	"	100	"
"	*deka-*	"	"	10	"
"	*deci-*	"	fraction of	$\frac{1}{10}$	part.
"	*centi-*	"	"	$\frac{1}{100}$	"
"	*milli-*	"	"	$\frac{1}{1000}$	"
"	*micro-*	"	"	$\frac{1}{1000000}$	"

* The Standards Act of 1855 declared that the researches of scientific men have thrown doubts on the accuracy of the method of reference to constants in nature. The standard metre is therefore no longer the theoretical metre or one ten-millionth of the elliptic quadrant of the meridian through Paris, but is simply the length of the bar of platinum-iridium called the "Mètre International," deposited with the International Committee of Weights and Measures at Paris. (See Mr. H. T. Chaney, 'Our Weights and Measures.')

Thus, a kilometre means a thousand metres, a centimetre means one-hundredth part of a metre, and a micrometre means one-millionth of a metre. The same prefixes are employed in the case of all other units. An ampere is (as will be explained later) the practical unit of electric current, and hence a milli-ampere is the one-thousandth part of an ampere, and a deka-ampere is ten amperes.

Since one metre is equal to 39·37011 English inches or to 3·2808 feet, it follows that one centimetre is equal to ·3937 of an inch. Hence we arrive at the following useful rules.*

To convert inches to centimetres, multiply by 2·54.

To convert feet to centimetres, multiply by 30·48.

To convert centimetres to inches, multiply by ·3937.

To convert centimetres to feet, divide by 30·48.

The following are the roughly approximate equivalents of some of these metrical units in British measure.

One kilometre is nearly $\frac{5}{8}$ of a British mile.

One metre is nearly 3 feet 3 inches and $\frac{3}{8}$ of an inch.

One inch is very roughly $2\frac{1}{2}$ centimetres, or more nearly $25\frac{1}{2}$ millimetres.

One foot is nearly $30\frac{1}{2}$ centimetres.

Collecting the results, we have the following frequently required equivalents :—

1 metre . .	= 39·37011	inches	= 3·2809	feet.
1 centimetre ·	= ·3937	inch	= ·0328	foot.
1 English foot	= ·3048	metre	= 30·4797	centimetres.
1 English inch	= ·0254	"	= 2·5399	"

Usual abbreviations are, m. for metre ; cm. for centimetre ; sq. m. for square metre ; sq. cm. for square centimetre ; μ for micrometre.

* The student should provide himself with a pocket rule having one side or edge divided into inches and fractions, and the other into centimetres and millimetres.

In expressing very large numbers, it is convenient to do so by writing them as multiples of powers of 10; thus 10^2 stands for 100, 10^3 for 1000, and 10^6 for 1,000,000. 3×10^9 then stands for 3,000,000,000. An earth quadrant is very nearly 10^9 centimetres. The velocity of light is 3×10^{10} centimetres per second.

In expressing fractions we can in the same way express them decimally. Thus, 10^{-1} means one-tenth, 10^{-2} means one-hundredth, 3×10^{-3} means three one-thousandths, and so on.

From the unit of length is derived the unit of bulk or capacity. If a cube is constructed the side of which is one decimetre, the cubic capacity of this space is called one *litre* or cubic decimetre. Multiples and fractions of the litre are expressed by the employment of the above mentioned prefixes.

A cubic centimetre (c.c.) is then one-thousandth part of a litre, or is a millilitre. The cubic centimetre is the usual practical unit of volume for measurements made on the C.G.S. system.

It is useful to know that 1 cubic inch is 16·387 cubic centimetres, and that

To convert cubic feet to cubic centimetres, multiply by 28316·77.

To convert gallons to cubic centimetres, multiply by 4546.

To convert cubic inches to cubic centimetres, multiply by 16·387.

Frequently required equivalents are :—

1 square inch	=	6·4516 square centimetres.
1 cubic inch	=	16·387 cubic centimetres.
1 cubic foot	=	28,317 " "
1 gallon .	. =	4,546 " "

Usual abbreviations are, cub. m. for cubic metre, cub. cm. or c.c. for cubic centimetre.

The unit of mass in the metric system is defined to be the mass of a certain piece of platinum-iridium kept at

Sèvres, near Paris, and called the "*Kilogramme International.*" The original standard was made by Borda, and was intended to be equal in mass to the mass of a litre of pure water taken at its temperature of maximum density, namely, at 4° C. This is not precisely the case, but it is near enough for practical purposes. The mass of a cubic decimetre, or litre, of pure water at 4° C. is now known to be 999·840 grammes. Hence the mass of one cubic centimetre of pure water at 4° C. is very nearly one gramme.

The kilogramme is equal to 2·204 British standard pounds avoirdupois.

It is convenient to have the following equivalents at hand :—

1 British standard pound avoirdupois	=	453·59 grammes.
1 ounce avoirdupois	=	28·3495 „
1 kilogramme	=	2·204 lbs. nearly.

Usual abbreviations are, grm. for gramme; kgr. for kilogramme.

The advantage of establishing this simple relation between the unit of volume and the unit of mass by means of pure water, is that the mass of any volume of water can be at once approximately deduced, and a knowledge of the density of any material gives us at once the means of knowing the mass of any volume of it.

§ 7. **The C.G.S. System.**—In the absolute system of units which is employed in scientific measurements, the fundamental unit of length is the *Centimetre*, and that of mass the *Gramme*. This is completed by taking as the unit of time the *mean solar second*, and this system is called the C.G.S. system (centimetre, gramme, second) of measurement.

The time during which the earth makes one complete revolution on its axis is called a *sidereal day*. The interval between the two successive passages of the centre of the sun across the meridian of any place is

called a *solar day*. The duration of solar days is not the same, but by taking the mean length of all the solar days in the year, we obtain the *mean solar day*.

A pendulum which makes in one mean solar day 86,400 swings (from side to side, and not backwards *and* forwards) beats mean solar seconds. The sidereal day is 86164·09 mean solar seconds. The mean solar second is the generally accepted scientific unit of time.

Hence we have the following fundamental units of space, mass and time.

1 centimetre . .	30·4797 cm.	=	1 British foot.
1 gramme . . .	453·59 grm.	=	1 lb. avoirdupois.
1 mean solar second	86,400 sec.	=	1 mean solar day.

§ 8. **Derived Units.**—From these fundamental units of length, mass and time, we can pass on to consider certain derived units, such as those of *velocity*, *acceleration*, *momentum*, *force*, *work* and *power*.

If any small portion of matter, which may be called a material particle, is in motion along any path, it passes over a certain space in a certain time. If the length of path described in a very short interval of time is measured, and if the time of describing this path is also observed, the quotient obtained by dividing the numerical value of the length described by the time of describing it is a measure of the mean *linear velocity* of the body during that interval. The unit of linear velocity is that of a body which passes over one centimetre per second when moving uniformly.

If a solid body rotates round any axis, and if the time taken for it to rotate through a very small angle is observed, and if the angular displacement of the body in that time is measured in *radians*, the quotient obtained by dividing the numerical value of the angle described by the time of describing it is called the mean *angular velocity* of that body round that axis at that instant. The *radian* is an angle such that its *arc* is equal to the radius, and is equal to 57·29°.

The unit of angular velocity is that of a rotating body which passes through one radian per second described uniformly.

Velocity may otherwise be defined as rate of change of position.

If the change of position of a very small material particle is a change of place along any path, this change may be a uniform change, such that equal spaces are passed through by the particle in equal times. In such cases the velocity is called a uniform velocity. If the velocity is not uniform, then it has different values at successive instants, and it can be defined at any moment by stating the space in centimetres which the body would pass over in one second if it continued to move uniformly in a straight line with the velocity it had at the instant considered.

If the velocity of a body is not uniform, then it must be changing either in magnitude or in direction. The rate at which velocity is changing at any instant is called the *acceleration* of the body, and acceleration means a change in velocity either in amount or in direction. If a material particle is in motion in a straight line, and if the velocity is measured at two instants of time, separated by a very small interval, then the value of the increment or change in the velocity, divided by the numerical value of the time interval in which this change is effected, is a measure of the mean *acceleration* of the body during that instant. The acceleration may otherwise be defined as the rate of change of velocity, and the unit of acceleration is a velocity of one centimetre per second added per second.

Acceleration is, therefore, related to velocity just as velocity is to path-distance or path-direction measured from a certain datum point or line.

The *momentum* of a body moving without rotation is measured by the product of its linear velocity and its mass. If the body is rotating round an axis, the product of its angular velocity and its *Moment of inertia*

round that axis is called its *Angular Momentum.* The moment of inertia with respect to any axis is obtained by supposing the whole mass of the body divided into very small elements of mass, multiplying the mass of each of these elements by the square of its distance from the axis of rotation, and then taking the sum of the values of all these products.

Momentum, like velocity, may be either uniform or changing in magnitude or direction.

It is the fundamental property of matter that it cannot change its own momentum. If left to itself momentum continues unchanged, that is, the body moves on without change in the magnitude or direction of its motion. This is called the *First Law of Motion.* If momentum is changing either in direction or in magnitude, this change is said to be due to impressed force; and *Force* is defined as the cause, whatever its nature, which creates a change in momentum in a body. Force is measured by the *rate at which it changes momentum in the direction in which it acts.* Hence the unit of force is that which alters the momentum of a body at the rate of one unit of momentum per second in its own direction. Suppose a small particle having a mass of one gramme is moving uniformly with a velocity of one centimetre per second. This body has a unit of momentum. If a force acts on it in such manner as to change its velocity to a uniform velocity of two centimetres per second in the same direction, the momentum has been increased to two units. If this change in momentum takes places in one second, then the force causing the change has a magnitude of one unit in the direction in which the body is moving. This unit of force in the C.G.S. System has received a name. It is called *One Dyne.*

Consider, for instance, a body falling from rest freely under the action of gravity. If a mass of one gramme is allowed to fall, at the end of one second it will be found to be moving with a velocity of 981 centimetres

per second in the direction of the earth's centre. At the end of two seconds it will be found to be moving with a velocity of 1962 centimetres per second in the same direction. It has, therefore, had its momentum increased by 981 units in one second. Hence the force due to gravity acting on the gramme mass is 981 dynes. It is an experimental fact that the acceleration of all bodies, large or small, or of whatever substance they may consist is the same *in vacuo*. Hence it follows that the force due to gravity acting on any body is proportional to its mass. The process of weighing bodies on a balance consists in making a comparison of these bodies as regards equality in mass, by testing the equality or inequality of the forces on them due to gravity.

Change in the direction of momentum, as well as change in the magnitude of momentum, is due to force. Thus, if a body revolves uniformly round a centre like a planet round the sun, although its speed, as we generally call it, may remain the same, the direction of its velocity, and hence of its momentum, is continually being changed, hence this is due to force. Force, therefore, is measured by the rate at which it changes the momentum of a body, whether that change is one in numerical magnitude, or in direction, or in both together.

When some external agent causes a body to move in a direction opposite to that in which another force would move it if allowed to operate freely, this agent is said to do *work against the force*. Thus, if a man lifts up a stone he moves it against the direction in which the force of gravity if allowed to act would move it, and he does *work* against the force of gravity. Work is measured by the product of the distance moved through, and the mean value of the force against which work is done, estimated in the direction of the movement. The unit of work is the work done in moving a body a distance of one centimetre against a force of one dyne acting in that direction. This unit of work may be called a dyne-centimetre, but is generally called

One Erg. An erg is a very small amount of work, hence a multiple of it is taken as the usual practical unit of work, viz. 10^7 ergs or 10 million ergs, and this is called *one Joule.* 3,600,000 joules are called one *Board of Trade unit of work* or one B.T.U. The Board of Trade unit of work is, therefore, equal to 36 billion ergs.

When any agent is working against a force, the most important question is generally to determine *the rate at which it works.* This rate of working is called its *Activity* or *Power.*

The C.G.S. unit of *power* is an activity of one erg done per second. The work done in lifting a mass of one gramme one centimetre against gravity, is 981 ergs. The practical unit of activity or power is equal to 10^7 or 10 million ergs done per second, and this is called a Power of *one Watt.*

One kilowatt is 1000 watts, and a kilowatt is nearly equal to 1·3 horse-power.

One horse-power is equal to 33,000 foot-lbs. per minute, or to the power required to lift 33,000 lbs. 1 foot high in 1 minute against the force of gravity.

If a body is at rest or moving uniformly, and we act upon it by any force so as to change its velocity, we are said to do work against the force of inertia, and the work so done is measured by the increase produced in the value of the kinetic energy of the body. This kinetic energy is measured at any instant by the product of the mass of the body and half the square of its velocity.

The usual British unit of work is the *foot-pound*, and it is defined as the work done in lifting one lb. one foot high against the force of gravity. As, however, the force of gravity is different at different places on the earth's surface, being greater by about one-half per cent. at the poles than at the equator, the foot-pound is not an absolutely defined unit unless some place is named at which the pound is lifted.

We have, however, the following practical equivalents:—

1 joule = 10^7 ergs = ·7373 foot-lb.
1 watt = 1 joule per second = 10^7 ergs per second.
1 kilowatt = 1·3 horse-power nearly.
746 watts = 1 horse-power = 550 foot-lbs. per second.

The energy or work required to heat 1 gramme of water 1 degree C. in temperature in the neighbourhood of 10° C. has been determined to be 42 million ergs, or 4·2 joules or 3·096 foot-lbs. This amount of heat is called 1 gramme-centigrade degree unit of heat, and the value 4·2 joules is called the *Mechanical Equivalent of Heat.* An amount or quantity of energy *of any kind* is measured in ergs or joules.

The student will notice that in this case most of these derived units, viz. in the case of velocity, acceleration, force and activity, the measure of them is *a rate* at which some other quantity is changed. Velocity is the time-rate of change of position. Acceleration is the time-rate of change of velocity. Force is the time-rate of change of momentum. Activity or power is the time-rate of change of work or energy.

There are some other terms which it will be convenient to define at this point. When any body is acted upon by two equal and parallel forces not acting in the same straight line, these constitute what is known as a *couple* or *torque*, and the mechanical value of the couple or torque in producing *twist* or *rotation* is measured by the product of the value of either force and the vertical distance between the forces. The effect of a couple acting on a body is to cause rotation or *twist* round an axis. The unit couple is that due to two forces each of one dyne acting at a distance of one centimetre and causing twist round an axis perpendicular to their own direction.

In many problems in physics we have to deal with the case of a body rotating or swinging round an axis,

such as a fly-wheel rotating or a pendulum swinging. An important quantity which then presents itself is that of the *Moment of Inertia* of the body. As above stated, this quantity is calculated by assuming the body to be divided into equal and very small parts, taking the product of the mass of each part and the square of its vertical distance from the axis of rotation, and adding together all these products. The moment of inertia and the angular velocity of a rotating body enter into such rotational problems just as the mass and linear velocity do into the corresponding problems in the movement of a particle of matter. Thus the product of the moment of inertia and the angular velocity is called the *Angular Momentum.* The product of the moment of inertia and half the square of the angular velocity is called the *Angular or Rotational Energy*, and the rate of change of angular momentum is the measure of the *torque* or *couple* causing rotation. The dynamical measure of the torque or couple is the rate at which it changes the angular momentum of the system estimated round the axis of the torque.

The moment of a force with respect to any axis perpendicular to its direction is measured by the product of the numerical value of force and the vertical distance between the axis and the line representing the force.

In any system of bodies rotating round a centre, such as the planets and the sun round their common centre of mass, no interaction between the different bodies can change the total angular momentum of the system. This last can only be altered by some torque acting on the system from without. This fact is one of fundamental importance. It is called the principle of the *Conservation of Angular Momentum*, or of the Moment of Momentum. It is in reality only the fundamental fact concerning matter, viz. that it cannot change its own state of rest or motion, applied to the particular case of rotation of a number of masses.

TABLE OF FUNDAMENTAL UNITS.

	C.G.S. System.	Practical System.	British or Foot-pound-second System.	Relations.
Unit of length ..	One centimetre	.. ..	One foot	One foot = 30·48 centimetres.
Unit of time	One mean solar second ..	.. ..	One mean solar second.	
Unit of mass ..	One gramme	.. ..	One pound	One pound = 453·59 grms.
Unit of velocity ..	One centimetre per sec.	.. ..	One foot per second.	
Unit of acceleration	A velocity of one centimetre per second added per second.	.. ..	A velocity of 1 foot per second added per second	Acceleration of gravity on C.G.S. system = 981. On foot-pound-second system = 32·2.
Unit of force ..	One dyne	One kilodyne $= 10^3$ dynes.	One poundal	13825·42 dynes = one poundal. "Weight of one pound" = 453·59 × 981 dynes = 445,000 dynes nearly.
Unit of work or energy.	One erg	One joule $= 10^7$ ergs.	One foot-pound	One joule = ·737 foot-pound. One horse-power-hour = 2,685,600 joules = 1,980,000 foot-pounds.
Unit of power or activity.	One erg per second ..	One watt $= 10^7$ ergs per second.	One foot-pound per sec. or one horse-power = 550 foot-pounds per sec. = 33,000 foot-pounds per min.	One horse-power = 746 watts.
Unit of heat	One calorie = one grm. of water raised one deg. Centigrade in temperature at about 10° C	One calorie = 4·2 joules or 42×10^6 ergs or 42 million ergs.	One pound of water raised one degree Fahrenheit in temperature.	779 foot-pounds = one pound-degree-Fahrenheit unit of heat.
Board of Trade unit of electrical energy.	$= 36 \times 10^{12}$ ergs = 36 billion ergs.	One kilowatt-hour or 1000 watt-hours.		

CHAPTER III.

MAGNETIC FORCE AND MAGNETIC FLUX.

§ 1. **A Unit Magnetic Pole.**—Let a long thin steel wire or knitting needle be uniformly magnetised, and then broken in the middle. At the point of rupture it will, as already described, develope new, equally strong north and south magnetic poles, which would, if held near to each other, mutually attract.

Imagine these poles placed one centimetre apart, and that the pull or attractive force between the poles is measured by a very delicate spring balance. Furthermore, suppose that this force proves to be just one dyne or a dynamical unit of force, as defined in the previous chapter. Then these two equal magnetic poles, which attract each other with a force of one dyne when placed one centimetre apart, are called *Unit Magnetic Poles*. The *strength of a magnetic pole* is numerically measured by the number of unit magnetic poles to which that particular pole considered is magnetically equivalent. Thus any given magnet may have a pole of strength equal, say, to 10 units; and this would denote that in all respects the given pole was equivalent to ten unit magnetic poles placed together; assuming that these did not in any way affect each other in power by being put together.

§ 2. **Moment of a Magnet.**—By the phrase *moment of a magnet* is meant the value of the product of the strength of either pole of the magnet and the shortest distance between the poles. Thus, consider a strongly magnetised steel wire. Let it be 10 centimetres long, and let it have pole strength represented by 10; that is,

let each pole have a strength equal to ten unit poles. Then the product of the pole strength and the shortest distance between the poles is 10 times 10 = 100, and the number 100 represents therefore the moment of that magnet. In general it is not possible to fix the exact position of the poles of a magnet, because they do not reside at definite points and are not placed exactly at the extremities of the bar. In the case of straight bar magnets it has been found that the distance between the poles may be considered to be approximately equal to five-sixths or to ·82 of the whole length of the magnet. This distance is called the *Magnetic Length* of the magnet. The determination of the true magnetic lengths of magnets is always subject to some uncertainty. It is possible, however, by certain known methods, to measure the *moment of a magnet* as a whole without knowing precisely the value of the two separate factors, pole strength and pole distance, *or* magnetic length, the product of which is the value of the magnetic moment of the magnet. A magnet is said to have a unit magnetic moment when its moment is equal to that of a small magnetic needle having unit poles, and the shortest distance between those poles being one centimetre.

§ 3. **Magnetic Fields and Magnetic Couples.**—It has already been explained that the region round about a magnet within which its influence is felt is called a *Magnetic Field.* If a small exploring magnet, like a compass needle, is held anywhere in a magnetic field, the direction in which this needle sets itself is called th *Direction of the Field* at that point.

Next suppose a very long magnetised steel wire having at each end a pole of unit strength, is held with its north pole in a magnetic field, the other pole being so far removed that the field has no influence on it, the tendency of this north pole is to move in a certain direction under the action of the field, and that direction is called the *Positive Direction along the Field* at that point. Again, if the unit pole were held at that point it would

require a certain mechanical force applied to it to prevent it moving in the direction of the field, and this force, measured in dynes, is the measure of the *strength of the field* at that point.

If, therefore, the force on a pole of strength 10 units is 100 dynes, the strength of the field is 10 units, and generally speaking the product of the pole strength and the field strength is a measure of the force in dynes required to hold the pole at that point in the field. The student must notice that neither the pole strength nor the field strength alone are measured in dynes, because they are not in themselves of the nature of mechanical forces, but their *product* is a quantity which is of the nature of a mechanical force, and is therefore measured in dynes as its appropriate unit. The field strength at any point is also commonly called *the Magnetic Force* at that point.

In the next place, suppose a small magnet suspended in any position in a uniform magnetic field; since the two poles of a magnet are always of equal strength, the forces acting on the poles of the magnet are equal and parallel, but opposite in direction. A pair of equal parallel and oppositely directed forces acting on a body, but not acting in the same straight line, constitutes what is called in mechanics a *Torque* or *Couple*, and hence the effect of a magnetic fi·ld on a small magnet is to produce a couple, tending to turn it round its centre. There is no tendency to pull the magnet bodily along in any direction. This can be proved by floating a small magnet on a cork on water, and holding a large magnet some little way from it. It will be found that the floating magnet turns round, as on a pivot, until it sets in one direction, but it does not move bodily towards the larger magnet.

Hence the action of a uniform magnetic field on a small magnet is a couple, and is called a *Magnetic Couple*. The action of the earth, as a great magnet, is to cause a couple to act in every suspended magnet or compass

needle, and this is called the *Terrestrial Magnetic Couple.* If the magnet is held with the line joining its poles at right angles to the direction of the field, the mechanical couple or torque acting on the magnet is numerically equal to the product of the strength of the field and the moment of the magnet. The magnetic moment of a magnet may therefore be measured by the torque or couple required to hold it in a unit-field with its axis at right angles to the direction of the field.

§ 4. **Lines of Magnetic Force.**—A very small compass needle or magnet having a unit magnetic moment can, therefore, be used to map out a magnetic field in the following manner. Let the little needle be placed at any point in the field; its direction at that spot indicates the direction of the field. Let it then be carried forward by little steps, always moving in the direction of its own length; the centre of the needle will move along a curve which is called a *Line of Magnetic Force.* If this experiment is tried with a small needle and bar magnet, it will be found that the line of magnetic force starts from some point on one half of the magnet, and extends through the surrounding space in the form of a curved line. We may even use for this purpose a small non-magnetised steel needle, and it will still set itself in the same manner in the field of the larger magnet, because it becomes immediately magnetised permanently under the influence of the magnetic force of the field. Taking advantage of this fact, the lines of magnetic force may be delineated in the following useful manner much used by Faraday.

Lay a bar magnet underneath a sheet of stiff cartridge paper, or very thin cardboard, which is perfectly flat. By means of a muslin sieve sprinkle *steel* filings lightly but uniformly all over the card. Tap the card very gently at one corner, and it will be seen that the filings have arranged themselves in a series of curved lines, branching out from the two polar regions of the magnet. (See Fig. 11.) The lines are easily fixed as follows.

Purchase from any mathematical instrument maker some of the photographic paper known as ferroprussiate paper, which is used by engineers' draughtsmen for making *blue print* copies of their tracings. This paper is photographically sensitive, and if exposed to light for some time and then washed with water, turns blue where it has been so exposed. If, however, any opaque object is placed on the paper, and shields it at that point from

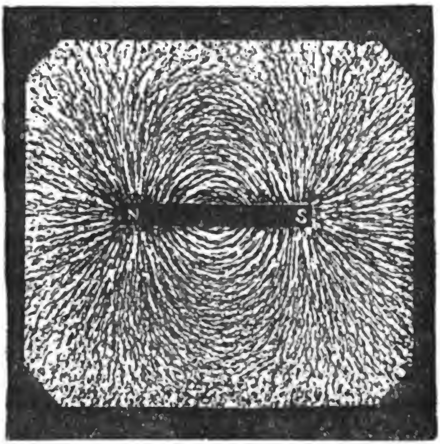

Fig. 11.—Lines of Magnetic Flux round a Bar Magnet delineated by Steel Filings.

the light, then the paper when washed appears white at that spot. If then a sheet of this paper is laid on the card, and held flat by clips or pins, we can secure in the following way a permanent record of the form of these magnetic lines of force.

In a partly dark room place the sheet of prepared paper on the card, and laying a magnet or magnets on a

board, place over them the card and paper, the sensitive side of the paper being uppermost. Then sprinkle the steel filings and obtain the curves. Carry the board carefully into the light, and let it remain in the sunshine for ten or fifteen minutes. At the end of that time, throw off the steel filings, and wash the paper with water ; this will develope and fix the curves. The photograph when dry can be mounted on cloth. The student will find it to be a most instructive and useful exercise to prepare in this way a series of permanent photographs on blue paper, showing the forms which the steel filings assume when sprinkled over magnets of various forms, such as bar and horseshoe magnets arranged in various ways. The formation of these curves may also be shown to a class or large audience by means of a vertical lantern. For this purpose small magnets, bar or horseshoe, are mounted between two thin glass plates, $3\frac{1}{4}$ inches square, and the plates may conveniently be held in a wood or metal frame. This is then placed on the stage of a vertical projector attached to an optical lantern, and an image of the magnets focussed upon the screen.

By using a very small sieve of muslin, steel filings may then be peppered over the plate, and on gently tapping the plate, will arrange themselves in the magnetic curves.*

In order to understand the reason why filings settle themselves in these curves, it is necessary to consider the action of a large magnet N S upon a small one *n s*, held with its centre at a point P in the field of the large magnet N S. Let a bar magnet N S (see Fig. 12) have a small exploring needle *n s* held somewhere in its field in the same plane. Then between the poles N *s* and S *n* there is an attractive force, and between N *n* and S *s* a repulsive force. Hence there are four forces acting

* A very good form of teacher's lantern for this purpose is made by Messrs. Newton, opticians, of Fleet Street, London, and also by Mr. Chadwick, of Deansgate, Manchester, for use with either oil-lamp, lime light, or electric arc as illuminant.

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upon the poles *n* and *s* of the small needle in the direction represented by the arrow-headed lines in Fig. 12.

When two forces act together upon a point, it is shown in treatises on mechanics that their joint effect is equivalent to a single force, called their *resultant*.

A force has always *direction* as well as *magnitude*, and hence can be represented by a straight line so drawn that its direction corresponds to that of the force, and the length of the line is to some suitable scale proportional to the magnitude of the force. If two forces acting on a point are represented by two lines, and if we complete

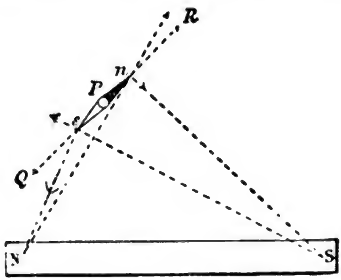

Fig. 12.—Forces acting on a Small Magnet when placed in the field of a larger one.

the parallelogram formed on these two lines as adjacent sides, then the diagonal of that parallelogram will represent to the same scale the resultant of the two forces. Hence, compounding in this way the two pairs of forces acting on the poles of the small needle, we see that they are equal to two resultant forces—to *n* R, *s* Q—which will be very nearly equal to one another if the needle *n s* is small.

These two forces constitute a *couple*, and the small needle *n s* will not be in equilibrium or at rest unless the two resultant forces *n* R *s* Q, and the needle *n s*, are in the same straight line. Hence the action of the resultant

forces is to pull the little needle round until it sets in the same direction as the resultant forces, and it indicates, therefore, by its direction the direction of the resultant magnetic force R Q at the point P in the field. Imagine, therefore, the large magnet surrounded by an immense number of very small pivoted magnetic needles. Each one of these will set itself so as to indicate the direction of the magnetic force due to the large magnet at the place where that particular small needle is situated. If instead of using small magnetic needles we sprinkle steel filings on paper laid over a magnet, each little particle of steel, which resembles in shape a minute steel compass

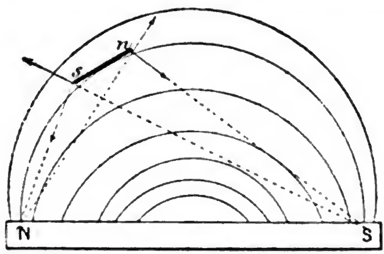

Fig. 13.—Small Exploring Needle N S, placing itself Tangent to the Lines of Magnetic Flux of a Large Magnet N S.

needle, becomes magnetised by being placed in the field. When we tap the paper, the little filings jump up into the air, and turn round in the air so as to set their magnetic axes or greatest length in the direction of the magnetic force at that point, due to the large magnet. Accordingly the whole collection of filings acts just as would hundreds of little compass needles distributed around the large magnet, and they delineate by their combined action the direction of the magnetic force at all points in the field.

The student will find it to be instructive to take the small exploring needle, or a little pocket compass, and having formed the magnetic curves with steel filings on

paper laid over a bar magnet, to place the small compass needle at various places in the field. (See Fig. 13.) He will see that wherever it may be placed the small compass needle always stands *tangent* to, or along the direction of, the lines delineated by the steel filings. Hence these steel filings show us at a glance the direction of the magnetic force at all points in the field, and we see that magnetic force is distributed in curved lines, and that, broadly speaking, these lines emanate from the north half of the magnet, and pass through surrounding space to the south half.

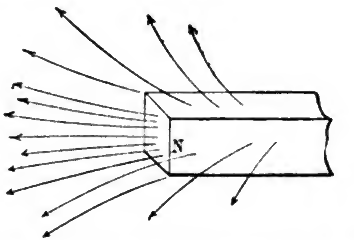

Fig. 14.—Lines of Magnetic Flux proceeding from a North Magnetic Pole.

The reader must think of these lines as proceeding *out* of the north magnetic pole, and returning *into* the south magnetic pole, not only in one plane, but in all directions (see Fig. 14), like the branches of a palm springing from the stem; and he must furthermore think of this system of magnetic force as moving with the magnet and carried about with it wherever it goes.

The *positive direction of a line of magnetic force* is the direction *from* the north pole *to* the south pole through surrounding space. This, however, is merely a convention or arrangement, and there is nothing except the convention to fix which pole should be selected as the one *from* or *to* which we reckon.

A north magnetic pole, if placed anywhere on a line

of magnetie force, tends, if free, to move along that line in the positive direction. A south magnetic pole tends to move in the negative or opposite direction. Hence a small compass needle suspended in a field sets itself so that its north pole is as far as possible in the positive direction of the field, and its south pole as far as possible in the negative direction.

§ 5. **Magnetic Flux.**—So far we have considered the facts of magnetic attractions and repulsions as if they were due to mechanical forces acting at a distance between the magnetic poles of magnets. At this point we have to enlarge our conceptions by considering another magnetic measurable quantity.

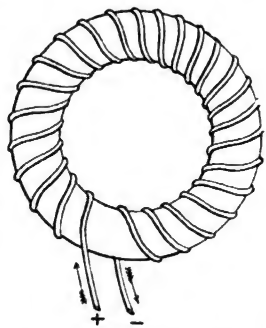

Fig. 15.—A Ring or Poleless Electromagnet.

We have already described the construction of a straight solenoid or magnetising coil, and shown how by a coil of wire traversed by a current an iron bar can be magnetised. Let the student now procure an iron ring like an anchor ring (see Fig. 15), of circular form and section, and wind it over closely and uniformly with covered copper wire. The ring may conveniently be six inches or 15 centimetres in mean diameter, and one centimetre in cross sectional diameter. The covering wire may be one layer of No. 16 cotton-covered copper wire. Such an endless coil of wire is called a *Circular Solenoid*. If an electric current from a battery is sent through this wire it causes it to exert a *magnetic* or *magnetising force* upon the iron, and the direction of this magnetising force is everywhere along the direction of the circular axis of the ring. This magnetic force magnetises the iron, and yet since the iron ring is endless, if it is tested

with a compass needle it will be found that there are no magnetic poles, and hence it has no magnetic moment, and no external magnetic effect. There is no question, however, that the ring *is* in a magnetic state, and is magnetised, because if this experiment is performed with a steel ring, which can afterwards be cut open and bent out into a bar, it will be found to be a powerful bar magnet, with a north pole at one end, and a south pole at the other, no matter where it is cut.

Similarly, if the ring is cut in two places, two bar magnets can be made out of the steel ring so treated by dividing it into two parts.

We cannot describe the condition of the iron ring therefore, when inside the endless magnetising solenoid, by stating its magnetic moment, because it has no free poles, and therefore no moment. We can, however, express its magnetic state by saying that the iron is traversed by *magnetic flux* or, as it is sometimes called, *magnetic induction.* The student may assist his conceptions by thinking of this magnetic flux as the physical state produced in the iron by the magnetic force of the solenoid, just as a flux or a flow of liquid is a physical state produced in a liquid in a pipe or channel by a hydrostatic force or pressure, or a flow of heat in a conductor is a physical state produced by a difference of temperature.

The terms *force* and *flux* are used in this connection to signify respectively a certain kind of physical cause and its effect, and the *magnetic flux* is thus considered to be the result or effect due to the action of *magnetic force* on a magnetisable body.

Magnetic flux belongs to that class of physical effects which can only take place in complete circuits. The path in which magnetic flux takes place is called a *magnetic circuit.* The direction of the flux is considered as marked out by endless lines called *Flux Lines.* We cannot have a terminated or finite magnetic flux. Its path always must form a closed loop or ring, or endless path.

Consider for instance an incompressible fluid like water, existing in an endless pipe or self-closed channel. If we try to set it in motion, it cannot flow at one place unless it *circulates*, or moves everywhere at the same time in the whole channel. The characteristic of this kind of motion is that if we select two points on the self-closed channel and note the quantity of fluid which passes each section in a given time we find these quantities to be identical. The same kind of statement holds good with regard to electric currents. Generally speaking, those physical effects of which the above fact is true are called *circuital quantities*. Hence we must think of magnetic flux as a physical state produced in a magnetisable body all along certain self-closed lines or paths which constitute what is called the magnetic circuit. This magnetic flux is due to the operation of magnetic force existing at all points in the circuit.

Magnetic circuits may be of two kinds. They may be homogeneous, or consisting of material of the same nature everywhere. Thus magnetic flux may take place all round and in an iron ring, in lines parallel to the circular axis of the ring, and such a circuit is called a closed iron magnetic circuit.

The circuit may also consist of air, wood or brass, of uniform material of identical or nearly identical magnetic qualities. On the other hand, the magnetic circuit may be heterogeneous, or not all of the same magnetic material. It may be, for instance, partly of iron or steel, and partly of air, and the path of the magnetic flux may be through a circuit which has very different magnetic qualities in different parts. Whatever may be the nature of the circuit, there can, however, be no difference in the total quantity of the magnetic flux across various sections of a magnetic circuit bounded on all sides by flux lines. The total flux across any one section of the *whole* circuit, bounded by what are called the flux lines, is the same as that across all other sections. In a magnetic circuit we have to consider therefore as important quantities,

the *length* of the circuit, the *cross section* of the circuit, and in addition the specific magnetic quality of the material of which the circuit is made at each point.

Although, through all sections of the magnetic circuit bounded by flux lines in which the magnetic flux is taking place, there must always be the same total flux, yet nevertheless at various parts of the circuit, owing to the different sectional areas, there may be a difference in the magnetic flux passing perpendicularly through a unit of area at these places. This is expressed by saying that the *magnetic flux density* may differ at various points on the magnetic circuit.

The student must therefore distinguish between the *total magnetic flux* through a section of a magnetic circuit, and the *magnetic flux density* or flux per square centimetre of normal section at any point in the circuit.

We shall explain later on how magnetic flux is practically measured; meanwhile we may here state that it is consistently reckoned in a unit called a *Weber*. Since, however, this unit is for most purposes too large, a smaller unit called a *Microweber*, which is the one-millionth part of a weber, is conveniently employed.

The term *magnetic induction* is used by some writers instead of *magnetic flux*, and hence the student should notice that the following phrases mean the same thing, viz.: *the magnetic flux; magnetic flux density; flux density; the magnetic induction; magnetic induction density; magnetic flux per square centimetre; magnetic induction per square centimetre;* and these are all equivalent terms employed by different writers to denote one and the same physical quantity. The terms most usually used are either, *the flux density*, *the magnetic flux*, or *the magnetic induction*, to describe the physical state in the magnetic circuit produced at any point by the magnetic force acting there, and flux density is best measured practically in *microwebers per square centimetre*. A phrase also sanctioned by custom to denote what we

shall hereafter call the *flux density* is, *the number of lines of force per square centimetre.*

Magnetic flux, like magnetic force, has direction as well as magnitude at any point where it exists, and we may mark out its path by *lines of magnetic flux or induction.* Thus the lines of magnetic flux in the iron ring above considered are concentric circles parallel to the circular axis of the ring.

§ 6. **Magnetic Reluctivity and Reluctance.**—Magnetic bodies differ very much in regard to the magnetic flux density which can be produced in them by any given magnetic force.

Consider for instance the two cases of wood and iron. Imagine a wooden and an iron ring of the same size wound over with the same number of turns of wire, forming a circular solenoid, and the same current sent through each wire. There would then be the same magnetising force in both cases acting on the iron and wooden magnetic circuits. The magnetic flux density, which is produced at any section of the ring, would, however, by no means be the same in the two cases. The flux density may be hundreds, or even thousands of times greater in the iron than in the wood. Wood, brass, air, any non-magnetic or very feebly magnetic bodies do not, however, differ much from one another in respect of the flux density produced in them by a given force. A term is wanted to express the nature or quality of the material of which the circuit is composed, considered in respect of the magnetic flux density which can be created in it by a given magnetising force, and this quality is called the *magnetic reluctivity* of the material.

As a material of comparison we take air as a standard, and call its reluctivity unity. Then the reluctivity of any material forming a magnetic circuit is expressed by stating the multiple or fraction which its reluctivity is of that of air.

If we multiply the number representing the value of the reluctivity of the material of any homogeneous and

uniform magnetic circuit by the length of the circuit, and divide this product by the numerical value of the cross section of the circuit, we obtain the *Reluctance* of that magnetic circuit. Hence the reluctivity of a material is the reluctance per cubic centimetre of it.

§ 7. **Magnetomotive Force.**—We have next to introduce another notion, viz. that of *magnetomotive force.* Consider again the iron ring above mentioned, magnetised in the circular solenoid. Measure the length of the circular axis or mean length of the ring. This is the *length of the magnetic circuit.* The magnetic force inside the solenoid or wire windings, supposing the iron removed, has everywhere the same value, and the value can be measured as described at the beginning of the chapter. It is otherwise called the interior field of the solenoid. This magnetic force is at all points directed along the circular axis of the ring. If we take the product of the numerical value of this magnetic force and that of the mean length of the magnetic circuit, we obtain a measure of the *magnetomotive force round the circuit.* In other words, the magnetising or magnetic force is the magnetomotive force per unit length or per centimetre of the magnetic circuit. The magnetic force may be called the *Slope* of the magnetomotive force. It is also called, more properly, the *magnetomotive intensity.*

If the magnetic force along any circuit is not everywhere of the same magnitude, or if it is not everywhere parallel to the direction of the circuit, we can still obtain the magnetomotive force round that circuit by taking the sum of a number of products, each of which is the length of a short part of the magnetic circuit mutiplied by the magnetic force at the centre of that short part of the circuit estimated in the direction of that element of length. Hence generally we have the rule, when the magnetic force at all points of a uniform homogeneous magnetic circuit is the same, and is directed everywhere along the circuit, then

Magnetomotive force = magnetic force × length of circuit ;

or,

Magnetomotive force per unit of length of the circuit = magnetic force or magnetising force.

There are six important quantities concerned in defining the magnetic condition of a magnetic circuit, and these are :—

1. The total magnetic flux.
2. Magnetic flux or flux density, or magnetic flux per square centimetre.
3. The magnetomotive force.
4. The magnetic force, or magnetomotive force per linear centimetre.
5. The reluctance of the circuit.
6. The reluctivity of the material.

In the case of a uniform and uniformly magnetised magnetic circuit the above quantities are related in the following way :—

$$\left.\begin{array}{c}\textit{The total magnetic flux} \times \\ \textit{the reluctance of the circuit}\end{array}\right\} = \left\{\begin{array}{l}\textit{The magnetomotive} \\ \textit{force acting round} \\ \textit{the circuit.}\end{array}\right.$$

$$\left.\begin{array}{c}\textit{The magnetic flux density} \times \\ \textit{the reluctivity}\end{array}\right\} = \left\{\begin{array}{l}\textit{The magnetic force} \\ \textit{acting on the cir-} \\ \textit{cuit at that point.}\end{array}\right.$$

$$\textit{The total magnetic flux} = \begin{array}{l}\textit{The magnetic flux} \\ \textit{density} \times \textit{the cross} \\ \textit{section of the mag-} \\ \textit{netic circuit.}\end{array}$$

The reluctivity of any material is the reluctance of one cubic centimetre of it when forming part of a magnetised homogeneous and uniform magnetic circuit, and otherwise we may say :

$$\left.\begin{array}{c}\textit{The reluctance of} \\ \textit{a magnetic circuit}\end{array}\right\} = \left\{\begin{array}{c}\textit{The reluctivity of} \\ \textit{the material}\end{array}\right\} \times \frac{\textit{The length of the magnetic circuit}}{\textit{The cross section of the magnetic circuit}}.$$

All measurements of dimensions being in centimetres.

The student may here note that by common consent, words ending in the syllable *-ance* refer to a quality of a whole circuit, and words ending in *-ivity* refer to the particular or specific quality of the material of which it is made.

It is usual, and useful, to employ two other terms which respectively signify the reciprocal or inverse of reluctance and reluctivity, and these are *permeance* and *permeability*. Hence a magnetic circuit of *large reluctance* has *small permeance*, and a material of *small reluctivity* has high or *large permeability*.

The reader will find it easy to remember the relation between magnetic flux, magnetomotive force and reluctance or permeance, and thus between magnetic flux density, magnetic force, and reluctivity or permeability, by the following mnemonic rule:—

Commit to memory the following arrangements of terms:

$$\frac{\textit{Magnetomotive force}}{\textit{magnetic flux} \times \textit{reluctance}},$$

and

$$\frac{\textit{Magnetic force}}{\textit{magnetic flux density} \times \textit{reluctivity}}.$$

If the finger is placed over any one of the terms in the above fractions, the relation of the covered quantity to the other two terms left will be shown by the position of the terms left. Thus, in the first fraction cover over the words *magnetomotive force:* the terms left are *magnetic flux* × *reluctance;* which show that magnetomotive force is equal to the *product* of magnetic flux and reluctance. In the same way *reluctance* is seen to be the *quotient* of magnetomotive force by magnetic flux.

The rules may be expressed in terms of *Permeance* and *Permeability* as follows:

Magnetic flux = permeance × magnetomotive force.

Magnetic flux density = permeability × magnetic force.

Or as fractions for the mnemonic rule :

$$\frac{\textit{Magnetic flux}}{\textit{permeance} \times \textit{magnetomotive force}},$$

and

$$\frac{\textit{Magnetic flux density}}{\textit{permeability} \times \textit{magnetic force}}.$$

We shall defer until a later chapter the discussion of the practical units in which the above quantities are measured, in order to make the relations above given available for calculation.

One remark must, however, here be made. The reluctivity of the so-called non-magnetic or feebly magnetic substances, and the reluctance of any magnetic circuit which does not consist wholly or partly of a strongly magnetic substance like iron, nickel or cobalt, is a constant quantity, and for all non-magnetic substances like air, wood, brass, &c., the value of the reluctivity and, hence, of the permeability of the material, is taken as unity. Hence the reluctivity of air is 1 The reluctance of an air circuit 10 centimetres long and 2 square centimetres in cross section is, therefore,

$$\frac{1 \times 10}{2} = 5, \text{ since}$$

$$\textit{Reluctance} = \frac{\textit{reluctivity} \times \textit{length of magnetic circuit}}{\textit{section of magnetic circuit}}.$$

The reluctivity or permeability of iron, and hence the reluctance of a magnetic circuit partly or wholly of iron, *varies with the magnetic flux density in it*, and hence cannot be stated absolutely unless we know the value of that flux density. The numerical ratio, however, of the magnetic force to the magnetic flux density or of the magnetomotive force per unit of length of the circuit to the magnetic flux per unit of section of the circuit, gives us the value of the reluctivity of the material at that point in the circuit. If the material of

which the circuit is made is homogeneous, then the product of this reluctivity and the ratio of the length to the section of the circuit, gives us the reluctance of the whole circuit.

In selecting our units for practical work we arbitrarily take the reluctivity of air to be unity, and hence the reluctance of a uniform air circuit is simply measured by the quotient of its length in centimetres by the cross section in square centimetres.

It follows from this that the magnetic force and the magnetic flux density have the same direction and same numerical value at all points and in all places when the circuit is made wholly of non-magnetic material.

Iron filings sprinkled on a card held in the region outside a permanent magnet really delineate and show us *the direction of the magnetic flux in the air* round the magnet in the plane of the card, and ought properly to be called the magnetic flux lines, although they also show the identical direction of the magnetic force at these places, and hence, following Faraday, have always been more generally called "the lines of magnetic force."

§ 8. **Lines of Magnetic Flux.**—Faraday originated the method of representing the distribution of magnetic flux in a magnetic field by *lines*, the direction of which delineate the direction of the magnetic flux at each point, and which are so spaced out that the *number of lines of magnetic flux* which pass perpendicularly through every square centimetre are numerically equal to the flux density at the centre of that area. These are also called *the number of lines of induction* per square centimetre. Hence in any part of the field where the flux density is large the flux lines are closely packed or most numerous, and where it is small they are more scattered or diffused.

The phrase "*number of lines of induction per square centimetre*" is in some respects less useful in practical work than the phrase "*flux density.*" The student should, however, understand that they mean the same

thing, and that when an iron circuit is strongly magnetised it may have across its section a magnetic flux density as great as 100 to 200 microwebers per square centimetre, or 10,000 to 20,000 lines or units of induction per square centimetre, reckoning on the centimetre-gramme-second system.

Whatever magnetic flux enters or leaves iron or any other strongly magnetic body immersed in a medium of lesser permeability, it creates magnetic poles at the places where it enters or leaves.

Thus, for instance, let us consider the case of a piece of iron placed in the field of another magnet. It is there subjected to the action of magnetic force. It has produced in it, as part of a magnetic circuit, magnetic flux. This flux flows through the iron, entering it at one end, where it creates in the iron a South pole or south polar area, leaving it at the other, where it creates a North pole, and flowing also round the space outside the iron. Every line of magnetic flux is a closed loop, and the flux itself is called, as already stated, *a circuital quantity*, because it takes place in a circuit. If iron filings are sprinkled over the iron so situated in the field, they will delineate and show the direction of this magnetic flux in the space round the iron. The flux does not leave the iron entirely at the end surfaces, but leaks out, as it were, at all sides of the iron. Hence, if sections be taken across the iron bar at different places, a different flux density would be found at each section, even if the iron is subjected to an originally uniform field of force. This flux density is the greatest in the centre of the iron bar, and tails off towards the ends just in proportion as the total magnetic flux in the iron is weakened by the lateral escape of part of the flux. The flux apparently tends to get back or complete its circuit by the easiest route, or to take the shortest cut in completing its circuit; and in so doing it selects the *path of least reluctance*. Hence, if a movable piece of iron is suspended in a magnetic field so as to be free to move, it turns itself so

as to make the total magnetic reluctance of the circuit a minimum. Wherever the flux leaves the iron to go into a less permeable region it creates a north magnetic pole or polar area, and wherever it enters the iron again it creates at that place a south pole or area. Hence, as the flux must complete its circuit, it always must *re-enter* the iron again somewhere if it leaves it at one part, and therefore north and south poles must occur in pairs of equal strength. If, however, a magnetic circuit consists wholly of iron, it may be that the flux is confined entirely within the iron, and in that case there are no poles at all. No substance is known which is entirely impermeable to magnetic flux, or which cannot have magnetic flux created in it by magnetic force. As a consequence of this, there is no such thing as an *insulator* for magnetism.

The student must think of a permanent steel bar magnet as traversed by magnetic flux permanently attached to it which proceeds inside the magnet from the south pole *to* the north pole. It then leaves the steel, branching out in all directions through the air, and turning back, passes through the space round the magnet and enters it again at the south pole. Each line of magnetic flux is, as above stated, a closed loop which passes through the magnet in some part of its course, and through the air or space outside during the remainder of its path. The steel filings or small exploring needle held anywhere near the magnet shows us the direction of the magnetic flux in the air or space outside.

The magnetic flux density at any point outside the magnet has a numerical value equal to that of the magnetic force at that point, and the magnetic force has the same direction as the magnetic flux.

The free magnetic poles only exist at places where the magnetic flux passes from one medium to another of different reluctivity; and hence, if the magnetic flux is wholly confined to the iron as in a ring magnet, there are no free poles. The free magnetic poles, when they exist in a bar of iron or steel, exert a reverse or demag-

netising action on the rest of the bar; and hence, if a straight piece of iron is placed in a uniform field of magnetic force, the actual resultant magnetic force creating magnetic flux in the iron is *less* than that which exists when the iron is not there. It is very important that the student should grasp clearly the significance of this last statement. He is otherwise apt to imagine that the actual magnetising force acting on a piece of iron placed in an originally uniform field of magnetic force is always the same as when the iron is not in that field. This, however, is not the case: it may be much less.

If a piece of iron, nickel or other magnetisable material is placed in a field of magnetic force, the total

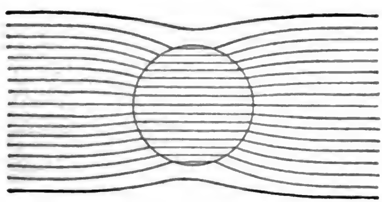

Fig. 16.—Paramagnetic Body placed in a Field of Magnetic Flux.

magnetic flux is not only increased, but is concentrated considerably into the iron or nickel, and hence these last materials are said to be more *permeable* than the air or original space. On the other hand, bismuth and diamagnetic materials placed in a field are slightly less permeable than empty space. If, therefore, a disc of iron is placed in a uniform field of magnetic force, the flux concentrates itself in the iron, as if it thereby found an easier path, and the flux lines take the form shown in Fig. 16.

Again, if a disc of diamagnetic material, such as bismuth, is placed in a uniform field of magnetic flux, the flux lines are slightly widened apart, as if they ex-

perienced more difficulty in getting through the bismuth than the air, as shown in Fig. 17.

The reader should carefully notice the sense in which the terms, a *magnetic* body and a *magnetised* body, are employed.

A magnetised substance is one which has a circuit of magnetic flux permanently attached to it, like a permanent steel magnet. A magnetic substance is one which *may be* magnetised, but is not necessarily so at the time considered.

This attached magnetic flux has been originally produced by a magnetic force acting on the material, but the magnetic flux continues to exist, in the case of

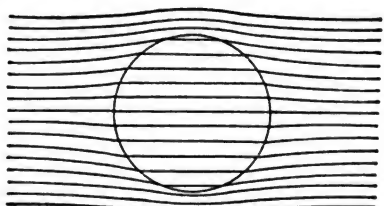

Fig. 17.—Diamagnetic Body placed in a Field of Magnetic Flux.

certain substances, in virtue of their *retentivity* and *coercivity*, long after the magnetic force is withdrawn. A magnetic substance, on the other hand, is one which permits the production of magnetic flux through it more easily than does a vacuum, or has a less reluctivity than empty space. To magnetise a magnetic body, or to create magnetic flux in a circuit, always requires an expenditure of energy, but in the case of those circuits or substances like steel which possess retentivity and coercivity, the flux, when once produced, continues wholly or in part without further expenditure of energy.

§ 9. **Action of a Magnetic Field on a Magnetic**

Substance.—When a magnetic body is placed in a non-uniform magnetic field, it is acted upon by a mechanical force tending to move it from weak places in the field to strong ones, and hence it follows that it is attracted by a magnetic pole from which proceeds a diverging field. This attraction is mutual. Not only does the single magnetic pole attract a magnetic body, but the magnetic body attracts the single magnetic pole. Hence a piece of a magnetic body, if sufficiently magnetic, will attract either end of a compass needle if held near it.

On the other hand, if the body is permanently magnetised, one or other end of it will *repel* the north pole of a suspended compass needle. Hence the indifferent attraction of either pole of a compass needle by a body merely shows the body to be *magnetic*, but the repulsion of one or other pole of the test needle by one part of the body shows that the body is *magnetised*.

It is important that the student should, even at this stage, realise that the mechanical actions occurring between magnetic poles are not due to action at a distance, as it has been called, or to magnetic poles pulling or pushing other magnetic poles across empty space without intermediate machinery; but they must be regarded as the visible effects of operations taking place in a *medium* called the *electromagnetic medium* or *ether*, which fills all space. The mutual dynamical action of magnetic poles can be accounted for by the assumption that this medium, when traversed by magnetic flux, tends to contract or shrink along the direction of the lines of magnetic flux, and tends to expand or swell out in a direction at right angles to them.

§ 10. **Practical Measurement of Magnetic Flux and Magnetic Force.**—In the practical measurement of any physical quantity, our method of procedure is to select some effect or change due to it, and to utilise this as a basis for measurement. Thus, to measure temperature, we select one change out of many others caused by change of temperature, viz. the change in volume pro-

duced in substances; and we obtain a practical measure of temperature-change by observing the apparent expansion of a standard substance such as mercury in a glass vessel, produced by change in temperature. In order, therefore, to measure magnetic flux, we must select some physical effect due to it, and make use of that as a means of measurement. As we shall see in a subsequent chapter, Faraday made the important discovery that if magnetic flux passes *through* or is *linked with* a conducting circuit, when that flux is annulled, reversed or changed in amount, an electromotive force is set up in the conducting circuit. This electromotive force is at any instant proportional to the rate at which the magnetic flux is being changed. Hence, if we consider a magnetic circuit as linked with or embraced by one turn of a conducting circuit, and if we consider the flux in this magnetic circuit as uniformly removed or destroyed, an *electromotive force* is set up in the conducting circuit which is proportional to the rate of removal of the magnetic flux. If the flux is uniformly removed at such a rate that an electromotive force of *one volt* * is set up in the single turn of the conducting circuit, then the magnetic flux is being removed at the rate of *one weber per second.*

This gives us a practical definition of what is meant by a magnetic flux of one weber. In order to create a flux of one microweber per square centimetre in a uniform endless air magnetic circuit, it is found that a magnetic force has to be applied to it in the same direction which is equal to that due to a current of one ampere flowing 80 times round the magnetic circuit for every centimetre in length of the magnetic circuit. In other words, we have to apply 80 ampere-turns per centimetre, or 100 ampere-turns per half inch of length of the magnetic circuit.

* For the definition of the *Volt* see Chapter V. An electromotive force is defined as that which causes or tends to cause an electric current in a conducting circuit.

This uniform magnetic force may be regarded as the uniform space-slope of the magnetomotive force required to produce a magnetic flux of one microweber per square centimetre in an air circuit.

It has been suggested that the C.G.S. unit of magnetomotive force which is equal to 0·8 of an ampere-turn should be called a *Gauss*.*

It would be more convenient, perhaps, if the magnetomotive force represented by 80 ampere-turns (or by $0{\cdot}8 \times 100$ ampere-turns) were called *one microgauss*. We should then have this simple rule:

A magnetomotive force having an intensity of one microgauss (80 *ampere-turns*) *per linear centimetre acting on an air magnetic circuit produces in it a magnetic flux in the same direction having a density of one microweber per square centimetre of section of that circuit.*

Magnetic language would probably be improved if the term *Magnetic Force* were disused, and we were to replace it by the term *Magnetomotive Intensity*. We should then have only two conceptions to deal with, viz. *Magnetic Flux* as the name for the physical state produced in substances by *Magnetomotive Force*. When considering the specific qualities of materials, we are concerned with the flux per square centimetre, or *Flux Density*, and the magnetomotive force per linear centimetre, or the *Magnetomotive Intensity*. The numerical ratio of the magnetomotive force and the magnetic flux is the measure of the *reluctance*. The numerical ratio of the magnetomotive intensity and the magnetic flux density is the measure of the *reluctivity*.†

* Numerous different definitions have been given of the term *gauss*. The above is that suggested in 1895 by the British Association Committee on Electrical Standards.

† There are great objections to the use of the term *Force* in connection with any other idea than that of the change in momentum of material substances, as defined in Chapter II. Instead of employing the term *Magnetomotive Force*, it would be better to express the same notion by speaking of it as the *Gaussage* of the magnetic circuit, as suggested by Mr. Oliver Heaviside. The magnetomotive intensity or gaussage per linear centimetre might be called the *Gaussivity* acting on the circuit at the

If the magnetic circuit, instead of being of air, consists of iron or material of higher permeability than air, then the value of the magnetic flux density produced in a homogeneous magnetic circuit of that material is obtained by dividing the magnetic force measured as above by the reluctivity of that material. This can only be obtained from a table showing the reluctivity corresponding to each particular value of the flux density for the material in question.

Hence we have the following rule for the determination of the total magnetic flux in a uniform homogeneous magnetic circuit of known dimensions when acted upon by a known magnetomotive force:

$$\left.\begin{array}{r}\text{The total magnetic}\\ \text{flux reckoned in}\\ \text{microwebers}\end{array}\right\} = \frac{\begin{array}{c}\text{The magnetomotive force acting on}\\ \text{the circuit reckoned in microgausses.}\end{array}}{\begin{array}{l}\text{The reluctivity}\\ \text{of the material}\end{array} \times \dfrac{\text{The length of the circuit}}{\text{Cross section of the circuit}}}$$

As an illustration, let us consider a circular ring-shaped magnetic circuit of material having unit reluctivity, such as air or wood. Let the mean diameter of the circuit be 10 centimetres, and the cross sectional area be 2 square centimetres. Let us then inquire what magnetomotive force will create in the circuit a magnetic flux density of 5 microwebers per square centi-

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specified place. The principal magnetic facts could then be expressed by the simple statement that gaussage (measured in ampere-turns) produces magnetic flux (measured in webers) in magnetic circuits. The *intensity* of this cause and effect is described by stating that the *gaussivity* (in ampere-turns per linear centimetre) produces in the circuit a certain *flux density*, or flux per square centimetre. The *Reluctance* of the magnetic circuit is that quality of it in virtue of which gaussage is required to produce *change in magnetic flux*. The *Reluctivity* of a circuit at any place is the reluctance of a cubic centimetre of it at that place and under the assigned conditions. The *Retentivity* is that magnetic quality of the material in virtue of which magnetic flux persists after gaussage is withdrawn. The *Coercivity* of the material is that magnetic quality of the material in virtue of which a reversed gaussage is required to annul or wipe out magnetic flux.

metre, or 500 C.G.S. units of induction per square centimetre. The total magnetic flux is $2 \times 5 = 10$ microwebers. The length of this magnetic circuit is $\frac{22}{7} \times 10$ centimetres, and its section is 2 square centimetres.

Hence we have

$$10 = \frac{\text{Magnetomotive force in microgausses}}{1 \times \dfrac{\frac{22}{7} \times 10}{2}}.$$

From which equation we see that the magnetomotive force in microgausses required is 157·14, and in ampere-turns is represented by the number $80 \times 157 \cdot 14 = 12{,}571$.

The case of iron magnetic circuits presents special difficulties, which will be considered in the chapter on Electromagnets.

§ 11. **Rational and Irrational Magnetic Units.**— We have given at the beginning of this chapter the ordinary definition of a Unit Magnetic Pole as a pole which, at a distance of one centimetre, repels another similar and equal pole with a force of one dyne. In selecting this as the fundamental definition of quantitative magnetism, the framers of our present system of magnetic units were dominated rather by the notion of attraction at a distance than by the fundamental ideas of Faraday, who regarded all electric and magnetic phenomena as being the result of operations taking place in a *medium* everywhere present. The total magnetic flux which proceeds from the pole of a magnet is a more important physical quantity than the attraction exercised at a distance by this pole on another equal and opposite pole, and the former should have been considered as taking precedence over the latter in creating a definition. The most obvious and natural definition of a unit magnetic pole is to define it as

a pole from which proceeds a total magnetic flux of one unit, and to take such a definition of a unit magnetic flux as shall make it create, if annulled, a flow of a unit quantity of electricity round a conducting circuit of one turn, and having a resistance of one unit, which is linked with the flux. The starting-point of the *present and irrational system* of magnetic units is the unfortunate selection of the definition of the unit magnetic pole. A magnetic pole is, under this irrational system, said to have a strength of m units if the force between it and another equal pole placed at a distance of d centimetres, is equal to $\frac{m^2}{d^2}$ dynes. In other words, the attractive force f, measured in dynes, between two opposite magnetic poles of strengths m and m^1 placed in air at a distance d centimetres apart, is given by the equation $f = \frac{m\,m^1}{d^2}$ (dynes), and the unit of magnetic pole strength is selected so as to fit in with this equation.

A consequence of this definition is that the magnetic flux proceeding from a pole of strength m is $4\pi m$ units; and this constant 4π $(= 4 \times 3 \cdot 1416)$ makes its appearance in numerous other magnetic formulæ, obscuring their physical meaning and greatly adding to the difficulties of the student.

The reason for the appearance of this 4π may be seen as follows: Imagine a long and very thin steel wire permanently and uniformly magnetised. Let this be called a magnetic filament. The poles of this filamentary magnet may be considered to be exactly at its extremities. From these poles proceeds magnetic flux radiating out into the air space round the magnet. Round one pole conceive a very small sphere of radius r to be described. Within this small sphere the magnetic flux lines are nearly straight radial lines uniformly spaced out like radii of the sphere. In the air space outside the magnet the numerical value of the flux density is the same as that of the magnetic force, because the permea-

bility of air is arbitrarily taken as unity. Hence, if the strength of the pole of the magnet is m, and we take the usual definition of pole strength, the magnetic force due to this pole at the surface of the sphere is $\frac{m}{r^2}$ dynes; hence this is also the numerical value of the flux density at that point. Accordingly the total magnetic flux over the whole surface of the sphere is $4\pi r^2 \times \frac{m}{r^2} = 4\pi m$ units, since the surface of a sphere of radius r centimetres is $4\pi r^2$ square centimetres. The whole flux out of the magnet pole must be equal to the whole flux through the surface of the embracing sphere. Hence the flux proceeding from a pole of strength m is $4\pi m$ units of flux (or $4\pi m$ lines of magnetic induction). Furthermore the same difficulty with the 4π follows us into other definitions. If the length of the magnetic filament is l centimetres and the section is s square centimetres, then the product $m l$ (or pole strength × length) is the measure of the magnetic moment of the magnet. The volume of the magnet is $l s$ (or length × section) cubic centimetres, and the quotient of the magnetic moment by the volume or $\frac{m l}{l s}$, or the moment per unit of volume, is called the *Intensity of Magnetisation* or simply *the Magnetisation* of the magnet. It is always denoted by the letter I. Hence $\frac{m l}{l s} = \mathrm{I}$, or $m = \mathrm{I}s$.

It therefore follows that the total magnetic flux coming out of the pole of a magnet of strength m, cross section s, and magnetisation I, is equal to $4\pi m$ units or to $4\pi \mathrm{I} s$ units. If the filament is a very long or endless soft iron filament and is magnetised by being placed in a uniform magnetic field the strength of which is H units when the iron is not there, then the total magnetic flux along the filament is the sum of the flux due to the intrinsic magnetisation of the iron and to the magnetic

flux in the air in the same space when the iron is removed. If s is the section of the filament, then, since in air magnetic flux density has the same numerical value as magnetic force, the total flux through a section s is H s units when the iron is not present. When the iron occupies that space and takes a magnetisation I, which we will suppose is uniform along the filament, the flux added by, and due to, the iron itself is, as we have seen, $4\pi I s$ units. If we denote as usual the resultant flux density in the body of the iron by B, then we have the equation

$$B s = 4\pi I s + H s, \quad \text{or} \quad B = H + 4\pi I.$$

Expressing the fact that the flux produced in the iron is added to the flux already existing in the air, and creates the total flux, all reckoned per square centimetre.

The ratio of the numerical value of the resultant flux density B to the magnetising force H producing it, is called the *permeability*, and is denoted by the letter μ. The ratio of the intensity of magnetisation of the iron to the force producing it is called the *susceptibility*, and is denoted by the letter k. These facts are in symbolical language expressed by the fundamental magnetic equations

$$B = \mu H \qquad I = k H,$$

which are in reality definitions of μ and k.

Substituting the values for B and I as given in the above equations in the equation

$$B = H + 4\pi I,$$

we arrive at the expression

$$\mu = 1 + 4\pi k \quad \text{or} \quad \mu - 1 = 4\pi k$$

as an equation connecting magnetic permeability (μ) and susceptibility (k).

The above equations and relations between magnetic quantities are in general very puzzling to beginners. They do not easily see the reason for the appearance of

this intrusive constant 4π entering into all these physical equations and greatly adding to their difficulties in seeing the real meaning of the symbols.

The true remedy for this confusion has been suggested by Mr. Oliver Heaviside to be the substitution of *rational* for *irrational* formulæ and definitions.

He has restated the definition of a unit magnetic pole in such a way that the subsequently derived definitions of important practical magnetic quantities are free from this disfiguring 4π and physically more intelligible. Mr. Heaviside's starting-point is a new definition of the unit magnetic pole as follows:

A magnetic pole is said to have a strength of m units if it attracts or repels another equal pole placed at a distance of d centimetres with a force of $\frac{m^2}{4\pi d^2}$ *dynes.*

The above definition furnishes us with a unit magnetic pole which is not the same in magnitude as the unit pole previously defined Mr. Heaviside's unit pole is called a *rational pole.* Hence a rational pole of strength m attracts another rational pole of strength m' placed at a distance of d centimetres in air with a force of f dynes, so that

$$f = \frac{m\,m'}{4\pi d^2}.$$

It follows from this that a *rational unit magnetic pole* attracts another equal and opposite rational unit pole placed at a distance of one centimetre with a force of $\frac{1}{4\pi}$ of a dyne, whereas the irrational, or C.G.S., unit poles are of such a magnitude or strength as to attract each other with a force of 1 dyne under the same conditions.

Hence the rational unit pole is weaker or smaller than the irrational or present unit pole in the ratio of $\frac{1}{\sqrt{4\pi}}$ to 1, or $\cdot 28205$ to 1.

The magnetic force due to a rational pole of strength m, or having a strength of m in rational units, at a distance of d centimetres, is $\frac{m}{4\pi d^2}$ units. Returning, then, to our magnetic filament, let us suppose as before a small sphere of radius r described round its pole of strength m (reckoned in rational units). The magnetic force at the surface of this sphere is $\frac{m}{4\pi r^2}$ units, and this is also, therefore, the numerical value of the magnetic flux density at that surface. Hence the total magnetic flux through the surface of the sphere is $4\pi r^2 \times \frac{m}{4\pi r^2}$ units $= m$ units, and therefore the number which denotes the total magnetic flux coming out of the pole of strength m in rational units is also m. The rational system thus gives us an obvious and natural definition of a unit magnetic pole, viz. that it is a pole from which proceeds a unit of magnetic flux. It follows, therefore, that if the intensity of magnetisation of the magnet is I, the flux traversing any transverse section s of the magnet is $\mathrm{I}\,s$ units; and that if the filament is an endless or poleless iron filament, magnetised uniformly in a field by a resultant external magnetic force H, we have the equation

$$\mathrm{B} = \mathrm{I} + \mathrm{H}$$

as a rational equation expressing the fact that the resultant magnetic flux per square centimetre of cross section along the iron is equal to the intrinsic flux I produced in the iron *per se*, added to the flux H produced in the same space if the iron is removed. It follows also then, that we have the equation

$$\mu = 1 + k$$

as a rational equation, connecting μ and k. The meaning of this equation is, that taking the permeability of air as unity, the susceptibility k of the iron may be regarded

as the amount by which the metal increases the permeability of the space which it occupies.

On the rational system, since the unit pole strength has been decreased in the ratio of 1 to $\frac{1}{\sqrt{4\pi}}$, or 3·5441 to 1, when compared with the magnitude of the present irrational unit pole, and since the unit of magnetic flux is the total flux proceeding from a unit pole, it follows that Mr. Heaviside's unit of magnetic flux is larger than the C.G.S. unit of magnetic flux in the ratio of 3·5441 to 1. By adoping a similar rational definition of a unit of electric quantity, a complete rational system of electrical units has been framed by Mr. Heaviside, in which the magnitude of the rational units is related to those of the C.G.S., or present practical units, as follows:

*Relation of the Rational to the existing Electric and Magnetic Units.**

1 "rational" unit magnetic pole ..	=	·28205	of present or C.G.S. unit pole.	
1 ,, ,, of magnetic force	=	3·5441	present units.	
1 ,, ,, of magnetic flux	=	3·5441	,,	,,
1 ,, ,, of magnetic flux density	=	3·5441	,,	,,
1 ,, ampere	=	2·8205	,,	amperes.
1 ,, volt	=	3·5441	,,	volts.
1 ,, ohm	=	1·2566	,,	ohm.
1 ,, henry	=	1·2566	,,	henry.
1 ,, coulomb	=	2·8205	,,	coulombs.
1 ,, farad	=	·79577	,,	farad.
1 ,, watt	=	10	,,	watts.
1 ,, joule	=	10	,,	joules.
1 ,, ampere-turn	=	2·8205	,,	ampere-turns.

Where

$$3{\cdot}5441 = \sqrt{4\pi}, \quad 2{\cdot}8205 = \frac{10}{\sqrt{4\pi}}, \quad 1{\cdot}2566 = \frac{4\pi}{10},$$

$$\cdot 79577 = \frac{10}{4\pi}, \quad \text{and} \quad \cdot 28205 = \frac{1}{\sqrt{4\pi}}.$$

Although the rational system has obvious and immense advantages from a theoretical point of view, it

* From the 'Electrician,' vol. xxxv. p. 774.

may yet be some considerable time before it is introduced into practice. This latter step would involve remaking or reconstructing all the thousands of ampere-meters, voltmeters, and resistance coils in actual use, and would necessitate a practical revolution almost akin to that resulting from a proposal to change the actual lengths and weights of the standards denominated respectively a yard or a pound.

CHAPTER IV.

ELECTRIC CURRENTS.

§ 1. **Electric Currents and Electromotive Force.** —We have already pointed out that a non-magnetic material like a copper wire can, under some conditions, exhibit magnetic qualities. If we find under any circumstances a wire of any material whatever exhibiting the two properties of being more or less hot, and having a magnetic flux taking place round it in closed loops or paths, we say that *an electric current is flowing* along this wire. As a matter of fact, we do not know that anything flows, or if it does, in what direction or with what velocity it flows. We use, however, the phrase *electric current*, sanctioned by custom, to express the sum total of all the properties possessed by the wire under these conditions, the principal ones being that the wire, or conductor as it is termed, is in some degree warm, and has a magnetic flux of greater or less strength taking place round it and within it in closed lines. The conductor is in fact the *axis* round which a magnetic flux takes place, the lines of which are all closed loops having their planes perpendicular to the wire. Since this flux may take place in one direction or in another, may be of any strength, and may be constant in strength or variable, we have as a principal fact about electric currents that they have *direction*, and *magnitude* or *strength*, and may be *varying* or *unvarying* in strength. If the embracing magnetic flux regularly changes its direction periodically, being first directed one way and then the other,

the current is said to be *alternating*. Hence, as a first classification, electric currents may be either

1. Of various strengths ;
2. Continuous or unvarying ;
3. Alternating ;
4. Fluctuating or variable.

If continuous, they may be in one direction or the other.

It is important that the student should bear in mind that, although we are accustomed to speak of the current as *flowing in the wire* in one direction or the other, this is a mere form of words. What we call *the current* in the wire is, to a very large extent, a process going on in the space or material outside the wire. Just as we familiarly speak of the sun as rising and setting, when the effect is really due to the rotation of the earth, so the ordinary language we use in speaking about electric currents flowing in conductors retains the form impressed upon it by older and erroneous assumptions as to their nature.

Electric currents, like magnetic flux, can, however, only exist in *circuits*. The cause, whatever it may be, which gives rise to an electric current in a circuit is called an *Electromotive Force.*

It is found that a given electromotive force produces currents of very different strengths in circuits of various kinds, as measured by the magnetic flux existing round the circuit at an assigned place.

That quality of the circuit, in virtue of which it permits an electric current to be produced in it by electromotive force, is called the *conductance* of the circuit, and a circuit in which a current is taking place is called an *Electric Circuit.*

It is usual and convenient to define the chief property of an electric circuit as its *resistance*, and to say that the resistance of an electric circuit is a quality of it, in virtue of which electromotive force is required to create in it an electric current.

An electric current can no more start itself in a circuit than a material substance can set itself in motion or change its own momentum. When momentum is created or destroyed, it is always due to mechanical force. When an electric current is created, it is invariably a consequence of the existence of electromotive force in some part of that circuit.

To create an electric current in an electric circuit requires an expenditure of energy, and hence the current when flowing is a form of energy.

The peculiar property of electric circuits as we know them is that they continually absorb and dissipate the energy of their surrounding magnetic field as heat in the conducting circuit, and hence some source of energy has to be continually drawn upon to maintain a current, that is to maintain the magnetic flux embracing the conducting circuit.

The student may assist his ideas by thinking of the energy processes at work in starting a heavy fly-wheel in rotation. To get up a certain speed the inertia of the wheel has to be overcome. This involves doing work, or expending energy of some kind on the wheel. When the wheel is set in rotation, it has associated with it a certain store of energy. The wheel will not, however, keep on rotating for ever by itself, because the friction at the bearings dissipates this energy gradually as heat. Hence, to keep this wheel rotating at a certain speed, we have not only to expend energy in the first place on it to get up speed, but we have to keep on expending energy on it to supply the amount continually being frittered away into heat in the bearings. The wheel has, therefore, two qualities: it has *inertia*, in virtue of which energy has to be spent on it to *begin* or *produce* rotation, and it has an *energy-dissipating quality*, due to the friction at the bearings, in virtue of which energy has to be continually supplied to the wheel to *maintain* rotation at a constant speed.

Electric circuits are found to have two qualities

exactly analogous to *inertia* and *friction* in their effects. The first of these is called the *inductance of the circuit*, in virtue of which a current cannot be started at full strength at once in a circuit, but it requires the expenditure of energy to produce it. The second quality is called the *resistance of the circuit*, and in virtue of this the energy initially given to the circuit is progressively dissipated as heat; and hence electromotive force has to be continually applied to maintain the current. We shall see later how each of these qualities is measured.

§ 2. **Production of an Electric Current.**—There are many known methods by which an electric current can be produced; but from what has been said in Chapter II. it will be evident that in each case, since an electric current represents energy of a particular kind, some other existing form of energy must always be transformed in manufacturing an electric current. The three principal ways in which the creation of a current takes place is by a transformation of—(1) Chemical energy; (2) Heat, or thermal energy; (3) Mechanical energy.

The first of these methods was discovered by Alessandro Volta, who at the close of the last century was led by following out certain physiological experiments, made by Galvani in 1790, to invent the *voltaic cell, battery* or *pile.* Volta was Professor of Physics in the University of Pavia, in Italy, and on March 20th, 1800, he communicated an account of his discovery to the Royal Society in England.

Let us attend first to the facts observed in the case of a single voltaic *cell*, *couple* or *element*, as it is also called.

In a glass vessel place some water slightly acidulated with sulphuric acid, or, in fact, almost any acid. Place in this vessel a rod or plate of perfectly pure zinc. Ordinary zinc may be rendered pure enough on the surface by rubbing it with a little mercury and dilute

sulphuric acid until it becomes *amalgamated* on the surface, with a mixture or alloy of zinc and mercury, which is as effective as pure zinc. It will be seen that the acid in the vessel has no apparent action on this amalgamated or pure zinc, and produces no change in it. A copper, silver, platinum, or a hard carbon plate is now to be placed in the same acid, but not touching the zinc. No visible action takes place on either body. So far, the whole arrangement might be left an indefinite time without any apparent or appreciable changes taking place.

Fig. 18. Fig. 19.

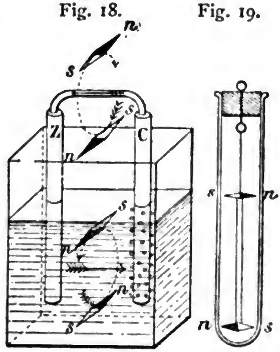

Fig. 18.—A Voltaic Cell consisting of Zinc and Copper Rods placed in dilute Sulphuric Acid. The dotted arrows show the direction of the magnetic flux and the firm-line arrows the direction of the electric current.
Fig. 19.—Pair of Astatic Needles.

In the next place let a wire be arranged so as to touch both the zinc and the copper or other plate, the wire being outside the liquid. Immediately a torrent of bubbles will be seen to ascend from the surface of the copper plate. (See Fig. 18.) The bubbles prove on examination to be hydrogen gas. At the same time, if we examine the connecting wire, we find this wire becoming warm, and on testing it with a small exploring magnetic needle it will be found that there is a magnetic flux all round this wire, embracing it at every part, the lines of the flux being closed loops surrounding the wire. The wire, therefore, proves to be the *axis* of a magnetic flux. We also find that the magnetic flux embraces the liquid in the cell, and that the axis round which the magnetic flux is formed is a closed line or circuit formed partly of the liquid and partly of the wire.

Unless the current given by the cell is strong the student may have some difficulty in verifying the above statements. It is, however, more easy to do so by using a pair of *astatic* exploring needles made as follows. To a finely drawn-out glass fibre, about six inches long, affix two fragments of magnetised watch-spring, each half an inch long, and made by breaking in half a piece of highly tempered watch-spring an inch long, previously carefully magnetised. These two little magnets are fastened transversely, as in Fig. 19, to the glass fibre, but with their poles in opposite directions. The whole arrangement is then hung up by a fibre of cocoon silk in a long glass tube. Such a pair of equally strong magnets fastened to the same stem with poles in opposite directions is called an *astatic system of needles*, because it has no tendency to stand in any particular direction under the action of the earth's magnetic force, and therefore may be regarded as out of the control of the terrestrial magnetic couple.

Having then prepared the voltaic cell, hold the astatic system of magnets so that the lower needle lies just *above* the wire, at its centre; the little needle will set itself transversely to the wire, thus showing that the magnetic flux is in that direction at that point. Next hold the same needle just *below* the wire, and it will set itself with its poles in the opposite direction, but the needle still standing transversely to the wire. By bending the wire over sideways, the magnetic flux on the sides of the wire may be explored, and the student can convince himself that the magnetic flux round the wire is everywhere at right angles to the wire, and is directed round it in closed loops.

If he then lowers the needle into the acid between the plates, he will find that it will set itself transversely or across the direction joining the plates, and that at the surface of the liquid it will place its north pole in one direction; but that if lowered to the bottom of the liquid the needle will place its north pole in the opposite direction, and it is possible to show that the liquid in the cell between the plates is surrounded by, and includes a magnetic flux.

In trying this last experiment it is essential that the wire connecting the plates should either be bent out of the way, or else that the glass connecting fibre joining the upper and lower needles should be so long that only the lower needle of the astatic pair is affected by the magnetic flux we are exploring.

Hence the result of experiment is to show that the circuit formed partly of the wire and partly of the liquid in the cell is an electric circuit, or, as it is commonly called, a galvanic circuit, and thus it is embraced everywhere by the lines of a magnetic flux linked with it. This flux passes through the aperture of the electric circuit and returns to complete its circuit over the outside.

The circuit is said to be traversed by an electric current *generated* by the cell. Much discussion has taken place as to the location or origin of the *electromotive force* in the cell, but to follow this out would lead us into matters hardly suitable for an elementary treatise. The plain facts concerned are not, however, difficult to understand. A brief discussion of the theory of the voltaic cell will be given in Chapter X.

In order to determine whether a circuit is traversed by an electric current, and if so, in what direction, we make use of the magnetic property of the circuit so traversed, and construct what is called a *Galvanometer* to detect its presence.

The student will find it desirable at this stage to construct a simple form of galvanometer in the following way:—

Obtain two pieces of hard wood (see Fig. 20) ½ an inch thick and 4 inches square, and cut in the centre of each a hole 1 inch in diameter. Make a stout paper tube 1 inch in diameter and 2 inches long, and glue the wooden pieces on to the ends of this, so as to make a short bobbin with square ends or cheeks. Into the interior of the paper tube fit a large wine-bottle cork which slides easily into the tube. Over the outside surface of one cheek fasten a watch-glass or piece of thin mica to close the opening to the tube. Procure half a pound of double-cotton covered wire, of size known as No. 32, and carefully wind this

on the bobbin. Before so doing it is better to increase the insulating power of the cotton, and prevent it absorbing moisture by boiling the covered wire for some time in an iron saucepan in melted paraffin wax, which can most easily be obtained in the form of a paraffin candle. Fix to one of the wooden cheeks two brass binding screws or terminals, and in winding on the wire bring out the inner end of the wire through a small hole made in the cheek quite close to the paper tube and attach the bared end to one terminal. When the winding is complete connect the outside end of the wire to the other terminal.

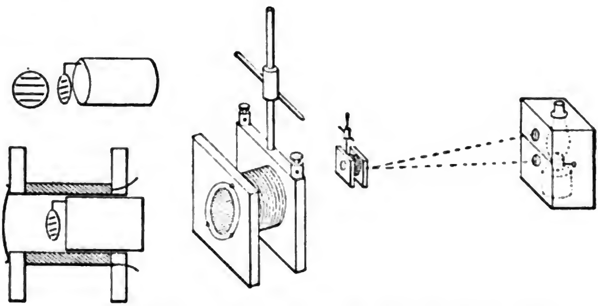

Fig. 20.—Simple form of Mirror Galvanometer and Lamp Case.

This constitutes what is called the coil of the galvanometer. In the next place obtain from a scientific instrument maker a light silvered mirror, made by silvering a small disc of microscopic cover-glass half an inch in diameter. To the back of this fasten three small fragments of magnetised watch-spring by a little touch of shellac varnish *placed in the centre of the mirror only*; the poles being placed in the same direction. The mirror and magnets are then to be suspended from a pin stuck in the end of the cork by means of a short and very fine fibre of cocoon silk. Some little dexterity is needed to do this successfully. The cork, mirror and magnets are then to be

inserted in the opening of the paper tube, so that the little magnets come exactly in the centre of the coil. Note that if the mirror back is covered with shellac varnish *all over*, it will on drying distort the mirror.

The instrument so made is called a *mirror galvanometer*. Place the instrument on a table, and turn it round so that the magnetised needles hang parallel to the coils of the wire and the mirror is square across the tube. Obtain a large biscuit tin and place in it a small paraffin lamp, cutting a hole at the top for the chimney, and a hole at the side on a level with the flame about the size of a sixpence. Place this lamp about a yard from the galvanometer, so that the light of the lamp falls through the hole on to the mirror. By means of a magnifying glass or convex lens, held between the hole and the mirror, it will be found possible to make a sharp image of a fine wire stretched across the illuminated hole appear upon a sheet of card placed against the biscuit box above the hole. This arrangement forms a lamp, lens and scale as used with a mirror galvanometer. It is convenient to add to the galvanometer a short vertical brass wire stuck in one cheek. On this wire is made to slide a cork, traversed by a short length of magnetised knitting needle. This *controlling magnet* can be turned round into various positions, and by making a field in the interior of the bobbin in different directions, it serves to make an adjustment of the position of the spot of light, or to cause the galvanometer needle to stand in any desired direction, irrespective of the direction of the earth's magnetic field at that point.

The galvanometer forms a means of detecting the presence of an electric current in a circuit into which it is inserted. If a current flows through the wire of the galvanometer it creates round that wire a magnetic flux. The magnetic needle in the coil turns itself so that, as far as the controlling forces will permit, it stands with its axis more or less in the direction of the interior flux of the galvanometer coil, and, as we have seen, that flux is in a direction everywhere transverse to the wire. Hence if the galvanometer is so arranged that the needle,

when at rest, stands in the plane of the wire windings, the passage of any current through this wire will make it turn so as to stand more or less across the plane of the windings in one direction or the other, according to the direction of the current in the wire. We have in this instrument a means of detecting the direction as well as the presence of a current in a circuit.

Let wires be joined to the galvanometer terminals, and the following experiments tried.

Take a piece of zinc, a piece of iron, and a piece of silver, say a zinc rod, a steel knife, and a silver spoon. Join the zinc rod by a wire to one terminal of the galvanometer, and the iron to the other, and dip the ends of the zinc and iron into water to which a drop of acid, say of vinegar, has been added. Notice which way the galvanometer needle moves. It will fly to one side. Then try replacing the zinc and iron by iron and the silver. The needle will move in the same direction. Next exchange the positions of the metals, and the current will be in the reverse direction. By examining carefully in this way a series of metals it will be found that for any one liquid there exists an order or series amongst the metals, called the *Electro-chemical order*, such that any pair of metals in the series being taken and used as plates in a voltaic cell, the current, when the metals are connected, is *from* the one standing the higher of the two on the list *to* the one standing the lower of the two on the list, through the liquid in the cell.

We have here used the term direction of the current, and the student must note how the *direction of the current* is determined. Consider a conducting wire in which an electric current exists, and suppose this wire is looked at endways; if then the magnetic flux round that wire is in the same direction as that of the rotation of the hands of a watch, the current is said to flow *from* the observer in the wire.

The reader must carefully fix this relation in his

mind by the aid of the diagram in Fig. 21, and by thinking of the way in which the thrust and twist of a corkscrew are related. In putting in a right-handed screw, we *twist round* in the *clockwise* direction, and *push away* from us. We then arrange the convention that the *positive direction* of the magnetic flux round the wire, and the *positive direction* of the current (so-called) in the wire, shall be related to each other like the twist and thrust of a corkscrew. By considering the direction of the current on this convention flowing in the wire connecting a pair of metals in a voltaic cell, it is found that the

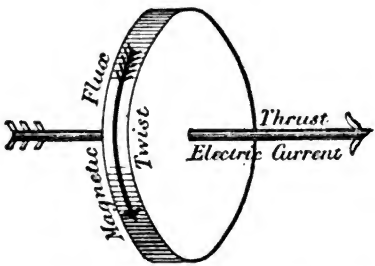

Fig. 21.—Diagram showing the relation between *Positive Twist* and *Positive Thrust*, and also between *Magnetic Flux* and *Electric Current.*

current flows *in the cell* from the metal standing the higher of the two in the electrochemical series to the metal standing the lower of the two, through the liquid.

Thus, the current flows *from* zinc *to* copper *through* liquid, or in the direction, zinc, acid, copper (Z.A.C.). The copper is said to be the *positive pole* of the cell, the zinc the *negative pole.* The electrochemical order of the metals depends, however, to some extent, upon the nature of the liquids used as excitant, and if an alkaline liquid excitant like potassium persulphide is used, the order is somewhat different from that in dilute mineral

acids, as may be seen by comparing the two following lists:—

Electrochemical order of the metals in dilute acids.	Electrochemical order of the metals in alkaline persulphides.
Zinc.	Zinc.
Cadmium.	Copper.
Lead.	Cadmium.
Tin.	Tin.
Iron.	Silver.
Nickel.	Lead.
Bismuth.	Antimony.
Antimony.	Bismuth.
Copper.	Nickel.
Silver.	Iron.
Gold.	Gold.
Platinum.	Platinum.
Hard carbon or graphite.	Hard carbon or graphite.

If two rods, plates or wires, made respectively of any of the bodies mentioned in the above list, are placed in any conducting acid or alkaline solution, the arrangement is called a *Voltaic Couple*. Even if the plates are not connected there is an *electromotive force* due to the couple, and this, when the plates are joined by a conductor, or metallic wire outside the liquid, sends a current through the conductor *from* the metal standing the lower of the two in the electrochemical series *to* the metal standing the higher of the two in the list.

The conditions to be complied with to obtain a current are that chemical action must be possible between one at least of the metals and the liquid used. Moreover, both the plates or *elements* in the cell, as well as the exciting liquid, must be conductors. The liquid must therefore be an *electrolyte*, that is one which conducts by chemical decomposition.

Consider, for instance, the case of zinc and copper placed in dilute sulphuric acid.

The sulphuric acid consists of hydrogen chemically combined with an *acid radicle* called the sulphuric acid radicle, and denoted in chemistry by the symbol SO_4, meaning a union of one atom of sulphur (S) with four atoms of oxygen (O_4).

Sulphuric acid is denoted chemically by H_2SO_4, and it is strictly speaking a sulphate of hydrogen. Sulphuric acid mixed with water is a good conductor of electricity, and, since its conductivity depends upon its chemical decomposition, it is called an *electrolyte.*

Compounds of SO_4 and metals, such as sulphate of zinc ($ZnSO_4$) and sulphate of copper ($CuSO_4$) also exist, and are well known. If a piece of metallic zinc is placed in a solution of sulphate of copper (bluestone) it is found that the zinc expels the copper from its combination with SO_4 and takes its place, thus forming sulphate of zinc (white vitriol) and depositing metallic copper. Hence it is said that the chemical affinity of the zinc for the sulphuric acid radicle SO_4 is greater than that of copper. If two pieces of copper are put into dilute sulphuric acid, no electric current can be obtained by connecting them by a wire, because both plates are electrochemically identical. If a piece of zinc and copper are placed, say, in paraffin oil, no current is obtained because paraffin oil is not an electrolytic conductor of the current, and has no chemical action, either with zinc or copper. If plates of zinc and copper are placed in an electrolytic conducting liquid, such as dilute sulphuric acid, the acid radicle of which has a greater chemical affinity for one metal than for the other, an electric current is produced when the plates are connected. Since a particular kind of hard carbon, called gas retort carbon, or the hard carbon used for making arc-light carbons, is a fairly good conductor, and has no chemical action with most acid or alkaline solutions, it is generally used, in the form of carbon plates, for one of the elements in a voltaic cell, and then forms the *positive pole* of the cell, with zinc as the other

element which forms the *negative pole*. If the two metals are merely placed in the exciting fluid or *electrolyte* and not connected, the cell or couple is said to be *on open circuit*. Under these conditions an electromotive force exists *tending to urge* a current through the liquid from one metal to the other, which is called the *Open Circuit Electromotive Force*, but no *current* exists until the metals are connected by a wire closing the circuit outside the cell. The energy of the current produced by a voltaic cell is the equivalent of a portion of the chemical potential energy of the chemical substances taking part in the reactions going on in the cell. This chemical potential energy in whole or part disappears, and an equivalent for some of it reappears in the form of the current. The zinc and sulphuric acid in the above instance taken together possess *potential chemical energy*. When they combine into sulphate of zinc and free hydrogen, the resulting compounds possess less total potential chemical energy than the zinc and acid forming them. The difference is the equivalent of the energy represented by the heat produced in the cell and the electric current energy generated in the circuit.

The voltaic cell therefore is a contrivance whereby some of the energy liberated when chemical bodies combine to form compounds possessing less potential chemical energy than the constituents, is transformed into the energy of an electric current. The heat produced in the cell, together with the heat equivalent of the electric current energy taken together, is equal to the whole energy set free during the combination.

Simple illustrations of *voltaic couples* as they are called, may be obtained by inserting a steel knife and a silver fork into an orange or lemon, and then connecting the metals with the galvanometer. A current will be generated proceeding *from* the knife *to* the fork through the *acid* juice of the orange.

A piece of zinc placed under the tongue, and a silver coin placed on the top of the tongue, but not touching,

will also be found to give a current when connected with the galvanometer, proceeding from the zinc to the silver through the tongue, the moisture of which acts as the acid. The zinc or steel in these cases forms the negative pole, and the silver the positive pole in the above combination.

§ 3. **Practical Forms of Voltaic Cells. Standard Cells.**—In the various forms of cells in use for producing electromotive force and currents (usually called *Primary Cells* or *Primary Batteries*) one of the elements is generally zinc or a metal of similar character, and the other element is generally copper, silver, platinum or hard carbon.

Cells are called *Dry cells* when the liquid is mixed with some substance so that the electrolyte is in such a pasty condition that the cell can be sealed and used in any position. They are called *single fluid cells* when only one liquid is used with two metals, and *two-fluid cells* when two liquids are used. We shall describe more in detail the construction of various primary cells in the chapter on the Generation of Electric Currents; meanwhile the student will find that for small currents not required for any length of time, the most convenient cell to use is a dry cell called the Obach cell. For other purposes, when a stronger current is required, some form of the Bichromate cell (so called because the electrolyte contains Bichromate of Potash or Soda), and for fair constancy of electromotive force, the Daniell cell is to be preferred.

There are two cells in special use as *Standards of electromotive force*. These are called respectively the Clark Standard Cell and the Helmholtz, or Calomel Standard Cell.

The *Calomel cell* is made as follows:—Procure a wide test-tube about an inch in diameter and 3 or 4 inches deep, and into the bottom of this place a little pure mercury, occupying say half an inch of the tube in depth. A piece of very fine glass tube must then have

a platinum wire sealed into one end, and the wire itself brought up to the top of the tube. A cork must be provided, fitting the test-tube, and through this cork two holes made, one in the centre, through which is passed a rod of pure zinc about one-quarter of an inch in diameter. (Such rods of cast zinc chemically pure can be obtained from any wholesale chemist.) The zinc rod must be passed through the cork tightly, and a short

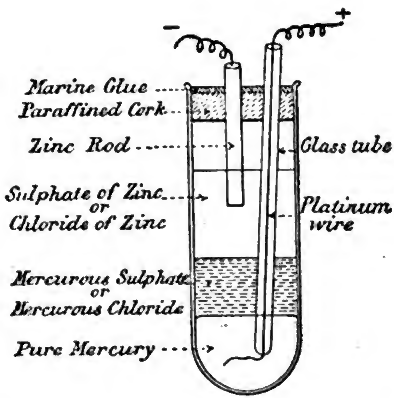

Fig. 22.—Board of Trade Pattern of Calomel or of Clark Standard Cell.

length of copper wire must be soldered to the top end of the zinc rod (see Fig. 22). A hole is then to be made in the side of the cork through which the glass tube containing the platinum wire can be easily passed. The cork itself should be first well boiled in paraffin wax. Obtain next from the chemist some calomel or mercurous chloride, put this white powder to a depth of one inch on the top of the mercury, previously inserting into the mercury the end of the glass tube through

which the platinum wire protrudes, so that the platinum wire is underneath and in contact with the mercury, but so protected that nothing else placed over the mercury touches the platinum wire. Into the test-tube then pour a saturated solution of zinc chloride made by putting the solid chloride of zinc in water until no more is dissolved. The test-tube containing the mercury and the calomel should then be supported by inserting its base into a wooden stand or into a hole bored in a large cork or bung. The saturated solution of zinc chloride having been poured on the top of the mercurous chloride, and the test-tube being very nearly filled with the solution, set the cork in the test-tube so that the zinc rod dips into the solution of zinc chloride, but does not extend far enough down to touch the mercury.

In putting in the cork, the glass tube containing the platinum wire must be passed through the side hole, so that when the cork is in its place the end of the zinc rod stands up through the centre of the cork, and the glass tube containing the platinum wire stands up through the hole in the side of the cork. Fine copper wires may then be soldered to the end of the platinum wire and to the zinc rod, and brought down to terminals fixed on the base. This arrangement constitutes a standard cell. It is not intended to be used for the purpose of producing an electric current, or at least only a very small one, but is employed as a *Standard of electromotive force.* The electromotive force of this cell is very nearly equal to a unit called a volt. To obtain an electromotive force of exactly one volt, the specific gravity or density of the zinc chloride solution must be adjusted to be 1·38.

The cell called the *Clark's Standard Cell* is made in a similar manner, with the exception that instead of using mercurous chloride, we use *mercurous sulphate,* and instead of using a solution of zinc chloride a solution of *zinc sulphate* is employed.

The following description is the specification given

by the Board of Trade for the construction of a standard Clark's cell :—

The Clark cell, when sending only a very feeble current, has an electromotive force of 1·434 volts at 15° Centigrade.

The cell consists of zinc or an amalgam of zinc with mercury and of mercury in a neutral saturated solution of zinc sulphate and mercurous sulphate in water, prepared with mercurous sulphate in excess. The materials to be used must be carefully prepared as follows :

(*a*) *The Mercury.*—To secure purity it should be first treated with acid in the usual manner, and subsequently distilled *in vacuo*.

(*b*) *The Zinc.*—Take a portion of a rod of pure re-distilled zinc, solder to one end a piece of copper wire, clean the whole with glass paper or a steel burnisher, carefully removing any loose pieces of zinc. Just before making up the cell dip the zinc into dilute sulphuric acid, wash with distilled water, and dry with a clean cloth or filter paper.

(*c*) *The Mercurous Sulphate.*—Take mercurous sulphate, purchased as pure, mix with it a small quantity of pure mercury, and wash the whole thoroughly with cold distilled water by agitation in a bottle; drain off the water, and repeat the process at least twice. After the last washing drain off as much of the water as possible.

(*d*) *The Zinc Sulphate Solution.*—Prepare a neutral saturated solution of pure ("pure recrystallised") zinc sulphate by mixing in a flask distilled water with nearly twice its weight of crystals of pure zinc sulphate, and adding zinc oxide in the proportion of about 2 per cent. by weight of the zinc sulphate crystals to neutralise any free acid. The crystals should be dissolved with the aid of gentle heat, but the temperature to which the solution is raised should not exceed 30° C. Mercurous sulphate, treated as described in *c*, should be added in the proportion of about 12 per cent. by weight

of the zinc sulphate crystals to neutralise any free zinc oxide remaining, and the solution filtered, while still warm, into a stock bottle. Crystals should form as it cools.

(*e*) *The Mercurous Sulphate and Zinc Sulphate Paste.* —Mix the washed mercurous sulphate with the zinc sulphate solution, adding sufficient crystals of zinc sulphate from the stock bottle to insure saturation, and a small quantity of pure mercury. Shake these up well together to form a paste of the consistence of cream. Heat the paste, but not above a temperature of 30° C. Keep the paste for an hour at this temperature, agitating it from time to time, then allow it to cool; continue to shake it occasionally while it is cooling. Crystals of zinc sulphate should then be distinctly visible, and should be distributed throughout the mass; if this is not the case add more crystals from the stock bottle, and repeat the whole process.

This method insures the formation of a saturated solution of zinc and mercurous sulphates in water.

The cell may conveniently be set up in a small test-tube of about 2 cm. diameter and 4 cm. or 5 cm. deep. Place the mercury in the bottom of this tube, filling it to a depth of say 0·5 cm. Cut a cork about 0·5 cm. thick to fit the tube; at one side of the cork bore a hole through which the zinc rod can pass tightly; at the other side bore another hole for the glass tube which covers the platinum wire; at the edge of the cork cut a nick through which the air can pass when the cork is pushed into the tube. Wash the cork thoroughly with warm water, and leave it to soak in water for some hours before use. Pass the zinc rod about 1 cm. through the cork.

Contact is made with the mercury by means of a platinum wire about No. 22 gauge. This is protected from contact with the other materials of the cell by being sealed into a glass tube. The ends of the wire project from the ends of the glass tube; one end forms

the terminal, the other end and a portion of the glass tube dip into the mercury.

Clean the glass tube and platinum wire carefully, then heat the exposed end of the platinum red hot, and insert it in the mercury in the test-tube, taking care that the whole of the exposed platinum is covered.

Shake up the paste and introduce it without contact with the upper part of the walls of the test-tube, filling the tube above the mercury to a depth of rather more than 1 cm.

Then insert the cork and zinc rod, passing the glass tube through the hole prepared for it. Push the cork gently down until its lower surface is nearly in contact with the liquid. The air will thus be nearly all expelled, and the cell should be left in this condition for at least 24 hours before sealing, which should be done as follows:—

Melt some marine glue until it is fluid enough to pour by its own weight, and pour it into the test-tube, above the cork, using sufficient to cover completely the zinc and soldering. The glass tube containing the platinum wire should project some way above the top of the marine glue.

The cell may be sealed in a more permanent manner by coating the marine glue, when it is set, with a solution of sodium silicate, and leaving it to harden.

The cell thus set up may be mounted in any desirable manner. It is convenient to arrange the mounting so that the cell may be immersed in a water-bath up to the level of, say, the upper surface of the cork. Its temperature can then be determined more accurately than is possible when the cell is in air.

In using the cell sudden variations of temperature should as far as possible be avoided.

One advantage which the Calomel cell has over the Clark's cell is that in the case of the Calomel cell there is a much smaller change in electromotive force with varying temperature, whereas in the case of the Clark's cell

the electromotive force of the cell decreases with temperature to a marked degree. On the other hand, variations in the specific gravity of the zinc chloride solution greatly affect the electromotive force of the calomel cell. The variation of electromotive force of the Clark's cell with temperature is shown in the table below. It is seen that a rise in temperature of one degree Centigrade *reduces* the electromotive force of the Clark's cell about 8 *parts in* 10,000. In the case of the Calomel cell the corresponding variation would only be 1 part in 10,000.

The temperature of the cell is best determined by placing it in water, and taking the temperature of the water after allowing a sufficient time for the cell to take the same temperature as the water.

TABLE SHOWING THE ELECTROMOTIVE FORCE OF A CLARK'S STANDARD CELL AT VARIOUS TEMPERATURES.

Temperature of the Cell in Centigrade Degrees.	Electromotive Force of the Cell in Volts.
5° C.	1·445
6°	1·444
7°	1·443
8°	1·442
9°	1·441
10°	1·440
11°	1·438
12°	1·437
13°	1·436
14°	1·435
15°	**1·434**
16°	1·433
17°	1·432
18°	1·431
19°	1·430
20°	1·428
21°	1·427
22°	1·426
23°	1·425
24°	1·424
25°	1·423

We shall refer to the use of these two cells as standards of electromotive force presently. Voltaic cells or couples may be joined up in various ways. They may be joined up *in series*, in which case the positive pole of one cell is joined to the negative pole of the next, and so on, forming what is called a battery, and in this case the electromotive force of each cell is added to that of the others, so that the electromotive force of the whole battery, if consisting of cells of the same kind, is equal to the electromotive force of one cell multiplied by the number of cells. Voltaic cells or couples may be also joined up *in parallel*, in which case the whole of the positive poles are joined together and the whole of the negative poles are joined together; the electromotive force of the battery is then only equal to that of one single cell of the same kind. Galvanic cells may be joined up partly in series and partly in parallel, to suit various necessities.

§ 4. **Thermo-electric Currents.**—It was discovered by Seebeck, in 1821, that an electromotive force could be generated by heating at one junction of a metallic circuit composed of two or more metals. If, for instance, a ring is made which is composed half of antimony and half of bismuth, and if one junction or place where the antimony touches the bismuth is slightly heated, while the other junction is kept at the ordinary temperature of the air, an electric current is produced which circulates round the ring in such a direction that it passes *from* the bismuth *to* the antimony across the heated junction. (See Fig. 23.)

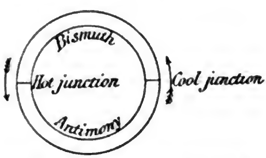

Fig. 23.—Bismuth-Antimony Thermocouple.

This discovery was completed by that of Cumming, who about the same time discovered another important fact, viz. that for every pair of metals there exists a

certain temperature called the *neutral temperature*, such that if the temperature of the hot junction is as far above the neutral temperature as the temperature of a cold junction is below the neutral temperature, then no current is produced in the circuit at all. Thus, for instance, the neutral temperature of copper and iron is about 275° C. Hence, if a ring is made half of copper and half of iron, and if one junction of the copper and iron is kept at 0° C., or at the melting-point of ice, and the other junction is heated to 100° C., or at the boiling-point of water, a current is produced which goes from the copper to the iron across the heated junction. If, however, one junction is heated to 450° C., whilst the other junction is kept at 100° C., then, since the temperature of the hot junction is 175° above the neutral temperature, and the temperature of the cold junction is 175° below the neutral temperature, it is found that no electric current is produced in the circuit. This current, however, when produced, is called a *thermo-electric current.* The two metals so used are called a *thermo-electric-couple*, and the electromotive force which produces this current is called a *thermo-electromotive force.* Since the current so generated represents energy, it is clear that, in order to create a thermo-electric current, some energy must be transformed, and the energy which is transformed is heat, which is absorbed in part at one junction of the two metals.

Lord Kelvin discovered, in 1851, that, in addition to the absorption of heat at the hot junction, there was also *an absorption of heat taking place all along the metals forming the thermo-couple* as a consequence of the difference in temperature between adjacent points in the metals. In other words, the electromotive force set up in the circuit is not only produced at the junctions where the two different metals touch one another, but there is also an electromotive force existing in the body of the metals themselves forming the couple, at all points, owing to the difference in temperature betwen adjacent

points in the metals. Thus, for instance, in the case of a copper-iron thermo-couple, part of the total electromotive force is due to the fact that *hot copper* is thermo-electrically different from *colder copper* in the same manner that *iron* is thermo-electrically different from *copper* at the same temperature.

In an unequally heated bar of copper there is at every point of the bar an internal electromotive force acting from the cooler part of the copper to the warmer part of the copper, and in an unequally heated bar of iron a similar electromotive force acting from hot iron to cold iron. Silver, zinc, cadmium and antimony resemble copper in this respect, whilst platinum and bismuth resemble iron. In lead, this internal electromotive force is zero.

This last effect is known as the *Thomson effect* in the metals, whilst the transformation of energy taking place at the junctions where the different metals meet is called the *Peltier effect.* Hence the resultant or total electromotive force generated in a thermo-couple is partly Thomson electromotive force and partly Peltier electromotive force, and is the joint effect due to the Peltier effect at the two junctions and the Thomson effect in the two separate metals. Pieces of two different metals may be joined up in such a way as to constitute what is called a *thermo-electric battery*, or *thermo-pile.* Thus, if a number of pieces of iron and German silver are joined together in a zigzag fashion alternately (see Fig. 24), and connected with a galvanometer, and if one set, say, the even junctions, are heated, whilst all the others, say the odd junctions, are cooled, then each thermo-electric couple will set up an electromotive force, and the elec-

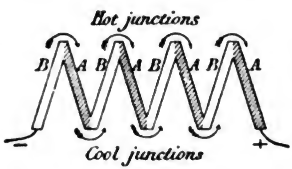

Fig. 24.—Bismuth-Antimony Thermopile.

tromotive force of the whole battery or thermo-pile is equal to the sum of all the electromotive forces of the separate couples of which it is composed.

A convenient arrangement of this kind, called a Gulcher thermo-pile, is useful in the laboratory. It consists of a number of bars of nickel and of a certain alloy of antimony, these thermo-couples being so arranged that one set of junctions are heated by a row of gas jets, whilst the others are kept cooled by the outside air. An instrument of this latter kind can be made to give a continuous electromotive force of 4 to 6 volts, as long as the gas jets are kept burning, and is very convenient to use for many purposes for which a small continuous electromotive force is useful.

§ 5. **Magneto-electric Currents.**—Faraday discovered in 1831 a third method for producing electric currents, of greater practical importance than those discovered either by Volta or Seebeck. Faraday found that if a metallic circuit, say a copper ring, is placed in the neighbourhood of magnets or of conductors conveying electric currents in such a position that a magnetic flux passes through the ring or circuit, the lines of magnetic flux being linked through with it, and if the total amount of the magnetic flux passing through the ring is altered or in any way changed, either increased, decreased, reversed, or destroyed, then under these circumstances an electromotive force is set up in the ring or circuit.

This electromotive force produces in the circuit of the ring an electric current if the circuit is complete. Any movement of the ring or any change of the magnetic field causing the total flux passing through and linked with the ring to change in any way, creates in the ring this electromotive force, which is called the *induced electromotive force.* By an elaborate series of experiments Faraday established that the electromotive force set up in the ring circuit is at any instant exactly proportional to the rate at which the magnetic flux through the ring is changing at that instant. The

student can verify the general facts by taking a coil of wire consisting (say) of one hundred turns of insulated wire formed into a small ring three or four inches in diameter, and connecting this wire coil with the galvanometer already described. Let the ring coil then be slipped over the pole of a permanent magnet, and it will be found that on so doing the galvanometer makes a deflection, indicating the presence of current in the coil. But if the coil is held steadily in any position near the magnet, then the galvanometer indicates no further current in the circuit. Any movement, however, of the coil to or from the magnet is accompanied by the pro-

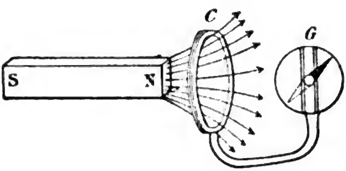

Fig. 25.—Coil C moving to or from a Magnet Pole N, so that a Current is induced in it.

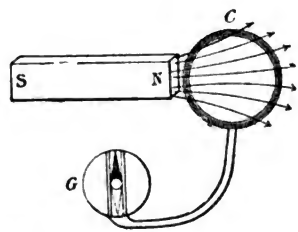

Fig. 26.—Coil C moving to or from a Magnet Pole N, so that no Current is induced in it.

duction of an electric current in the coil, which lasts just as long as the motion lasts, always providing that that motion is of such a kind as to continually change the magnetic flux passing through the coil. Thus, for instance, if the coil is held with its plane perpendicular to the axis of the magnet, but a long way from the magnet, and then moved suddenly close up to the pole of the magnet (as in Fig. 25), the result of this motion is to cause an increase in the magnetic flux passing through the ring, and hence to create an induced electromotive force in the ring. This electromotive force is greater when the ring is moved quickly than when it is

moved slowly. If, however, the ring coil is held edgeways towards the magnet, and moved up (as in Fig. 26) towards the magnet pole, this kind of motion does not change the magnetic flux passing through the coil; because no lines of magnetic flux are linked or unlinked with the circuit of the coil, and hence under these conditions we find no current produced in the ring.*

The student should experiment with a coil and a bar magnet, and convince himself that there are certain motions of the ring which result in the production of electromotive force in the ring, and hence of an electric current; and, on the other hand, there are certain other motions of the ring in the neighbourhood of the magnet which do not result in the production of an induced electromotive force in the coil circuit. In the chapter specially devoted to Electromagnetic Induction we shall examine still more in detail this remarkable discovery, and show how it leads, in addition, to a practical way of measuring magnetic flux whether existing in the air or in an iron circuit completely closed.

* It need hardly be said that neither here nor elsewhere in this book do the diagrams represent the relative sizes or distances of the apparatus illustrated. In the above experiments (Figs. 25 and 26) it would be necessary in practice to place the galvanometer a sufficient distance from the magnet to prevent the needle being directly influenced by the magnet.

CHAPTER V.

THE MEASUREMENT OF ELECTRIC CURRENTS.

§ 1. **The Magnetic Flux round an Electric Current.** Whatever may be the form given to an electric circuit or wire conveying an electric current, there is always for the same circuit a definite field of magnetic flux round that circuit when traversed by a current; and it is necessary to examine the character of the flux in various cases. The reader should construct, therefore, in the first place, a coil of wire of about No. 20 S.W.G. (Standard Wire Gauge), having the form of a flat ring, preferably of nearly square cross-section.

In buying wire for experimental purposes in electrical work, the student should purchase *double cotton covered copper wire* of the necessary size. There are certain standard sizes of wire in use, the diameters of the bare wires being stated by *numbers*, according to the scale called the *Standard Wire Gauge* (S.W.G.), or the *Birmingham Wire Gauge* (B.W.G.), but the sizes for very accurate work are best stated in *mils*, one mil being the one-thousandth part of an inch. The following tables (p. 107) give data for the sizes most in use in electrical work.

The wire for electrical purposes is sold insulated with either cotton or silk. Double-cotton covered (d.c.c.) copper wire is good enough for most purposes unless high insulation is required, when the more expensive double-silk covered wire should be used. The wire should always be of that quality called high conductivity (H.C.) copper wire.

The double-cotton covering adds about 10 mils to the thickness of the wire, and the double-silk covering adds about 5 mils to the thickness. When a hank or bobbin of wire is purchased it should be first baked in the oven for an hour at a temperature rather above that of boiling water, and then boiled

STANDARD WIRE GAUGE.

No.	Diameter in inches.	No.	Diameter in inches.	No.	Diameter in inches.	No.	Diameter in inches.
1	·300	11	·116	21	·032	31	·0116
2	·276	12	·104	22	·028	32	·0108
3	·252	13	·092	23	·024	33	·0100
4	·232	14	·080	24	·022	34	·0092
5	·212	15	·072	25	·020	35	·0084
6	·192	16	·064	26	·018	36	·0076
7	·176	17	·056	27	·0164	37	·0068
8	·160	18	·048	28	·0148	38	·0060
9	·144	19	·040	29	·0136	39	·0052
10	·128	20	·036	30	·0124	40	·0048

TABLE OF COPPER WIRE GAUGES. Round Bare Wire.

Number of Wire Gauge.	Diameter of Wire in inches.		B.W.G. Wire. Yards of Bare Wire which make 1 lb. in weight.	B.W.G. Wire. Resistance in ohms per lb.*
	Birmingham Wire Gauge. B.W.G.	Standard Wire Gauge. S.W.G.		
10	·134	·128	6·14	·0109
12	·109	·104	9·28	·0249
14	·083	·080	16·0	·0741
16	·065	·064	26·1	·1971
18	·049	·048	47·9	·6629
20	·035	·036	85·1	2·095
22	·028	·028	131·1	4·976
24	·022	·022	176·4	9·009
26	·018	·018	305·5	27·01
28	·014	·0148	430·8	53·72
30	·012	·0124	562·7	91·61
32	·009	·0108	765·9	169·7
34	·007	·0092	1103·0	351·9
36	·004	·0076	1767·0	903·5

* The figures in the last two columns are taken from the London Electric Wire Company's list. Other published values differ a little from those given above.

in white paraffin wax, to impregnate the cotton or silk, and prevent it from taking up moisture. To do this, procure a pound or two of pure white paraffin wax, and place it in a clean iron saucepan. Melt it carefully over the fire or a gas flame, and add a small lump or two of rosin. Then immerse the bobbin or coil of wire in this melted wax, and boil it until all bubbles cease to rise from the wire. Take care not to let the paraffin rise in temperature much above the temperature of boiling water, or so high as to smoke strongly. When the wire is impregnated pull out the bobbin or coil with a bit of bent wire and let it cool.

In making a flat ring coil of wire, the best way is to cut out a circular piece of wood, say of 4 inches in diameter, slightly conical, and ½ an inch thick. Screw on to each side of this disk a rather larger disk of thin wood or thick cardboard. On this flat circular bobbin wind a coil of say fifty or one hundred turns of No. 20 cotton-covered copper wire, bringing the ends out through holes in the side disks. Then boil this coil, boards and all, as made, in paraffin wax, in a saucepan (which is best kept for the purpose), and when well boiled take it out and let it cool. When *quite* cold the side disks may be taken off and the coil of wire detached. The hard wax will cause the turns of wire to stick together, so that they retain the circular shape. The ring coil is then best preserved by winding it over with tape, which may be finally painted with shellac varnish. The inner and outer ends of the wire must be brought out through the tape. In making a coil for experimental purposes always count the number of turns of wire, and mark it on the coil.

Fix this flat coil between two pieces of thin board joined together but having nicks or cuts made in them to let the coil through (see Fig. 27). The coil must be so arranged that it is inclosed by the boards—half the coil being above and half below—and the centre of the coil on a level with the upper surface of the wood. The wood must then be neatly covered with white paper on its upper surface. Pass a rather strong current (four or five amperes at least) through a ring coil so arranged, and sprinkle fine steel filings all over the paper. Tap the board, and the filings will arrange themselves in a

series of curves, as shown in the figure. These filings delineate the lines of magnetic flux round the circular current. It will be seen that the flux goes through the aperture of the coil and then divides and returns round the outside, thus completing its circuit. If a small exploring magnetic needle or pocket compass is held in various positions near the coil, it will be found that the small magnetic needle places itself in every position along, or *tangent to*, the line of magnetic flux ; and by following round a line of magnetic flux it will be seen to be *linked with* the wire of the electric circuit or axis of the electric current. It is therefore commonly said

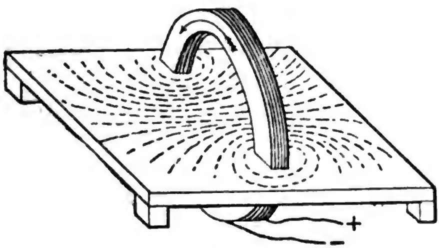

Fig. 27.—The dotted lines show the Direction of the Magnetic Flux round a Circular Conductor conveying a Current ; the lines being taken on a horizontal plane through the centre.

that a current flowing in a circuit generates lines of magnetic flux round the circuit which are linked with that circuit.

It is, however, better to think of the wire as simply forming the circular axis round which there is a re-entrant magnetic flux, or circuit of magnetic flux. It will also be seen that in no place except just at the very centre of the coil are the lines of magnetic flux parallel and straight.

The reader should, in the next place, explore the form of the magnetic flux round a long coil or solenoid,

made as described on page 13. Take such a long bobbin, and cut a piece of stout card just wide enough to fit easily into the hole in bobbin and form a *floor* half-way across. Sprinkle this card with steel filings, and then insert it into the hole in the bobbin and pass a strong electric current through the wire. Tap the card, and then stop the current. Withdraw the card gently and examine it, and it will be found that the steel filings are arranged (see Fig. 28) in a series of nearly straight lines, but branching out a little near to the mouth of the aperture. This shows us that the magnetic flux in the

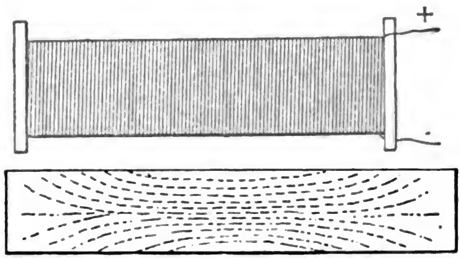

Fig. 28.—Lines of Magnetic Flux in the Interior of a long Solenoid delineated by Iron Filings.

interior of such a long coil is in nearly parallel lines, and the magnetic flux is said to form a *uniform magnetic field* in the interior of the bobbin. By fitting a piece of card outside the bobbin as well, the student may succeed in delineating the form of the exterior field of the bobbin or long coil, and it will be found to be *exactly similar to that of a bar magnet.* Hence it is shown that such a long solenoid or bobbin of wire traversed by a current produces a magnetic field outside the coil which is entirely similar to that of a bar magnet of about the same general shape; also that in the interior of the coil, at places not too near the mouth of the coil, it produces a

perfectly uniform magnetic field. It can be shown that the magnetic flux density in the interior of the long coil is in absolute (C.G.S.) units equal to $1\frac{1}{4}$ times the product of the number of turns of the wire per centimetre length of the coil and the current in *amperes* flowing through the coil. A definition of the *ampere* as the practical unit of current will be given later. Hence, if a long coil has a length between the cheeks of one hundred centimetres, and has on it five hundred turns, the turns per centimetre length are 5. If, therefore, one ampere is passed through the wire of this coil, it will make in the interior a uniform magnetic flux density of $1\frac{1}{4} \times 5$ C.G.S. units. This last product divided by 100 gives the flux density in *microwebers* per square centimetre.* A coil of this description is therefore useful in creating a *standard magnetic flux.*

There is another way in which a practically uniform magnetic flux may be created.

Let the reader prepare two such flat ring coils as described above, and fix them vertically on a board so that their planes are parallel to one another and their centres are separated by a distance equal to the mean radius of each coil. On sending a current through the two coils in the same direction, and placing a piece of cardboard in the plane of the centre and sprinkling iron filings over it, it will be seen that this arrangement of two parallel coils creates a very uniform magnetic field over a large space between the coils.

Based on this fact, we can construct an instrument for the measurement of electric currents called a tangent galvanometer.

§ 2. **Construction of a Tangent Galvanometer.**—A simple form of tangent galvanometer may be con-

* As a practical hint let the student note that for such a long solenoid the interior magnetic flux density is given by the rule—

Interior magnetic flux density in microwebers per square centimetre } = ampere turns per inch divided by 200.

structed in the following way. Let a wood-turner turn out of hard wood (say beech or sycamore) two disks of wood with a bevelled edge. The disks may be 8 inches in diameter on the wider side and 7 inches in diameter on the narrower side. These disks must be fixed by three *brass* screws to square pieces of wood nine or ten inches square, with the wide side of the circular disks outwards. The result is to form a V-shaped groove between the round and square boards. In this groove is to be wound covered copper wire No. 18 S.W.G. size. Fill up each groove with five turns of wire to form one coil, and then put thirty-five turns of a separate coil over the first coil, and bring out the ends of each coil. Each board will then have on it two overlaid circular coils of wire, one of five turns and the other of thirty-five turns. These boards are then to be fixed by a middle board between them, exactly a radius apart. This will make the inner surfaces of the circular disks about three inches apart. The middle board must be placed at such a height that its upper surface is in the line of the centres of the round disks. The ends of the wire coils are then brought to terminals; the ends of the two large coils being joined up so that they form one coil all wound one way, and the ends of the short coil being similarly joined and forming a separate circuit. In the centre of the middle board is then placed a circular card divided into degrees.* (See Fig. 29.)

Across the top of the square side boards is placed a small bar of wood, through the centre of which is passed a pin, and from this pin a silk cocoon fibre can be suspended to carry a small magnet needle as an indicator of the direction of the field in the centre. This needle is best made by bending a piece of softened knitting-needle into a little horse-shoe half an inch long, and with the legs a quarter of an inch apart. After making this

* Cardboard protractors, or circles divided into degrees, and also paper scales divided into centimetres and millimetres, can be obtained of Messrs. J. J. Griffin and Co., of 22 Garrick Street, Covent Garden, London.

horse-shoe, harden it again by heating red-hot and quenching in oil, and then magnetise it. Across the poles of this horse-shoe stick a fibre of glass 3 inches long as an indicator. Suspend this needle so that the poles nearly touch the surface of the divided card, and its indicating needle extends across the circular divided scale. It is then convenient to fix plates of window glass, cut the proper size, to the top and sides of the instrument so made, so as to prevent air currents disturbing the suspended needle.

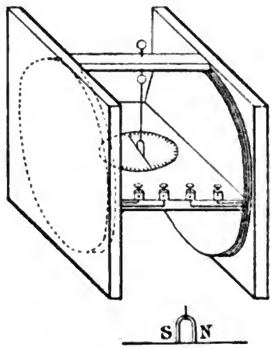

Fig. 29.—Simple Two-Coil Tangent Galvanometer.

If an electric current is sent through one of the two circuits it creates a magnetic flux or field in between the coils, the lines of which, in the central region, are parallel to one another and perpendicular to the plane of the wire windings. If the instrument is set on the table with the plane of the coils parallel to the direction of the earth's magnetic field, or to the magnetic meridian, then when a current is passed through the coils there will be in the central region between the coils two superimposed magnetic fields or fluxes—one due to the earth, parallel to the coils, and one due to the currents in the coils, perpendicular to the coils. The resultant field or joint-effect, therefore, is a field in an inclined position, and if the small suspended magnetic needle is placed in the centre of the coils, it sets itself in the direction of the resultant field. Since the resultant of two forces represented by the sides of a rectangle is in the direction of the diagonal, we can depict the relation of the coil field, the earth's field, and the resultant field as follows. Let

the line O E (see Fig. 30) represent in magnitude and direction the earth's magnetic horizontal field strength, or, which is numerically the same thing, the earth's magnetic horizontal flux density, at the centre of the coils. Let O C represent the magnetic field due to the coils, and which is at right angles to the earth's field. Then O R represents the direction of the resultant field, and will represent the direction in which the galvanometer needle will stand when placed at the centre of the coils. In a right angle triangle such as O E R the ratio of the length of the side E R to the side O E is called the tangent of the angle R O E, and since O C is equal to E R, the ratio of O C to O E is the tangent of the angle R O E. In the table of tangents (see Appendix) the value of the tangents for every angle is given. Thus, for instance, the tangent of 45° is 1. This means to say that if the angle R O E is 45°, then E R is equal to O E, or the *ratio of* E R *to* O E *is unity.* Hence, knowing the angle R O E and the value of O E, we can calculate O C. For it is obvious that in this case O C is equal to the product of O E and the tangent of the angle R O E. Hence, a tangent galvanometer is one in which the magnetic flux density due to a current in a coil at a certain assigned position is compared directly, as regards magnitude, with a known standard magnetic flux density at right angles to it, such as that of the earth, at the same place, by observing the deflection of a small magnetic needle in the resultant field, and thus obtaining the tangent of the angle of the deflection of the small needle, which number, multiplied by the number denoting the strength of the earth's magnetic flux density, gives a number denoting on the same scale of

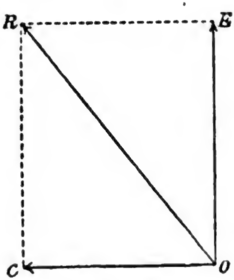

Fig. 30.

measurement the magnetic flux density due to the coils, both being measured at the central region half-way between the coils.

The current in the coils is proportional in strength to the magnetic flux density at any assigned or fixed point near the circuit. Hence such an instrument enables us to measure electric currents. The condition of success is that the instrument must obey the tangent law, and this is not the case unless the field due to the coils is perfectly uniform over a space larger than the length of the needle. Hence the indicating needle in a tangent galvanometer should always be short in length compared with the diameter of the coils, and at most not longer than one-tenth of the diameter of either coil. The needle of a tangent galvanometer may conveniently be made of a small horse-shoe magnet, with a long indicating needle formed of a glass fibre across the poles.

§ 3. **Definition of the Practical Unit of Current. One Ampere.**—We have not hitherto mentioned one important property of an electric current. If part of an electric circuit is composed of a liquid of certain kinds, it is found that this liquid is chemically decomposed when the current flows through it. In a glass or other vessel place a little solution of sulphate of copper, made by pouring boiling water on the blue crystals and decanting the clear solution. Scrape two lead plates, 1 inch wide and 6 inches long, quite clean and place them in the solution, near but not touching. Join to the lead plates the wires from a battery of three or four dry cells joined up in series, and let the arrangement stand a little while; then lift out the lead plates and examine them. It will be found that one plate, viz. that connected with the negative pole, or zinc end of the battery, is covered with a thin film of metallic copper; the other lead plate is merely slightly discoloured.

This process of electrically decomposing a liquid is called *Electrolysis*. Any liquid which can be thus

decomposed is called an *Electrolyte*. The elements or bodies into which an electrolyte is separated are called the *Ions*, and the plates or wires which are put into the electrolyte and against or on which the *Ions* are liberated are called the *Electrodes*. The electrode by which the current enters the electrolyte is called the *Positive Electrode*, or *Anode*, and the electrode by which it leaves the electrolyte is called the *Negative Electrode*, or *Cathode*. The vessel in which the experiment is done is called an *Electrolytic Cell*. These terms were invented by Faraday. It is not every liquid which is an electrolyte. For instance, paraffin oil is not an electrolyte, but salt water is an electrolyte. Dilute sulphuric acid is also an electrolyte, and the ions liberated from it are oxygen and hydrogen.

Solutions of metallic salts in water, such as nitrate of silver, sulphate of copper, or chloride of zinc, are electrolytes, and water acidulated with any acid is also an electrolyte.

Faraday discovered by laborious experiments that the passage of a given quantity of electricity through an electrolyte is always accompanied by the decomposition of a constant corresponding mass of that electrolyte, and that, in fact, without this decomposition the current cannot pass at all. Moreover, he found that if the same current is passed successively through two or more electrolytic cells in series having in them different electrolytes, the amount of electrolyte decomposed in each cell bears a definite relation to the chemical equivalents or chemical combining proportions of the ions composing the electrolytes.

The weight in grammes of any ion or body deposited at an electrode in one second by a unit current is called the *Electrochemical Equivalent* of that body. Great labour has been expended in determining the relative proportions by weight in which such bodies as silver, copper, zinc and hydrogen are liberated or deposited by a constant known current per unit of time.

It has been shown both by Faraday and by others

that the mass of any given ion deposited is exactly proportional to the product of the current strength, and to the time during which it has flowed. In other words, a given *quantity* of electricity passing through an electrolyte always deposits on the electrodes a definite corresponding *mass* of the ions. The *ion* which appears against or is deposited on the *positive electrode*, or electrode in connection with the positive pole of the battery, is called the *Electronegative ion.*

The other ion is called the *Electropositive ion.* Hydrogen and the metals are electropositive ions, and, so to speak, move *with* the current through the electrolytic cell. Oxygen and salt radicles are electronegative ions, and move *against* the current through the electrolyte.

The law of Faraday may be stated by saying that the deposition of a unit mass of any given ion against an electrode always requires the passage of the same quantity of electricity through the electrolyte. This at once provides a means of defining and measuring a current, and accordingly a unit of electric current has been selected which is called *One Ampere*, and which is legally defined as follows:—

The ampere is the name of an unvarying electric current which, when passed through a solution of nitrate of silver in water made according to a certain specification, deposits silver at the rate of ·001118 *of a gramme per second.*

The above is the Board of Trade definition of an ampere as ordered by a resolution of the Queen in Council in 1894.

If a current of one ampere flows for one hour through an electrolyte, the *quantity* of electricity which has passed is said to be one *ampere-hour.* It is easily seen that the ampere-hour of quantity would deposit 4·025 grammes of silver. Since there are 3600 seconds in one hour, and $3600 \times \cdot 001118 = 4 \cdot 025$.

The quantity of electricity which is conveyed past

any section of the circuit by one ampere per second is called a *coulomb*. Hence 3600 coulombs are equal to *one ampere-hour*.

Electrochemical equivalents may therefore be stated per coulomb or per ampere-hour, and these electrochemical equivalents are the weights in grammes of the ions liberated or deposited by the above quantities of electricity.

The following is a table of the electrochemical equivalents of many ordinary elements :—

Ions.	Electrochemical Equivalents in Grammes.	
	Per Coulomb.	Per Ampere-hour.
Hydrogen	·000010338	·03738
Potassium	·0004539	1·45950
Sodium	·00023873	·85942
Aluminium	·00009449	·34016
Magnesium	·00012430	·44748
Gold	·00067911	2·44480
Silver	·00111800	4·02500
Copper (from cupric salts)	·00032709	1·17700
" (from cuprous salts)	·00065418	2·35400
Mercury	·00103740	3·73450
Iron (from ferric salts)	·00019356	·69681
" (from ferrous salts)	·00029034	1·04521
Nickel	·00030425	1·09530
Zinc	·00033696	1·21330
Lead	·00107160	3·85780
Oxygen	·00008286	·29829

Having these data before us, the process of measuring an electric current in amperes by means of electrolysis is as follows :—

A saturated solution of sulphate of copper is placed in a glass vessel. This solution is made by pouring

boiling water on the crushed blue crystals of the sulphate of copper, and after stirring well and allowing it to stand, filtering off the cold solution. It is found desirable to add about 5 per cent., or one-twentieth part by volume, of strong sulphuric acid to this liquid. Two copper plates are then made chemically clean, which is done by immersing them in strong nitric acid, and letting the acid boil violently upon them.

This operation should be carried out in the open air or in a fume cupboard, as the red nitrous fumes which come off are very poisonous. The copper plates should be allowed to lie in the acid until, when picked out by a bent wire, they present a perfectly clean, bright, lustrous salmon-coloured appearance all over, and have no brown spots or patches anywhere. They should then be dropped into a large jug of cold water.

These plates having been chemically cleaned are then washed, dried and weighed.*

They are then placed in the sulphate of copper solution and supported so as to be about half an inch apart, and nearly all immersed in the liquid. If a constant steady electric current is passed through this electrolytic cell, one of the copper plates will gain in weight by the deposition of copper upon it, and one will lose in weight by copper being taken off it. If the time during which the current passes is noted by a good watch, and if the current is kept constant in strength during that time, and if at the end of the period the gain in weight of the negative electrode is noted, we can at once determine the strength of the current in amperes. To do this, divide the gain of weight of the plate measured in grammes by the number of seconds during which the experiment has lasted and by the number ·00032709, which is the electrochemical equivalent of copper per ampere per second. The result is the mean value of the current in amperes.

* For fuller particulars as to the precautions to be observed in carrying out this experiment the reader should consult the author's 'Electrical Laboratory Notes and Forms,' No. 24 Advanced Series.

The reader who has, in accordance with the foregoing instructions, made a tangent galvanometer, should then proceed to *calibrate this galvanometer*, that is to say, find out the deflection the needle makes when a current of one ampere is passed through the coils.

To do this, he should join up the tangent galvanometer and a copper electrolytic cell made as above described, and send the same current through both, adjusting the current to give a deflection of about 45° on the galvanometer scale. Then dry and weigh both copper plates, and with a good watch at hand start the current at a noted instant. Keep the current constant for four or five hours by slightly moving the copper plates to or from each other during that period. At the end of the experiment again dry, and weigh the plates on a delicate chemical balance, and from the gain in weight of the negative electrode and the known time of the experiment determine the mean value of the current in amperes. The *constant of the galvanometer* is the number by which the tangent of the angle of deflection of the needle must be multiplied to give the value of the current through the coils in amperes. Hence, having observed the deflection due to a known current, we can calculate this *constant* by dividing the value of the current in amperes by the tangent of the observed deflection, and always thereafter determine from any other observed deflection of the needle the value of the current in amperes it represents. If the reader has access to a good ammeter or instrument already calibrated to read direct in amperes, he will find it to be an instructive experiment to pass a number of different currents, measured in amperes, through the two-coil tangent galvanometer already described, and observe the deflections of the needle so produced. If he then compares the tangents of the angles of these deflections with the value of the known currents, it will be found that they are strictly proportional, over a range at least from 0° to 70° deflection.

This would not be the case if the tangent galvano-

meter were made with a single coil, as is usually the case with most shop instruments, and with a needle not very small in length compared with the diameter of the coils.

§ 4. **Practical Electric Units.**—We have already seen that to produce a current in a circuit an active cause called *Electromotive Force* must be present, and we have described the construction of a voltaic cell called a calomel standard cell, the electromotive force of which when not sending a current, or but a feeble one, is taken as a standard of comparison. The Helmholtz or calomel cell may be adjusted so as to have an electromotive force which is exactly equal to that called one volt. The electromotive force of a Clark standard cell is 1·434 volts at 15° C.

By the term *Resistance* of an electric circuit is meant that quality of it, in virtue of which it prevents the creation of more than a certain current in it by a given electromotive force, and it is found that this depends upon the form of the circuit, and upon the material of which it is made.

The electrical quality of the material of which the circuit consists, which determines in this respect the resistance of a standard form of it, is called its *resistivity*. The *resistivity* (ρ) is defined as the resistance of one cubic centimetre of the substance to conduction across opposed faces of the cube. Hence it follows that in the case of a uniform wire of cross section S square centimetres and length L centimetres, the resistance R in the selected units is obtained by multiplying ρ by L and dividing the product by S.

For the practical measurement of resistance we must select a standard substance, whose resistivity shall be the standard of comparison, and for this purpose *pure mercury* is chosen. The mercury is put into a tube of known form and length, and the unit of resistance is defined to be the resistance of a column of pure mercury at 0° C., the length of which is 106·3 cm., and cross

section one square millimetre. It is found best to define the section by means of the weight of mercury, and accordingly the Board of Trade definition of a unit of resistance is as follows:—

It is the resistance at the melting-point of ice of a column of pure mercury 106·3 *cm. long and weighing* 14·4521 *grammes, and of constant cross section. This resistance being the resistance to an unvarying electric current.*

The above unit of resistance is called *one Ohm.* As a matter of practical convenience it is customary to use, as a working standard of resistance, a copy made in metallic wire of the above mercury unit of exactly equal resistance, and this is called a *Standard Ohm Coil.* The resistivity of a substance is, therefore, the resistance of one cubic centimetre across opposed faces of the cube, measured in ohms or microhms.

The Board of Trade has also defined the unit of electromotive force, called *one Volt*, as follows:—

The unvarying electromotive force which creates in a circuit having a resistance equal to one ohm, an unvarying current of one ampere is called one volt. The *ampere*, the *ohm* and the *volt* are the three fundamental practical units of electrical measurement.

As regards the derivation of these names, the student must notice that the custom has arisen of calling the practical units of electrical measurements by names abbreviated from those of celebrated men. We have a similar usage in common life in which objects much in common use, such as a cab, carriage, hand-bag or cloak, are nicknamed after either their inventor or some illustrious person, and called *a Hansom*, *a Victoria*, *a Gladstone*, or *a Macintosh.* The practical electrical units are as follows:—

The unit of current is called an *ampere*, after Marie André Ampère. The unit of resistance is called an *ohm*, after George Simon Ohm. The unit of electromotive force is called a *volt*, after Alessandro Volta. The unit of quantity is called a *coulomb*, after Charles Augustin Coulomb. The unit of capacity is called a *farad*, after Michael Faraday. The unit of magnetic flux is

called a *weber*, after Wilhelm Weber. The unit of inductance is called a *henry*, after Joseph Henry. The unit of power is called a *watt*, after James Watt; and the unit of *work* or *energy* is called a *joule*, after James Prescot Joule.

The following is the legal definition of the Ohm, Ampere and Volt, as fixed by the authority of the Queen in Council.

At the Court at Osborne House, Isle of Wight, the 23rd day of August, 1894.*

Present, THE QUEEN'S MOST EXCELLENT MAJESTY IN COUNCIL.

Whereas by "The Weights and Measures Act, 1889," it is among other things enacted that the Board of Trade shall from time to time cause such new denominations of standards for the measurement of electricity as appear to them to be required for use in trade to be made and duly verified;

And whereas it has been made to appear to the Board of Trade that new denominations of standards are required for use in trade based upon the following units of electrical measurement, viz.:—

1. The Ohm, which has the value of 10^9 in terms of the centimetre and the second of time, and is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice 14·4521 grammes in mass of a constant cross-sectional area, and of a length of 106·3 centimetres.†
2. The Ampere, which has the value $\frac{1}{10}$ in terms of the centimetre, the gramme and the second of time, and which is represented by the unvarying electric current which, when passed through a solution of nitrate of silver in water, in accordance with the specifica-

* From the 'London Gazette,' Friday, August 24, 1894.

† Previously to this official definition of the Ohm, resistances were measured in terms of a unit called the *British Association Unit* (B.A.U.), and the relation between the B.A.U. and the Ohm as defined above is—

1 B.A.U. = ·9866 ohm.
1 ohm = 1·01358 B.A.U.

tion appended hereto, and marked A, deposits silver at the rate of 0·001118 of a gramme per second.

3. The Volt, which has the value 10^8 in terms of the centimetre, the gramme and the second of time, being the electrical pressure that if steadily applied to a conductor whose resistance is one ohm, will produce a current of one ampere, and which is represented by 0·6974 ($\frac{1000}{1434}$) of the electrical pressure at a temperature of 15° C. between the poles of the voltaic cell, known as Clark's cell, set up in accordance with the specification appended hereto, and marked B.

And whereas they have caused the said new denominations of standards to be made and duly verified;

Now, therefore, Her Majesty, by virtue of the power vested in Her by the said Act, by and with the advice of Her Privy Council, is pleased to approve the several denominations of standards set forth in the schedule hereto, as new denominations of standards for electrical measurement.

The Board of Trade specification for obtaining by the electrolysis of a silver salt the value of a current in amperes is as follows:—

In the following specification the term silver voltameter means the arrangement of apparatus by means of which an electric current is passed through a solution of nitrate of silver in water. The silver voltameter measures the total electrical quantity which has passed during the time of the experiment, and by noting this time the time average of the current, or if the current has been kept constant, the current itself, can be deduced.

In employing the silver voltameter to measure currents of about one ampere the following arrangements should be adopted. The cathode on which the silver is to be deposited should take the form of a platinum bowl not less than 10 cm. in diameter, and from 4 cm. to 5 cm. in depth.

The anode should be a plate of pure silver some 30 sq. cm. in area and 2 mm. or 3 mm. in thickness.

This is supported horizontally in the liquid near the top of the solution by a platinum wire passed through holes in the plate

at opposite corners. To prevent the disintegrated silver which is formed on the anode from falling on to the cathode, the anode should be wrapped round with pure filter paper, secured at the back with sealing wax.

The liquid should consist of a neutral solution of pure silver nitrate, containing about 15 parts by weight of the nitrate to 85 parts of water.

The resistance of the voltameter changes somewhat as the current passes. To prevent these changes having too great an effect on the current, some resistance besides that of the voltameter should be inserted in the circuit. The total metallic resistance of the circuit should not be less than 10 ohms.

Method of making a Measurement.

The platinum bowl is washed with nitric acid and distilled water, dried by heat, and then left to cool in a desiccator. When thoroughly dry it is weighed carefully.

It is nearly filled with the solution, and connected to the rest of the circuit by being placed on a clean copper support to which a binding screw is attached. This copper support must be insulated.

The anode is then immersed in the solution, so as to be well covered by it, and supported in that position; the connections to the rest of the circuit are then made.

Contact is made at the key, noting the time of contact. The current is allowed to pass for not less than half-an-hour, and the time at which contact is broken is observed. Care must be taken that the clock used is keeping correct time during this interval.

The solution is now removed from the bowl, and the deposit is washed with distilled water, and left to soak for at least six hours. It is then rinsed successively with distilled water and absolute alcohol, and dried in a hot-air bath at a temperature of about 160° C. After cooling in a desiccator it is weighed again. The gain in weight gives the silver deposited.

To find the current in amperes, this weight, expressed in grammes, must be divided by the number of seconds during which the current has been passed, and by 0·001118.

The result will be the time-average of the current, if during the interval the current has varied.

In determining by this method the constant of an instru-

ment, the current should be kept as nearly constant as possible, and the readings of the instrument observed at frequent intervals of time. These observations give a curve from which the reading corresponding to the mean current (time-average of the current) can be found. The current, as calculated by the voltameter, corresponds to this reading.

It is found by experiment that the resistivity of bodies is very much affected by their temperature. In the cases of all pure metals the resistivity is increased by heating them, and diminished by cooling them, and if they could be cooled to the temperature $-273°$ C., called the absolute zero of temperature, which is 273 degrees centigrade *below* the melting-point of ice, they would have no resistivity at all. In the case of alloys of metals, change of temperature has not nearly so great an effect, and in some alloys, for certain ranges of temperature the resistivity decreases instead of increasing as they are heated. In the case of carbon and some other bodies, particularly also in the case of electrolytes, the effect of heating them is to *decrease* their resistivity. The percentage change in resistivity produced by heating a body one degree centigrade in temperature is called its *temperature coefficient*, and is expressed as a percentage reckoned on its resistivity at 0° C. Thus, the mean temperature coefficient of pure metals *between* 0° C. and 100° C. is 0·4 per cent. per degree centigrade. It has been found by careful experiments that when proper correction is made for the change in temperature of a metallic electric circuit by the passage of a current through it, the resistivity of the material is the same whether the current through it is large or small. This is a very important fact, because it points to a great difference between the relation of magnetic reluctivity to magnetic flux density in magnetic circuits made wholly or partly of iron, and the relation of electric resistivity to electric current in electric circuits made of metals. The magnetic reluctivity of the iron is not independent of the magnetic flux density in it, but the electric resistivity is indepen-

dent of the current strength in the electric circuit, provided the necessary temperature correction is made.

The following tables * give the resistivity of most

TABLE SHOWING THE ELECTRICAL RESISTIVITY OF PURE ANNEALED METALS.

Metal.	Resistivity in Microhms per cubic centimetre at 0° C.	Mean Percentage Temperature Coefficient between 0° C. and 100° C.
Silver	1·468	·400
Copper	1·561	·428
Gold	2·197	·377
Aluminium	2·665	·435
Magnesium	4·355	·381
Zinc	5·751	·406
Iron	9·065	·625
Cadmium	10·023	·419
Palladium	10·219	·354
Platinum	10·917	·367
Nickel	12·323	·622
Tin	13·048	·440
Thallium	17·633	·398
Lead	20·380	·411
Mercury ‡	94·070	·389
Bismuth	108·000	..

* From experiments by Professors Fleming and Dewar, Phil. Mag., Sept. 1893, p. 271.

† From the determinations made by Professors Fleming and Dewar.

‡ The resistance of a column of pure mercury 100 centimetres long and 1 square millimetre in cross section, taken at 0° C., is equal to ·9407 ohm. The above resistance is called a *Siemens Unit.* Taking a column of mercury one square millimetre in section at 0° C., the following is the length of the column for various units of resistance:—

One ohm = 106·3 cm.
One British Association unit (B.A.U.) = 104·87 cm.
One Siemens unit = 100 cm.
One Siemens unit = ·9535 B.A.U.

TABLE SHOWING THE RESISTIVITY OF CERTAIN ALLOYS.

Alloy.	Composition.	Resistivity in Microhms per cubic centimetre at 0° C.	Mean Percentage Temperature Coefficient between 0° C. and 100° C.
Aluminium-copper .	94 : 6	2·904	·381
Aluminium-silver . .	94 : 6	4·641	·238
Gold-silver . . .	90 : 10	6·280	·124
Copper-aluminium .	97 : 3	8·847	·089
Platinum-rhodium .	90 : 10	21·142	·143
Nickel-iron . . .	95 : 5	29·452	·201
German silver . .	$Cu_5Zn_3Ni_2$	29·982	·027
Platinum-iridium . .	Pt_4Ir	30·896	·082
Platinum-silver . .	1 : 2	31·582	·024
Platinoid . . .	..	41·731	·031
Manganin . . .	..	46·678	·00
Iron-manganese . .	88 : 12	67·148	·127
Hadfield's Resista .	..	76·468	·110

ordinary pure metals and many alloys, expressed in microhms per cubic centimetre. This number is the resistance in millionths of an ohm of a cubic centimetre of the substance to conduction across opposed faces. The same tables give the *temperature coefficient* of the material, viz. the mean percentage change in resistance per degree centigrade.

The resistivity at any temperature t° C. can be calculated by adding to the resistivity at 0° C., as given in the tables, an amount numerically equal to the product of the resistivity at 0° C., the rise in temperature t°, and the mean percentage temperature coefficient, as given in the table above, divided by 100. Thus the resistivity of copper at 100° C. is equal to

$$1{\cdot}561 + 1{\cdot}561 \times 100^\circ \times \frac{{\cdot}428}{100} = 2{\cdot}229.$$

The above tables show that, with the exception of iron and nickel, the temperature-coefficients of pure metals are not very different, and are approximately equal to 0·4 per cent. per degree centigrade. As regards alloys there is no uniformity, and it is possible to form alloys which even have a *negative temperature-coefficient*, that is, which decrease in their resistance when heated within certain limits.

There are certain manganese copper and nickel alloys which, within certain ranges of temperature, thus have a negative temperature-coefficient: that is to say, the resistivity *decreases* or conductivity increases, with rise of temperature. In the case of carbon and electrolytes generally the same is also the case, and hence, by combining in series a suitable carbon and metallic resistance, it is possible to make a compound circuit which does not change its resistance at all within certain limits when heated or cooled.

The resistivity of copper wire is generally referred to a standard called *Matthiessen's Standard.* Matthiessen measured the resistance of a hard-drawn copper wire one metre long and weighing one gramme, and found that this *Metre-Gramme* of hard copper at 0° C. had a resistance of ·1450 ohm. This is called Matthiessen's standard. In converting it into volume resistivity, or resistance per cubic centimetre, he took the specific gravity of hard copper as 8·89; and hence the above value for the metre-gramme resistivity leads to the value 1626 C.G.S. units as the resistivity per cubic centimetre of hard copper wire, or 1·630 microhms per cubic centimetre. Matthiessen also found that the resistance of a wire of pure soft copper at 0° C. one metre long and weighing one gramme was ·1417 ohm. The specific gravity he took as 8·91, and hence the resistivity of soft copper per cubic centimetre was 1594 C.G.S. units, or 1·594 microhms per cubic centimetre. These numbers are not quite the same as those given on the authority of Fleeming Jenkin, by whom the resistivity of hard copper is given as 1620 C.G.S. units, and that of soft copper as 1584.

The ratio of the resistivity of hard or unannealed copper to

that of soft or annealed copper is given by Matthiessen as 1·0226 to 1.

The values which should be taken as deduced from Matthiessen's experiments with hard-drawn copper wire, and which are properly called Matthiessen's standards, are as follows:—

The resistivity of hard or unannealed copper wire of specific gravity 8·89 taken at 0° C. is 1626 C.G.S. units, or 1·626 microhms per cubic centimetre.

The resistivity of soft or annealed pure copper of specific gravity 8·91 taken at 0° C. is 1594 C.G.S. units, or 1·594 microhms per cubic centimetre.

At the present time soft annealed copper wire can be obtained with a resistivity of 1560 at 0° C., which is between 2 and 3 per cent. lower than Matthiessen's value.

The specific gravity of pure copper may be taken as 8·9 for all practical purposes. The resistance of the metre-gramme of unannealed or hard copper as ·1450 ohm at 0° C., and the resistance of the metre-gramme of soft annealed copper at 0° C. as ·1420 ohm.

§ 5. **Ohm's Law.**—We have then to consider more in detail the relation of the quantities above mentioned.

If a uniform unvarying electromotive force (produced say by a voltaic cell or battery) is allowed to act in an electric circuit, consisting of a metallic wire of uniform section, it produces in it a constant electric current.

If any two points on this electric circuit are selected and joined together by a very long and thin wire, this constitutes what is called a *Shunt Circuit* on these points. This shunt wire can be of such a nature, that when connected it makes very little change in the current flowing in the original circuit between these selected points, and for this purpose the shunt must have large resistance. The current flowing in this shunt wire can be detected by placing in the circuit of the shunt a sensitive mirror galvanometer, and the current so measured is called the *Shunt Current*. It is possible then to insert in the shunt circuit another electromotive force which shall oppose the tendency of the first to

produce a shunt current, and shall reduce this shunt current to zero. When this is the case, the electromotive force introduced into the shunt circuit is a measure of the *difference of potential* between the points selected on the original circuit. This potential difference (P.D.) between the points is, therefore, measured by the electromotive force reckoned in volts, which must be introduced into a shunt circuit of large resistance joining them, to just prevent the flow of any current through the shunt. Hence potential difference is measured in volts, just as is electromotive force.

The student must, however, notice that there is a physical difference between *electromotive force* (E.M.F.) and *difference of potential* (P.D.) There is as much distinction between these quantities as between the *water-moving force* of a pump and the mere *difference of level* between neighbouring points on a river. Instead of the phrase *difference of potential* we have also in use the equivalent terms *fall of potential*, or *fall in volts*, or *voltage*, to express the same idea. There may be a difference of potential between two points on an electric circuit, whilst at the same time there is no electromotive force in that portion of the circuit. We can, however, always measure the difference of potential between two points on a circuit, by measuring the electromotive force in volts which has to be inserted in a high resistance shunt circuit joining those two points, in order that no currents may flow through the shunt. It is to be noted that the current in a conductor is always *from* the place of higher potential *to* the place of lower potential, and in this connection the student may think of *potential* just as if it were *pressure* in a fluid in motion in a pipe.

If any conductor has produced in it steady unidirectional currents of different values, and if the corresponding electromotive forces in this circuit or potential differences between the ends of this conductor are measured, then the relation between these measured

values can be stated by a rule first given by Dr. G. S. Ohm, in 1827, and hence called after his name *Ohm's Law.* This law may be expressed as follows :—

If in any circuit there is a steady unidirectional electromotive force producing a corresponding steady unidirectional current, then, if the electromotive force changes in value, the current changes in the same ratio, provided there is no alteration in the temperature of the circuit.

The constant numerical ratio of the magnitudes of the electromotive force and current is the numerical value of the resistance of the circuit. Hence Ohm's law may be expressed symbolically thus :—

$$\frac{\text{Voltage}}{\text{Current}} = \textit{a constant}, \text{ or } \frac{V}{C} = \textit{const.}$$

Ohm's law applies not only to the circuit as a whole, but to each part of the circuit, when that portion does not contain a source of electromotive force.

It is then stated as follows :—

If in any conductor forming part of a circuit there is a steady unidirectional electric current, and a steady unidirectional difference of potential between the ends of that conductor, then if the difference of potential changes in value, the current changes in the same ratio, provided there is no alteration in the temperature of the conductor.

The constant numerical ratio of the values of the terminal potential difference and the current is the numerical value of the *resistance* of that portion of the circuit or conductor.

If the current is measured in *amperes*, and the electromotive force or difference of potential measured in *volts*, the resistance of the circuit or conductor will be measured in *ohms*.

We have, therefore, the following statement regarding steady unidirectional current flow in circuits :—

$$\left.\frac{\textit{The electromotive-force in the circuit measured in volts}}{\textit{The current in the circuit in amperes.}}\right\} = \left\{\textit{The total resistance of the circuit reckoned in ohms.}\right.$$

or

$$\left.\frac{\textit{The difference of potentials between the ends of the conductor measured in volts}}{\textit{The current in the conductor in amperes}}\right\} = \left\{\textit{The resistance of the conductor in ohms.}\right.$$

Ohm's law is not a mere truism. It is a physical fact, of great importance, that in the case of steady uni-directional current flow in conductors, the electromotive force or, as the case may be, the terminal potential difference varies in exact proportion to the current, as long as the conductor remains the same, and at the same temperature. It also gives us a definition of what is meant by *equality in resistance* between conductors made of different materials. Conductors are said to have the same or equal resistance if the numerical value of the ratio between the currents in them to the corresponding terminal potential differences is the same. Hence, if one volt terminal potential difference corresponds to 10 amperes of current, conductors in which this occurs have the same resistance.

The student may be assisted in remembering these relations between electromotive force, current and resistance, by fixing in his mind the letters R, C, E, in the following positions :—

$$\frac{E}{C\,R}.$$

Cover over with the finger any letter, say C (current), and the position of the remaining letters $\left(\frac{E}{R}\right)$ denotes that the numerical value of the *current* is equal to that of the *electromotive force divided by that of the resistance.*

In the same way it is seen that electromotive force (E) is equal to the product (C R) of current and resistance. The rule may be stated in ohms (O), amperes (A), and volts (V) by the symbol

$$\frac{V}{A\,O},$$

which is used in the same way.

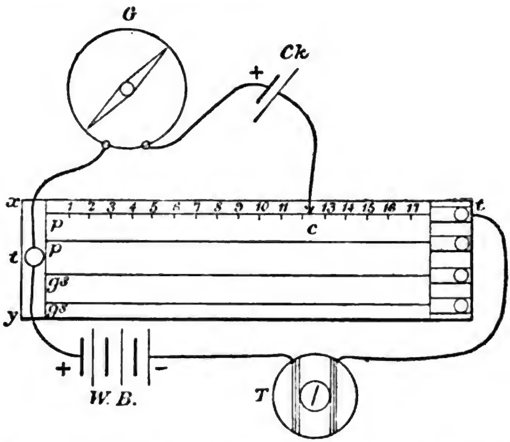

Fig. 31.—Potentiometer Apparatus for Experimental Proof of Ohm's Law. G, Mirror galvanometer; T, Tangent galvanometer; W.B., Working battery; C*k*, Clark cell.

In order to investigate experimentally the relation between the fundamental quantities, current, potential difference, and resistance, the student should construct the following piece of apparatus:—

On a hard straight wooden board (see Fig. 31), 4 feet long and 6 inches wide, paste a paper scale a metre in length and divided into centimetres and millimetres. If such a scale is not

procurable, the board must be covered with white drawing paper, and a scale in inches and tenths divided on it. The board should be stiffened by three transverse pieces, to act as feet, and prevent it warping out of shape. At one end of this board is fastened a strip of copper *x y* (see Fig. 31), the whole width of the board, and having on it a terminal *t*. To this strip are soldered four wires, called *slide wires*, of equal diameter, two of *platinoid*, *p*, and two of *German silver*, *gs*; and these wires should be about No. 32 gauge, and, as far as it is possible to obtain them, precisely of the same diameter. The other ends of the four wires are soldered to separate little strips of copper, each having a terminal *t*. The wires can be placed about half an inch apart on the board. This apparatus is then completed by joining in series with it a tangent galvanometer T, for measuring currents, made as described on page 111, and also a few battery cells, called the working battery W.B.

Three standard Clark cells C*k* must then be provided, and also a sensitive mirror galvanometer G. The instructions for making these instruments have been already given. The apparatus is arranged as shown in Fig. 31. The terminal of the Clark cell not attached to the galvanometer is provided with a wire, ending in a piece of brass with a chisel edge *c*, and which is wide enough to make simultaneous contact on two of the slide wires when placed across them. The Clark cell or cells must be joined up with the positive pole in connection, through the galvanometer, with the common terminal of the four wires, and to that terminal must also be attached the positive pole of the working battery. Each platinoid slide wire should have a resistance of about 6 ohms, and it will carry a current of half an ampere safely. The working battery should consist of 1 to 3 Obach dry cells, large sizes.

Proceed then to make the following experiments:—

Let a current be sent from the working battery W.B. through the circuit of the tangent galvanometer T and one of the slide wires *pt* (say a platinoid wire), and adjusted by resistance to such a value that the tangent galvanometer gives a steady deflection, say of 10° or 20°. Then find a place *c* on the slide wire, such that when touched with the chisel contact of the Clark cell C*k* it

causes no deflection in the needle of the mirror galvanometer G. When this is the case, it is clear that the fall of potential down the length of the slide wire, included between the place of the chisel contact and the common terminal, is equal to the electromotive force of one Clark cell.

Read off on the scale the length of slide wire included between the place of contact of the chisel and the common terminal bar. In the next place, insert *two* Clark cells in series in the shunt circuit, and increase the working currents through the slide wire, and observe the deflection of the tangent galvanometer when the mirror galvanometer makes no deflection, the chisel contact being preserved at the same point in the slide wire. It will be found that the tangents of the angles of deflection of the tangent galvanometer in the two cases are in the ratio of 1 : 2. Since these tangents are proportional to the respective currents in the slide wire, and since the fall in volts down the length of the slide wire is proportional to the electromotive force in the shunt circuit, we learn that for conductors of the *same resistance the current in them varies as the fall in volts down the resistance.* The experiment may be extended by taking *three* Clark cells in series, and again increasing the working current until the slide wire balance is obtained, and proving that the working current is now *three* times as large as in the first instance. In the third place, keep the working current constant, and vary the number of Clark cells in the shunt circuit, noting at the same time the varying lengths of slide wire required, as measured between the common terminal and the position at which the chisel contact must be placed, so that the mirror galvanometer may show no deflection.

It will be found that the length of wire varies as the voltage in the shunt circuit, or as the fall of potential down the section of the slide wire used. In the fourth place, join two of the platinoid wires in parallel, and, using the same working current as before, prove that

when *one* Clark cell is used in the shunt and *two* wires in parallel, the length of each slide wire employed is the same as if *two* Clark cells are used in the shunt circuit and *one* slide wire above. Lastly, by similar separate experiments with a German silver and a platinoid wire, using the same working currents and one Clark cell in the shunt, show that the slide wire balance cannot be obtained for the same length of slide wire in the two cases. For equal lengths and sections, and for equal working currents, the fall in potential down the *platinoid* wire will be about twice as great as that down the German silver wire. On carefully examining the results obtained it will be seen that these experiments may be made to prove:—

i. That the fall in potential down a wire of constant length, section and material varies as the current through it, or

$$V \propto C.$$

ii. That the fall in potential down a wire of constant section and carrying a constant current, varies as the length of the wire, or

$$V \propto L.$$

iii. That the fall in potential down a wire of given material and of given length, and carrying the same current, varies *inversely* as the cross section of the wire, or

$$V \propto \frac{1}{S}.$$

iv. That for wires of the same length and section, and carrying the same current, the fall of potential down them is dependent on the nature of the wire, and varies for each different material, or

$$V \propto \rho;$$

where ρ is a physical constant, called the *resistivity* of the wire.

When the magnitude of an effect depends upon the magnitude of several variable quantities, and we find that allowing each to vary in turn, but keeping all the rest constant in magnitude, causes the dependent quantity or effect to alter proportionately to the change in each varied quantity separately, then it is evident that the dependent quantity is proportional to the *product of all* the variables, when all vary together. In other words, since V varies independently as C, L, $\frac{1}{S}$, and ρ when each changes separately, V must be proportional to the product of the four when all vary together, or

$$V = \frac{C L \rho}{S}.$$

The resistance of a uniform wire varies as its length, inversely as its section, and directly as the resistivity of the material. Therefore the quantity $\frac{L\rho}{S}$ in the above equation is the measure of the resistance (R) of the wire. Hence we have

$$R = \frac{L\rho}{S},$$

and therefore

$$V = C R,$$

or

$$\frac{C}{V} = R = \rho \frac{L}{S}.$$

This last equation expresses the fact that the ratio of the current in the wire, to the corresponding fall of volts down the wire, is proportional to the length of the wire, inversely as the section of the wire, and directly as the resistivity of the wire. The ratio of C to V is accordingly constant, and this is therefore a proof of Ohm's law. It will be seen, therefore, that the above experiment, if carefully carried out, is sufficient

to give an experimental proof of Ohm's law. We make no assumptions, except that the electromotive force of a Clark cell remains constant during the experiments, and that the electromotive force of two or three cells is twice or thrice that of one cell.

We have also previously proved that the tangent of the angle of deflection of the tangent galvanometer is a measure of the current flowing through its circuit.

If necessary, we can prove the *additive law* of electromotive forces, by experimentally showing that the fall of potential down a wire carrying a constant current which

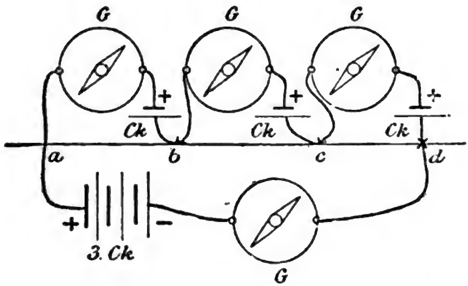

Fig. 32.—*a b c d*, Potentiometer wire ; G, Mirror galvanometers ; C*k*, Clark cells.

balances the electromotive force of three Clark cells in series is equal to three times the electromotive force due to one Clark cell. This can be shown by joining one Clark cell in series with the mirror galvanometer, and finding the length *a b* of the slide wire (see Fig. 32), which, when traversed by a current carefully kept constant, has a fall of potential on it just enough to balance the electromotive force of a single Clark cell. Then move on the two contacts, so that the shunt circuit again makes a balancing contact at two places *b c*, and a third time at a length *c d*. Then these lengths of wire, *a b*,

b c, *c d*, have equal drops of potential down them. Next take three Clark cells in series with the mirror galvanometer, and, with the same current in the slide wire, it can be shown that the shunt circuit must now be tapped cn to the slide wire over a length *a d*, to make a balancing contact, and hence the electromotive forces of the three Clark cells in series must be equal to the sum of their separate electromotive forces.

Very exact experiments have been made, to discover whether Ohm's law is strictly true for large variations of current in copper wire, and it has been found that when the necessary corrections have been applied for the change in resistance due to alteration of temperature, the numerical ratio of the current in amperes to the voltage producing that current is always the same, whether the current is large or small.

The student will notice, therefore, in comparing the production of electric currents in electric circuits by electromotive force, and the production of magnetic flux in magnetic circuits by magnetomotive force, that *all* electric circuits resemble *air* magnetic circuits, or circuits of unit permeability, in that the ratio of voltage to currents, or of magnetomotive force (gaussage) to magnetic flux, is independent of the particular magnitude of the current or flux. In the case of ferromagnetic circuits it is not so, the ratio of magnetomotive force to magnetic flux *is* dependent on the magnitude of the magnetic flux. This fact greatly simplifies electric calculations. They would be rendered much more difficult if it was not possible to name the resistance of a copper wire until we knew the current which was to be put through it.

§ 6. **Magnetic Field of a Straight and Circular Current.**—If a current flows through a straight conductor, with the return conductor at a considerable distance, the direction of the magnetic flux due to the straight current round its conductor is in a series of coaxial circles whose centres lie on the wire. This can be experimentally proved as follows :—

Take a copper wire, No. 12 size, and pass it through a hole in a piece of white cardboard. Support the wire so that it is vertical, and the card so that it is horizontal. Then pass through the wire a strong current (30 amperes is required to show the experiment well), and sprinkle the card with fine steel filings. These filings will arrange themselves (see Fig. 33) in a series of concentric circles.

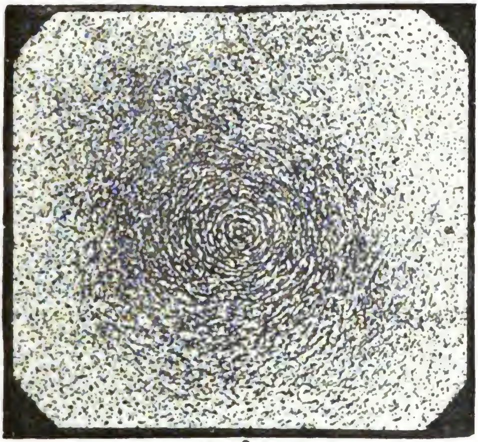

Fig. 33.—Circular Lines of Magnetic Flux surrounding a No. 12 Wire conveying an Electric Current of 30 amperes, taken in a plane perpendicular to the wire. Delineated by steel filings.

If a small magnetic test needle is held anywhere near the wire, it will set itself at all points at right angles to the wire, or stand tangent to the circular lines. (See Fig. 34.) These filings delineate the lines of magnetic flux. The direction of the flux is considered to be related to the direction of the current as is a right-handed *twist* to a forward *thrust*. It can be shown, that if the

wire is exceedingly long, the flux density, or which is the same thing numerically, the magnetic force, at any point distant *r* centimetres from the wire, is inversely proportional to *r*. It can also be shown that the magnetic flux density at any point is proportional to the current in the wire. The magnetic flux density, or magnetic force, at any place *r* centimetres from the wire, is numerically equal to twice the value of the current (in absolute or C.G.S. measure) divided by the distance *r*.

If a magnetic pole is placed, therefore, at any point in the neighbourhood of a straight current, it will be acted on by a mechanical force tending to make it re-

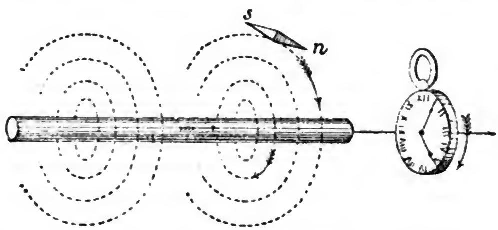

Fig. 34.—Relative Direction of Magnetic Flux and Current in the case of a Straight Conductor conveying the Current.

volve round the conductor in a circle. This mechanical force reckoned in dynes is numerically equal to the product of the strength of the pole and the flux density or magnetic force at the spot where this pole is held.

To show this experimentally requires special devices. Since in a magnet the North pole and South pole are inseparable, the North pole tends to revolve round the current in one direction and the South pole in the other. The only way to get out of the difficulty is to take off the current in the middle of the magnet. Let a couple of bar magnets N S, N S (see Fig. 35) be thrust through a cork *c*, and let a bent steel wire *a b* be put through the cork, so that one point rests in a little depression or cup filled with mercury, made in the top of a straight brass

rod B, and the other end of the steel wire dips into a circular channel, made in a ring R of cork or wood embracing the magnets, and which channel is filled with mercury. Then send a strong electric current up the brass rod, and out of the circular channel of mercury. The two similar bottom North poles of the magnets will begin to revolve round the current in the straight conductor. The upper or South poles, having no current to affect them, do not hinder their fellows from rotating.

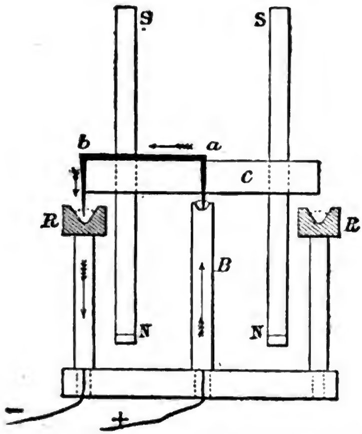

Fig. 35.—Apparatus for showing the Revolution of a Pole of a Magnet round a Current. R R is the section of a circular trough of ring shape with V groove in the upper edge.

If a unit pole is compelled to move round the straight conductor, conveying a current in a circle of radius r centimetres, in the direction opposite to that in which the mechanical force would move it if it were free, then *work* is done on the magnetic pole. The length of the path through which the pole is moved in each revolution is

$$2\pi r \; (= 2 \times 3{\cdot}1416 \times r)$$

and the mechanical force against which it is moved is $\frac{2C}{r}$ dynes where C is the magnitude of the current. Hence, the work done on taking the pole once round the current, is

$$2\pi r \times \frac{2C}{r} = 4\pi C = 4 \times 3 \cdot 1416 \times C.$$

It will be seen that this work is independent of the distance of the pole from the current. Hence we have this important proposition: The work done in taking a unit magnetic pole once round a current in a closed path—no matter what path it follows—is equal to 4π ($\pi = 3 \cdot 1416$) times the total electric current through the path.

The student will see that this obtrusive numerical quantity 4π again makes its appearance in a physical equation. If the unit magnetic pole selected is a *rational unit magnetic pole*, as explained in Chapter III., and if the unit of work is one erg, then it is clear that the unit of current can be so chosen that the number representing the work done in ergs, in taking a unit rational magnetic pole round the current, is the numerical measure of the current. In other words, the current is measured by the work done in taking a unit pole once round it. The above unit of current would be called a *rational unit current.*

§ 7. **C.G.S Absolute Unit Current.**—If a wire is bent into a circle and laid horizontally, and if a current is sent through the wire, the magnetic flux at the centre of the circular conductor will be in a vertical direction. The magnitude of the magnetic flux density or magnetic force at the centre of the circular current can be shown to be numerically equal to the value of 2π times the strength of the current in absolute or C.G.S. units divided by the radius of the circular conductor reckoned in centimetres. In other words it is proportional to the

length of the circuit, the current in the circuit, and inversely as the square of the radius of the circular circuit.

Imagine a very long magnet, with poles of unit strength, hung vertically with its North pole in the centre of a circular conductor having a radius of one centimetre, so that the magnetic field of the circular current acts only on the North pole. Let a current be passed through this circular conductor in such a direction that it tends to lift or repel the unit North pole with a mechanical force of 6·2832 dynes—that is to say, let every unit of length of the circular current act with a unit of force on a unit magnetic pole; then the whole circumference of the circular current will act with a force of 2π dynes ($= 6\cdot2832$ dynes) on the unit North pole. A current having this magnitude is called an *absolute unit of current on the C.G.S. system.* This absolute unit of current is equal to 10 amperes.

The magnetic force, or flux density, at the centre of a circular current of one turn, and having a radius r centimetres, is $\frac{2\pi C}{r}$ units, or $\frac{2\pi A}{10 r}$ units, according as the current is measured in absolute C.G.S. units (C) or in amperes (A). This magnetic force is numerically of the same value as the mechanical force on a unit magnetic pole held at the centre of the circular current.

Starting from this fact, we can construct what is called an *absolute tangent galvanometer* for measuring electric currents in the following way :—

Let a wire be bent into a circle of one turn, and let it have a radius of (say) 50 centimetres. In the centre of the circle hang a very small magnetic needle, and let the plane of the circle be the direction of the magnetic meridian of the place. Let H be the value of the earth's horizontal magnetic force at that spot. Pass a current through the circular conductor : it will cause the needle to be deflected through an angle (call it θ) from the meridian. Then, as explained in § 2 of this chapter, the tangent of the angle θ is the ratio of the magnetic

force due to the coil to the earth's horizontal magnetic force. The magnetic force due to the coil is equal to $\frac{2\pi A}{10 r}$ C.G.S. units, where A is the current in amperes flowing through the circular conductor and r is the radius of the conductor in centimetres. Hence

$$\tan \theta = \frac{\frac{2\pi A}{10 r}}{H}$$

or,

$$A = \frac{10 r}{2\pi} H \tan \theta.$$

Therefore, if we know the value of r in centimetres, and the value of the earth's horizontal force in C.G.S. units, we can determine the value of the current in amperes from the observed deflection of the needle. Thus, since $H = \cdot 18$ (nearly) in England, if the circular coil had a radius of 50 centimetres, and the deflection of the needle was observed to be 45° (tangent $45^\circ = 1$), we should know that the value of the current producing the deflection was

$$\frac{10 \times 50}{2 \times 3\cdot 1416} \times \cdot 18 \times 1 = 14\cdot 3 \text{ amperes.}$$

In this manner a current may be measured in amperes, if the value of the earth's horizontal magnetic force H * at that spot is known.

* For the method of measuring this quantity, see Appendix.

CHAPTER VI.

ELECTROMAGNETIC INDUCTION.

§ 1. **Faraday's Discovery of Electromagnetic Induction.**—In the autumn of 1831, Faraday made a discovery of far-reaching importance, and which has formed the foundation of much which has since been accomplished in electrical invention. He discovered that if a conducting circuit is traversed by magnetic flux, any change in the total amount of this flux passing through the circuit creates in that co-linked conducting circuit an electromotive force. Thus, the movement of a conducting circuit in a field of magnetic flux, is sufficient to set up in it a current, provided the movement is such as to change the total amount of the flux passing through the circuit. The fundamental facts can be best illustrated as follows. Let the student take the ring coil described on p. 108 and connect it by long and rather thick wires to a mirror galvanometer. The galvanometer should be placed at such a distance from the coil that the presence of a bar magnet will not disturb its indications. Place the coil, as in Fig. 36, close to the North pole of the magnet. In this position, magnetic flux proceeding from the North pole passes through the ring coil, and the lines of the flux are said to be *linked* with the conducting circuit. (See Fig. 37.) As soon as the galvanometer needle has come to rest, move the coil quickly away from the pole to a place farther off. It will be seen that the galvanometer needle makes a sudden deflection, and then returns to rest. This indicates the passage of a short or transient current through the galvanometer in one direction. In the next place, move the coil back to its

original position. The needle will make a single swing in the opposite direction. By placing against the tongue

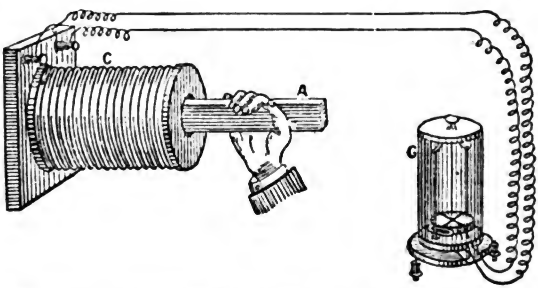

Fig. 36.—Current induced by a Magnet. C, Secondary Coil; A, Magnet; G, Galvanometer.

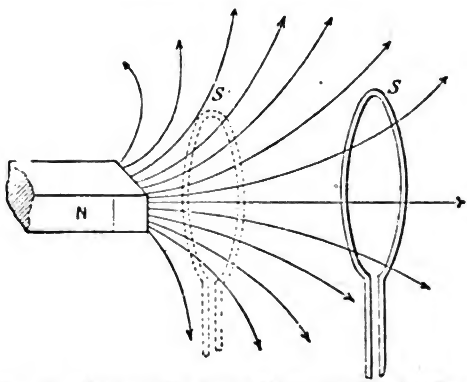

Fig. 37.—Magnetic Flux from a North Pole linked with, or perforating a Circuit S.

a small piece of zinc and a silver coin, attached to the leading wires of the galvanometer, the student should de-

termine the direction of the current which, when passing through the galvanometer, causes it to deflect in either direction. It will then be easy to determine the direction in which the current was set flowing in the ring coil, when it was moved from or to the magnet, and it will be found that, looking at the ring from the same side as that on which the North pole of the magnet is situated, the current *induced* in the ring coil flows round it *clockwise*, or in the same direction as the hands of a clock rotate, when the ring is moved *from* the North pole; and *counter-clockwise* when it is moved *to* the North pole.

The following rules must then be stored up in the memory :—

The *positive direction* of rotation is the direction opposite to that in which the hands of a clock appear to rotate, or *counter-clockwise rotation* is *positive rotation.* (See Fig. 38.)

The positive direction along a line of magnetic flux is the direction *from* the North pole *to* the South pole, through the space outside the magnet. Hence magnetic

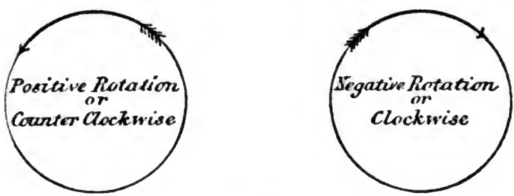

Fig. 38.

Magnetic Flux put in creates positively directed electric current.

Magnetic Flux withdrawn creates negatively directed electric current.

flux is said to come *out* from a North pole, and is always considered as proceeding *from* that pole. Then the student must remember, that if magnetic flux is put *into* or linked with an electric circuit, it creates a *positively*

directed current round that circuit, as seen from the side at which the flux enters the circuit. Similarly, if magnetic flux is *withdrawn* from a circuit, or unlinked from it, it creates in it a *negatively* directed current, as seen from the side of the origin of the flux. The reader will be able to fix these directions in his memory by looking at the diagram in Fig. 38. Let him place in front of himself a ring circuit, and imagine the front part of his body to be a North pole, then thrust the right hand through the circuit, to imitate a line of magnetic flux being *put into* the circuit. Give the hand a twist or screw in the opposite direction to that in which you twist it in putting a corkscrew into a cork, that is, give it a left-handed screw-twist; the relation of the *thrust* to *twist* of his arm will then be that of the positive direction of the line of flux put *into*, and the direction of flow of the induced current *round*, the ring coil. The current thus created in the ring coil is called an *induced current*, and the electromotive force causing it is called the *induced electromotive force*.

In the next place, note that the strength of the current, and therefore the magnitude of the induced electromotive force, is dependent upon the *rate at which the coil is moved.* Repeat the experiment just described; in the first place move the coil from a position far from the pole to a position nearer to the pole very quickly, and in the next place move it very slowly. It will be seen that, corresponding to the quick movement of the coil from one place to the other, the galvanometer needle makes a large swing, but that if the ring coil is moved slowly enough, the galvanometer needle hardly moves at all. This shows us that the induced electromotive force set up in the circuit essentially depends upon the *rate at which the magnetic flux* is inserted into, or removed from, the electric circuit. In the third place, the reader should verify the fact, that it is not every movement of the ring coil which sets up in it an induced electromotive force. Place the coil edgeways to the

magnet, and move it from a place near the magnet pole to a place farther away, and it will be found that this may be done quickly or slowly and yet it will not cause any induced current in the ring.

By many elaborate experiments of the kind, Faraday established two principal facts : *firstly*, that the induced electromotive force is only created in the ring coil when it so moves in a field of magnetic flux that there is a change in the total amount of the magnetic flux passing through it or linked with it ; and *secondly*, that when the total amount of the flux *does* change, the electromotive force set up in the ring is at any instant numerically measured by the rate at which the magnetic flux, linked with the ring circuit, is changing.

Before proceeding further, we must define more exactly the mode of measurement of the quantities with which we are dealing.

The rate at which a quantity is varying may always be represented by the length of a straight line. Thus, for instance, consider a railway train leaving a station and getting up speed.

At any instant we can say that the *rate* of the train is so many miles per hour. Meaning by that, that if the train continued to move uniformly at the same speed it has at that instant, it would in one hour pass over the stated number of miles.

Let the height of the dotted lines in Fig. 39 represent the *rate* at which a train is moving at the end of every minute or 60th part of an hour, starting from rest.

At the end of the 3rd minute, suppose it moving at the rate of 10 miles an hour, and at the end of the 4th minute, let it be moving at the rate of 12 miles an hour. Its mean rate *during* the 4th minute is 11 miles an hour, and the space it passes over during the 4th minute is obtained by multiplying the mean rate per hour by the time in fractions of an hour, viz. $\frac{1}{60}$th, during which it has so moved. Hence in the 4th minute, the train moves $11 \times \frac{1}{60}$ of a mile. This product also represents the area

or slice (cross hatched) of the curve, bounded by the lines drawn to represent the rate of the train at the beginning and at the end of the 4th minute. A little consideration will show, that if the ordinates or vertical lines, the tops of which define the curve, represent to some scale the *rate* at which the train is moving, at times represented by the lengths marked off on the horizontal line, then the whole area bounded by the curve, the base line, and the extreme ordinates, will represent the total distance moved over by the train in that whole time.

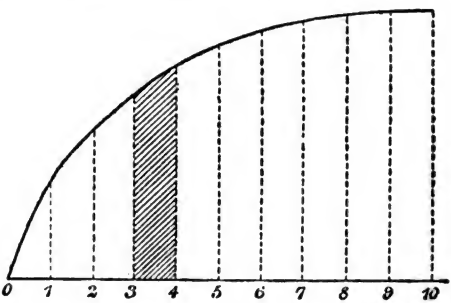

Fig. 39.—Velocity Curve.

Suppose, then, such a diagram to be drawn, the vertical lines in which represent, to a certain scale, the *rate at which the magnetic flux is being taken out of, or put into*, a ring coil or circuit, moved in that field, at different instants. It is evident that the total area of the diagram will represent the whole change which has been made in the magnetic flux passing through the circuit by that movement.

To fix our ideas, let us suppose that the flux passing through the circuit is 100 units at the instant of starting the movement of the ring coil, and that the ring coil is moved away from the magnetic pole in such a way that in each one-hundredth of a second of time *one* unit of mag-

netic flux is taken out of the circuit ; so that at the end of the first one-hundredth of a second there are 99 units of flux linked with the coil, at the end of the second one-hundredth of a second there are 98 units of flux linked with it, and so on. Then the rate at which the flux is removed is uniformly one unit of flux per one-hundredth of a second, or 100 units of flux per second. Hence the total flux of 100 units is removed in one second. In the above case the diagram will simply be a rectangular area, the equal vertical ordinates representing the equal rate at which the flux is being removed in each instant. The altitude of the rectangle must be taken to represent 100, viz. the rate of one unit of flux per one-hundredth of a second, or 100 units per second ; and the horizontal base of the rectangle must be taken to represent one, viz. one second. The whole area of the rectangle is then $100 \times 1 = 100$, and this number represents the total flux removed from the circuit by the motion.

By Faraday's law the rate at which the flux is being removed *from* the circuit is the measure also of the induced electromotive force set up *in* the circuit ; and, since by Ohm's law the electromotive force is numerically equal to the product of the current and resistance, each vertical ordinate of the above described curve must represent the product of the whole resistance of the circuit and the current flowing in it at that instant. If the current endures for a time represented by the horizontal line, and has always an unvarying value, the product of the current strength reckoned in amperes, and the time during which it lasts reckoned in seconds, gives us the value of the *total quantity of electricity*, measured in coulombs, which has flowed past any section of the conductor in that time. The quantity conveyed by *one ampere* in *one second* past any section of the circuit is called *one coulomb*. Even if the current is not unvarying, nevertheless if a diagram is drawn the vertical ordinates of which represent the current in amperes in a circuit at the instants represented by the length of the correspond-

ing horizontal distances from a datum point, taken to represent time, the whole area of this diagram bounded by the curve, the base line and the extreme ordinates will represent the whole quantity of electricity measured in coulombs which has moved past any section of that circuit. Thus, if one centimetre of height in the vertical ordinates represents one ampere, and one centimetre of length represents one second, then one square centimetre represents one coulomb, and the number of square centimetres in the area represents the total quantity of electricity in coulombs set flowing round the circuit. A little consideration, then, will make it clear that the area of the diagram drawn as described represents two quantities which are equal to one another, viz. the *total magnetic flux* removed from the circuit, and the product of the resistance of that circuit, and *total quantity of electricity* in coulombs which has flowed round that circuit.

Since the practical unit of electric quantity is *one coulomb*, it is easily seen that, to make our units consistent, we must take as the practical unit of magnetic flux that flux which, if removed from a conducting circuit having one turn and a unit resistance, sets flowing one coulomb of electricity round it.

If, for the sake of simplicity, we consider that the resistance of the circuit is one ohm, then we see that any movement of the circuit which changes the amount of the magnetic flux passing through the circuit also sets in motion a quantity of electricity flowing round the circuit of equal numerical value.

In reckoning what we have called above the whole area of the circuit, we must take the product of the mean area of the ring coil as seen by the eye, and multiply it by the number of turns in the ring coil ; because, if there are (say) ten turns in the coil, then the total magnetic flux which passes through the opening of the ring is really linked ten times with the circuit. If the reader has any difficulty in seeing this, let him take a red ribbon or tape, and coil it into a circuit of two turns, and fasten

the ends together. Let this represent an electric circuit of two turns. Then pass through the aperture of this circuit a blue tape to represent the magnetic flux, and join the ends together. Next open up the red tape into one large circuit of one turn, and let the blue tape take care of itself. It will be found that the blue tape is now *twice* linked with the red one.

Hence, if a stream of magnetic flux passes through an electric circuit or coil of 100 turns, it is as good as linked 100 times with it. Therefore, the total linkage of the flux with the circuit is obtained by multiplying the apparent area of the circuit by the number of turns of the circuit, and by the mean magnetic flux density over the apparent area of the circuit.

We have then the following definition of what is meant by a *unit of magnetic flux* :—

A unit of magnetic flux is that amount of magnetic flux which, if linked or unlinked once, *with a circuit of one turn having a resistance of one ohm, sets flowing round it a quantity of electricity equal to one coulomb.*

This unit of magnetic flux is called *one weber.*

No magnetic flux yet practically produced, has so great a value as one weber. The practical unit taken is therefore one millionth part of this, or one *microweber.*

If the student has been accustomed to think in C.G.S. units, or to speak of magnetic flux density as so many "lines of force" per square centimetre, then he must remember that one microweber is 100 C.G.S. units of flux or so-called "lines of force."

The practical electrical units are so chosen as to make them all consistent with one another.

Thus, a magnetic flux of one weber, if removed uniformly from a circuit of one turn in one second, will set up in that circuit an electromotive force of one volt.

If a magnetic flux of any number of webers is abstracted from or inserted into a circuit of any number of turns or linkages, then the result is to set flowing round the circuit a quantity of electricity measured in

coulombs, which is related to the whole resistance of the circuit measured in ohms by the rule—

Webers × *linkages* = *coulombs* × *ohms;* or
microwebers × *linkages* = *microcoulombs* × *ohms.*

§ 2. **Measurement of Magnetic Flux. Ballistic Galvanometer.**—In order that we may be able to measure magnetic flux, we require, therefore, some method of measuring electric quantity in coulombs or microcoulombs.

This is done by means of a *ballistic galvanometer.* A ballistic galvanometer is one so constructed that its magnetic needle is very little retarded in its movements, and which, moreover, has a very long period of swing.

This is done by weighting the needle, so as to make it take at least four or five seconds to make one complete vibration to and fro, if disturbed. The reader will find it best not to alter the mirror galvanometer he may have already made in accordance with the instructions in Chapter IV., but to make another galvanometer on a similar plan, but with a very long fine wire circuit, made with about half a pound of No. 36 covered copper wire, and a needle with a mirror not more than a quarter of an inch in diameter. When the magnetised needles have been fixed to the back of the mirror, the whole system should be weighted with little discs of tin foil fastened on the back until, when suspended by the cocoon fibre, it has a period of swing of about four seconds.

In constructing the 'needle' of the galvanometer, it is not well to stick the magnetised needles directly on the back of the mirror with shellac, because such a procedure nearly always warps the mirror out of shape. A small disc of very thin aluminium should be prepared of the same size as the mirror, but in cutting it out, three little lugs should be left, afterwards to be turned over and hold the mirror. The little fragments of magnetised watch spring, can be cemented to the back of this

aluminium disc by stiff shellac varnish, and when this is hard, the mirror of thin silvered glass is affixed by placing it on the side of the aluminium disc opposite to that on which the magnets are fixed, and then bending over the little projecting lugs so as to hold the mirror gently. The cocoon fibre is then added last of all, to suspend the whole system.

A galvanometer of this kind, having a needle with long periodic time, and but little retarded in its vibrations, is called a *Ballistic Galvanometer*, and is employed to measure *quantity* of electricity. If an electric current of very short duration is sent through such a galvanometer, the duration of the current being small compared with the periodic time of the needle—that is to say, if the discharge is all over before the needle has had time to move far from its position of rest—then the needle will receive an impulse or blow, which will cause it to make a quick swing to one side. It can be shown that the extent of the first swing, excursion or "throw" is proportional to the quantity of electricity, measured in coulombs or microcoulombs, which passed through the galvanometer. A ballistic galvanometer is said to be calibrated, when we know the swing of the needle caused by the passage through the galvanometer coil of one microcoulomb.

In order to calibrate it we require two other coils of wire—one for making a *Standard Magnetic Flux*, called a *Standard Primary Coil*, and one for gathering up a known quantity of this flux, called a *Standard Secondary Coil*. These coils are constructed as follows:—

Procure a long pasteboard tube, such as are used for sending drawings by post. This should be about one metre long and 5 centimetres in diameter. On the ends of this tube, glue square pieces of hard wood, having holes cut in them of the same diameter as the tube. These 'cheeks' may be 4 inches square and ½ an inch thick. When finished measure carefully in *centimetres* the length of the tube between the cheeks. Wind on this tube two layers of No. 16 double cotton-covered copper wire, closely and carefully wound. Count the number of turns

of this wire, and bring out the ends to terminals fixed in one cheek. Mark on the tube the number of turns per centimetre of length of the tube; that is to say, the whole number of turns (which should be about 800) divided by the length in centimetres, and the result should be about 8. The student has already learnt that, if an unvarying current is sent through the wire of such a long coil, it will make a very uniform magnetic field in the interior. In the next place, procure a round rod of hard wood $1\frac{1}{2}$ inches in diameter and 12 inches long. Measure carefully the *mean diameter* of the rod in centimetres, taking the mean of a number of measurements. Then wind on this rod one layer of No. 32 silk-covered copper wire laid on perfectly uniformly. Count the total number of turns, and bring out the ends to terminals fixed in the ends of the wooden rod or coil. Mark on both coils the length of the coil and the number of turns, and, in the case of the second coil, the *total area of all the windings.* This last number is obtained by taking the mean diameter of the wooden rod, added to the thickness of No. 32 covered wire, and finding from the tables the area of a circle of this diameter. Then multiply the number representing this area, reckoned in *square centimetres*, by the number of turns of the wire, and it gives the *total area enclosed by all the windings measured in square centimetres.* Let this be called the total area of the secondary coil.

If a constant electric current of known strength in amperes is sent through the long primary coil just described, it creates a magnetic flux in the interior of the coil, the density of which in C.G.S. units is equal to *one and a quarter times the ampere-turns per centimetre length of the coil.** Let the reader then note that the product of the number of turns of wire on a coil, and the value of the current in amperes sent through it, is called the *ampere-turns* of that coil. If the number representing the ampere-turns on the coil is divided by the length of

* The factor *one and a quarter* which comes in here is equal to $\frac{4\pi}{10}$. The 4π comes into the formula because of the use of *irrational units*, and the 10 because we have to divide by 10 to reduce currents measured in amperes to the same in C.G.S. units.

the coil in centimetres, we obtain the *ampere-turns per centimetre.*

Since the practical unit of magnetic flux, viz. the microweber, is equal to 100 C.G.S. units of magnetic flux or induction, we have the following important rule for determining the magnetic flux density in the interior of a long straight bobbin of wire, traversed by a continuous current.

The magnetic flux density in the interior of the coil, measured in microwebers per square centimetre, is equal to one-eightieth part of the ampere-turns on the coil per centimetre of length.

Or, if C.G.S. measure is used, then the rule is—

The magnetic flux density, or the induction in C.G.S. units, in the interior of a long solenoid, is equal to one and a quarter times the ampere-turns per centimetre of length of the coil.

Since 2·5 centimetres very nearly make one inch, or $1\frac{1}{4}$ centimetres are equal to half an inch, the above rule may be stated also as follows :—

The magnetic flux density in C.G.S. measure, in the interior of a long solenoid with air core, is measured by the ampere-turns per half inch of the coil length.

The above rules only apply if the material filling and surrounding the coil is air, or non-magnetic material, that is if the magnetic flux lines pass everywhere through a material of unit permeability or reluctivity in their course. These rules give the interior flux density over about three-quarters of the length of the coil, but do not give its value quite near the open ends.

If, then, the long primary coil, prepared as above described, is joined in series with the standardised tangent galvanometer, and with a battery of cells, we have the means of making a magnetic flux the density of which, in a particular region, can be exactly calculated. This constitutes a standard magnetic flux.

In the next place, let the secondary coil be placed in the interior of the primary coil, and let the secondary

coil be connected by long wires with the mirror ballistic galvanometer. This last must be placed so far away from the primary coil as not to be disturbed by the direct magnetic effect of the latter.

When the standard magnetic flux is produced in the interior of the primary coil, if we place in that interior the secondary coil, some of this flux is linked with the turns of the secondary coil, and the total amount of magnetic flux so linked is measured by the *product of the magnetic flux density* in the interior of the primary coil, and the *total area of all the windings of the secondary coil.* Hence the secondary circuit includes, or is linked, with a known magnetic flux.

If the primary current is now stopped all this flux disappears, and a known total flux is withdrawn from, or unlinked from, the secondary circuit. The galvanometer will then make a "throw," or deflection, which must be observed. The whole resistance of the secondary circuit, measured in ohms, must then be taken in the way described in the chapter on electrical measurements. We have then a known magnetic flux density reckoned in microwebers per square centimetre, linked with a circuit of known total area in a known number of linkages, and having a known total resistance. Hence, by the rules *microwebers* × *linkages* = *microcoulombs* × *ohms*, we can calculate the whole quantity of electricity in microcoulombs, sent through the galvanometer. Corresponding to this, we have the observed throw or deflection of the galvanometer needle. We can therefore calculate the galvanometer *ballistic constant*, that is to say, the deflection which would be given by the passage through the galvanometer of one microcoulomb. For, in a galvanometer of this kind, it can be shown by experiment that the "throw" or deflection of the needle is exactly proportional to the quantity of electricity which causes it. The student may easily prove this for himself. All that is necessary is to send different measured primary currents through the primary standardising coil,

and then observe the ballistic galvanometer "throw" when these primary currents are stopped. We know in this case that the galvanometer is traversed by quantities of electricity which, in the different experiments, are proportional to the strength of the primary currents. Hence, if the observed value of the scale deflection of the galvanometer is divided by the measured value of these primary currents, we shall obtain quotients which in every case are nearly the same. Every ballistic galvanometer has therefore its *ballistic constant*, enabling us to measure by it *quantity of electricity* in microcoulombs, just as every tangent galvanometer has its *deflection constant*, enabling us to measure by it *current strength* in amperes. For if we know the current in amperes which causes in a tangent galvanometer a deflection of 45°, or angle whose tangent is unity, then we can tell the *current* reckoned in amperes producing any other observed deflection. Similarly, if we know the "throw" or excursion of the needle of the ballistic galvanometer corresponding to the passage of a microcoulomb of electricity through it, we then know the *quantity* producing any other observed "throw."

§ 3. **Induced Electric Currents.**—Returning, then, to the subject of electro-magnetic induction, the student should notice that if a secondary coil is connected to a galvanometer, and if a primary solenoid is brought up towards the secondary coil, the primary coil being traversed by a current, we obtain inductive effects of the same character in the secondary coil that we did when employing a permanent magnet. Placing, for instance, the ring coil already described in connection with the galvanometer, bring up towards it a primary solenoid traversed by a current in such a direction as to make the forward end a North pole. It will be found that in every respect the galvanometer needle acts just as if the solenoid were replaced by a bar magnet held with its North pole towards the ring coil. If we regard the ring coil from the side at which we hold the magnet

or solenoid, then it will be found that the following statements hold good:—

(1) Moving the North pole of the bar magnet or North pole of the solenoid or coil *towards* the ring coil, creates a *counter-clockwise* induced secondary current in the ring coil.

(2) Drawing the North pole of the magnet or North pole of the solenoid *away from* the ring coil, creates a *clockwise* induced secondary current in the ring coil.

(3) Holding the solenoid or primary coil with the North pole just inside the ring coil, and *stopping* the primary current in it, creates an induced secondary current in the *clockwise* direction in the ring coil. That is to say, stopping the primary current has just the same effect as withdrawing the primary coil.

(4) Holding the primary coil with one end just inside the ring coil, and starting the primary current in it in such a direction as to make that end a North pole, creates an induced secondary current in the ring coil in the *counter-clockwise* direction, that is to say, in the same direction as if the solenoid traversed by the same current had been brought up suddenly to the ring coil.

If the student thoughtfully examines all these different cases, he will see that in every instance the guiding principle which determines whether a secondary current is produced or not, is the ascertained fact that, inserting or withdrawing magnetic flux into, or from, a secondary circuit, always creates an induced electromotive force in that circuit, provided that the change alters the total magnetic flux, or, in common parlance, 'the number of lines of magnetic force,' linked with the secondary circuit.

Hence we can sum up the whole of the facts in one general law, called Faraday's Law of Induction, which is as follows:—

If there be any conducting circuit, called a secondary circuit, which is placed in a field of magnetic flux, and if any change is made, either by motion of or alteration in

size of the secondary circuit, or by change in the strength or direction of the magnetic field, which alters the total amount of the magnetic flux linked with the secondary circuit; then an electromotive force is set up in that circuit, which in magnitude at any instant is measured by the rate at which the total linkage of the magnetic flux with the secondary circuit is being changed.

The direction of the induced electromotive force is determined by another general principle. If a coil, such as the above-mentioned ring coil, is traversed by a continuous current, then, if we picture to ourselves the current flowing in the wire, that side or face of the ring coil at which we must look, to see in imagination the current rotating in the opposite direction to the hands of a watch, acts in every respect like a North magnetic pole. It is easy to test this fact by a small exploring magnetic needle, and to prove that the ring coil traversed by a current is in fact a very short magnet, having one face a North pole and the other a South pole.

An excellent way to make evident the magnetic power of such a circular current, is to construct a circular coil of about twenty turns of No. 20 cotton-covered copper wire, in the form of a flat ring coil three inches in diameter. This coil is then fixed on the top of a large flat cork or bung.

Through the cork is then pushed a small rod of amalgamated zinc, and one of electric arc-light carbon, and the ends of the windings of the coil fastened to the top of these rods as in Fig. 40.

In order to attach a copper wire securely to a carbon rod, the rod has first to be *electrotyped.* For this purpose, prepare a saturated solution of sulphate of copper, and place the end of the carbon rod about half an inch deep in this solution. Place also in the solution, but not touching the carbon, a small copper plate. Connect the carbon and the copper to the terminals of a voltaic battery of three or four cells, the carbon being connected to the negative pole of the battery. Leave the arrangement for a few minutes. At the end of that time a thin deposit

of copper will have been made on the end of the carbon rod. Take out the rod and dry the coppered end. Then *tin* it by dipping it into melted solder. It will then be found easy to solder a wire to the coppered end of the carbon rod.

Rods of hard carbon suitable for this experiment are used for electric arc lamps, and are called arc lamp carbons. Usual sizes are 9 to 15 millimetres in diameter.

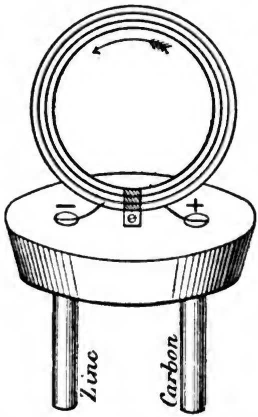

Fig. 40.—Floating Zinc-Carbon Cell and Ring Coil.

The coil and carbon-zinc rods are then floated on the surface of a solution of bichromate of potash and sulphuric acid, or bichromate battery solution (see p. 10), contained in a large basin. The copper-zinc couple then acts as a single voltaic cell, and sends a current through the circular coil of wire. This current flows from the end attached to the carbon, through the wire, to the end attached to the zinc. The cork being perfectly free to move, the coil can turn round in any way. Hold the North pole of a permanent magnet towards the floating coil. It will be found that the coil turns itself round, so as to place its South face towards the North pole of the magnet, and then is attracted towards the magnet. If the South pole of the magnet is presented to that same face, the circular coil is repelled. Then it recedes, turns right about face, and presents its North face to the South pole of the magnet, and is again attracted.

If we substitute for the permanent magnet another similar ring coil, carrying a strong continuous electric current, we can easily convince ourselves that when the two coils are so placed parallel to each other that the

currents in them are flowing in the *same direction* the coils *attract* each other, but when the currents are flowing in *opposite* directions the coils *repel* each other.

Returning then to the consideration of the direction of the induced current set up in a circular coil by moving a magnet pole towards the coil, the general principle governing the direction of the induced current is as follows:—

The secondary current or induced current is always in such a direction that it tends to oppose the motion of the magnet pole or solenoid creating it. Hence moving a North pole *towards* a coil induces a *counter-clockwise* current in it, because a current in the coil in that direction makes the coil-face opposed to the magnet a North face, and therefore tends to repel the advancing North magnetic pole or resist its motion.

The same fact governs the direction of the secondary or induced current created in one circular coil by another similar coil placed over it. If a primary current is *started* in one coil, it tends to make a secondary current in the *opposite* direction in the other or secondary coil, because oppositely directed currents in two parallel conductors cause the conductors to repel each other. Similarly, on stopping the primary currents, there is created a transient secondary current in the same direction as the primary one.

These facts regarding the direction of the induced currents created by starting or stopping a primary current may best be remembered by imagining two long wires stretched parallel to each other. Let one be brought round so as to form a closed circuit and be called the Secondary circuit, and let the other, called the Primary circuit be traversed by a continuous current which can be started or stopped.

Then starting the primary current creates an oppositely directed transient electric current in the secondary circuit, generally called the "inverse induced current." Stopping the primary current creates a similarly directed

transient electric current in the secondary circuit, called the "direct induced current."

Two circuits so arranged are said to exert mutual induction on each other. If instead of starting or stopping the primary current in its wire, we keep the primary current constantly flowing in one direction, and move the primary circuit parallel to itself up to the secondary circuit, then the advancing primary current induces or creates an oppositely directed or repelling secondary current, and a retreating primary current creates or induces a similarly directed or attracting secondary current. These secondary currents only last as long

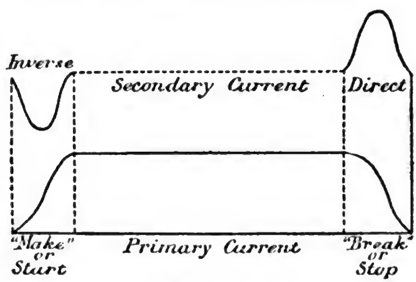

Fig. 41.

as the motion or change of the primary current lasts. The two secondary currents, direct and inverse, are equal in total electric quantity, but may be very different in maximum current strength.

If we take a horizontal line on which to mark off distances representing *time*, and set off perpendicular distances representing currents, then we can graphically represent the action of a primary current on a secondary circuit as follows. Let the ordinate or height of the bottom curve in Fig. 41 represent the strength of a primary current beginning gradually, growing up to a constant strength, and then after a time gradually stopped

or reduced to zero. In an adjacent parallel secondary circuit, the inverse and direct induced secondary currents can be represented by the two curves in upper part of Fig. 41. The inverse secondary current at starting the primary, is a short wave of current lasting only so long as the primary is increasing in strength, and in the opposite direction to the primary current.

The direct secondary current at stopping the primary, is a short wave of current in the same direction as the primary current. The total areas included by the secondary curves are equal, but the maximum value

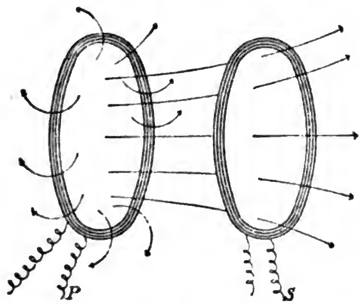

Fig. 42.—Lines of Magnetic Flux from a Primary Coil P traversing a Secondary Coil S.

of the *direct* induced current is generally greater than that of the *inverse* induced current.

If a primary and a secondary circuit are placed in the neighbourhood of one another, and if a steady current is passed through the primary circuit, the student must picture to himself some portion of the magnetic flux produced by and surrounding the primary current, as linked with the secondary circuit (see Fig. 42). Corresponding to the flow of one ampere through the primary circuit, there will be a certain total magnetic flux linked with the secondary circuit. This total link-

age must be understood to mean the total magnetic flux passing through the secondary circuit, and is, therefore, measured by the product of the number of turns on the secondary circuit, and the flux through the apparent aperture or area of the secondary coil. If there is no iron or magnetic material in or around the coils, then the total magnetic flux passing through the secondary circuit will be proportional to the strength of the primary current, assuming the position of the coils to remain unchanged.

§ 4. **Mutual Inductance. The Henry.**—The total magnetic flux due to the primary coil, which is linked with the secondary coil, when one ampere traverses the primary coil, is called *the Mutual Inductance* of the two coils, and is measured in terms of a unit called *one Henry.* The mutual inductance of two coils is said to be *one Henry* when the passage of *one ampere* through one coil causes a total magnetic flux of *one weber* to be linked with the secondary coil.

The mutual inductance of two coils changes with every change in position of the coils, but if the coils are not wound on iron cores, or if no iron is near, then the mutual inductance in any given position is a constant quantity, in that the total magnetic flux through the secondary circuit, per ampere of current through the primary coil, has value which is independent of the strength of the primary current. If, however, the coils are wound on iron cores, or have iron near them, then the mutual inductance of the coils cannot be defined in quite so simple a manner. It is then defined to be, the ratio between the electromotive force set up in the secondary circuit at any instant, and the rate at which the primary current producing it is changing in strength. This ratio is not constant, but is different for different absolute values of the primary current.

§ 5. **Conductors cutting Magnetic Flux. Faraday's Disc.**—Returning, then, to the subject of induced electromotive force, let the ring coil above described be

placed in front of a bar magnet, with its plane perpendicular to the axis of the magnet. In this position we must think of a certain group or bunch of the lines of magnetic flux proceeding from the pole of the magnet, as passing through the opening of the ring coil. If the coil is moved nearer to the magnet, more lines of magnetic flux will be crowded into the aperture. These lines can only come in by *cutting sideways through* the circuit of the ring coil. Similarly, if the coil is moved farther away, the lines of flux can only be withdrawn from the secondary circuit by cutting again through the conductor. Faraday showed that this "cutting" is the effective cause of the induced electromotive force, and the increase or diminution of the total flux *included by* the complete secondary circuit is only a consequence of the fact that lines of magnetic flux cannot spring into existence suddenly in any place. If two long straight wires are placed parallel to each other, we have seen that if a current is started in one wire, the magnetic flux round it is distributed in circles co-axial with the wire.

These lines of flux must not, however, be considered as springing into existence suddenly in the places occupied by them, but as growing out from the wire, just as the water-ripples on the surface of a lake, grow outwards from the place where a splash is made by throwing in a stone. In their progress outwards to their final positions, these magnetic flux lines will "cut through" any parallel secondary conductor, and induce in it a momentary electromotive force, and therefore, secondary current. Hence, if a conducting loop or circuit is at any time found to be including a greater amount of magnetic flux, it can only be because the flux lines have become concentrated, by additional lines crowding into the circuit from outside, and to do this they must *cut across* the circuit.

The induced electromotive force set up by the cutting of a conductor across magnetic flux lines is well

shown by the effect of the rotation of a copper disc between the poles of a magnet. Faraday constructed, in the following manner, the first machine capable of producing a continuous electric current by the rotation of a conductor in a magnetic field. A copper disc (see Fig. 43)

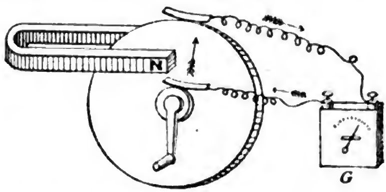

Fig. 43.—Faraday's Disc Induction Machine. (The Disc revolves Clockwise.)

is placed between the poles of a horseshoe magnet, so that the North pole is one side of the disc, and the South pole on the other, and the line joining the poles passes through the disc about half way between the centre and the edge. A small metallic spring is placed so as to touch the circumference of the disc, and another to touch the metallic axis of the disc. When the disc is turned round like a grindstone, each radius of the circular copper sheet passes in succession between the poles of the magnet and "cuts through" the flux of magnetic induction proceeding from the North pole to the South pole. If the springs are joined by a wire, in the circuit of which is placed a galvanometer, G, then a steady revolution of the disc in one direction, creates a steady induced current, which flows from the centre of the disc to the edge of the disc, or *vice versa*, completing its circuit through the external wire. The machine so constructed was the parent of all subsequent dynamo machines in which electric current energy is produced from mechanical energy by rotating a conductor in a suitable field of magnetic force.

In order that we may find the value of the electromotive force so created, in terms of the dimension of the disc, and its speed, it is necessary to examine a more simple case of induction by motion of a conductor.

Let a copper wire be bent, as shown in Fig. 44, into a pair of parallel lines, closed at one end, forming thus three sides of a rectangle, A B C B. Let a second wire, D E, be placed across the bent wire and be supposed to slide along parallel to itself and to the end wire. Imagine this conductor placed in a field of magnetic flux, the lines of which are perpendicular to the plane of the paper, and are represented by the black dots. Then in any position of the sliding piece, a certain total magnetic flux passes through the secondary circuit formed of the two rails, the blind end of the bent wire, and the cross or sliding piece.

Let the cross bar move parallel to itself with a certain velocity. It is easy to see that the rate at which the area B C D E gets larger, is represented by the product of the velocity of the cross piece and its length.

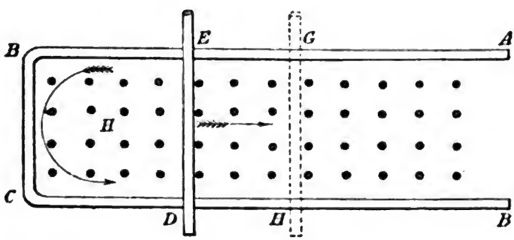

Fig. 44.—Electromotive Force produced in a Bar sliding across Magnetic Flux Lines.

The total magnetic flux passing through the area B C D E is represented by the product of the mean magnetic flux density all over the area, and the value of the area measured in square centimetres. Hence the

rate at which the total magnetic flux included in the area B C D E changes when the cross bar D E moves parallel to itself one way or the other, with a uniform speed, is given by the product of the mean magnetic flux density all over the area, the length of the sliding piece, and the velocity of the bar parallel to itself.

By Faraday's law the rate of change of the total flux is the measure of the induced electromotive force set up in the circuit B C D E. Hence we have the following rule:—

The electromotive force set up in the circuit B C D E, by the uniform motion parallel to itself of the sliding bar, is numerically given by the product of the length of the bar, its velocity, and the magnetic flux density of the field in which it moves.

The above rule provides us with a means of defining the practical unit of electromotive force. *If the length of the bar C D is one centimetre, and if it moves parallel to itself with a velocity of one centimetre per second in a field of magnetic flux having a density of one weber per square centimetre, then the electromotive force set up in the circuit will be one volt.*

The above is the definition of what is meant by an electromotive force of one volt, and it can be practically created by causing a disc of known dimensions to spin in a field of magnetic flux of known density.

Since the earth's vertical magnetic field is a field having a magnetic flux density, in England, of about four one thousand millionths of a weber per square centimetre, or ·004 of a microweber per square centimetre; we can easily calculate the electromotive force set up in a conductor moving transversely across the direction of this field, that is to say, horizontally. Let such a conductor be 1 metre long, or 100 centimetres in length, and move with a velocity of 3000 centimetres per second, or about the speed of an express train. The electromotive force set up in this conductor can then be calculated by the rule :—

Magnetic Flux density × *length* × *velocity* = *electromotive force* or

$$\frac{4}{10^9} \times 100 \times 3000 = \frac{12}{10^4} = \frac{12}{10000} \text{ of a volt};$$

hence it is rather more than one-thousandth of a volt.

§ 6. **The Direction of the Induced Electromotive Force. The Hand Rule.**—We require a rule to tell us, not only the magnitude of the induced electromotive force, but also the direction of it with reference to the motion of the conductor, and the direction of the field, and it is found in the following useful *hand rule*.

Hold the forefinger, the middle finger, and the thumb of the right hand, in such a position that they are as nearly as possible at right angles to each other (see Fig. 45.) Then make the following associations. Let the direction of the forefinger represent the direction of the magnetic flux (*F*ore and *F*lux). Let the direction of the thumb represent the direction of the motion of the conductor (Thu*M*b and *M*otion). Finally let the direction of the middle finger represent the direction of the induced electromotive force (M*I*ddle and *I*nduced). The rule is then as follows:—

If a conductor, represented by the middle finger, be moved in a field of magnetic flux, the direction of which is represented by the direction of the forefinger, the direction of this motion, being in the direction of the thumb, then the electromotive force set up in it will be indicated by the direction in which the middle finger points. The associations once made, it is easy, by twisting the hand about, with the two fingers and thumb held rigidly in the rectangular positions, to find at once the direction of the induced electromotive force in a conductor moved parallel to itself in a field of magnetic flux.

Thus in the case of the sliding bar in Fig. 44; if the direction of the magnetic flux is *into* the paper away from the reader, and if the sliding bar moves to the right hand parallel to itself, then the direction of the

induced electromotive force is *upwards* on the sliding piece, and the current induced is counter clockwise in the circuit, A B C D.

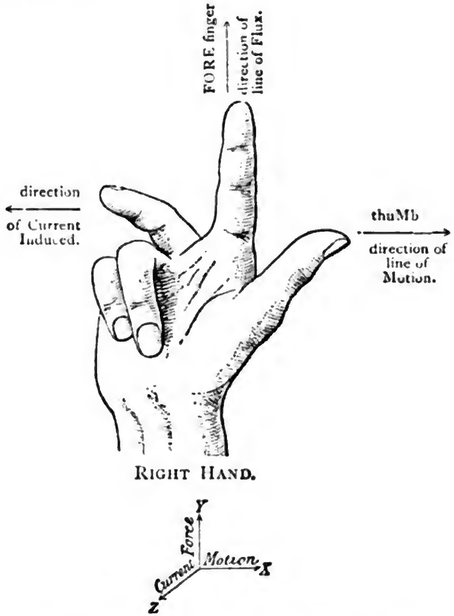

Fig. 45.—Three Axes at Right Angles, indicating respectively the Direction of the lines of Magnetic Flux or Force Y, the Line of Motion of Conductor X, and the Direction of Current Induced Z.

A final and important case of electromagnetic induction which we must examine, is that of a circular or rectangular coil, rotating about an axis in a uniform field of magnetic flux. Let A B C D represent (see Fig. 46) such a rectangular circuit or wire rotating about an axis

X Y. Then, if the field of magnetic flux in which it is immersed is represented as regards direction by the arrow-headed lines, and if the circuit rotates round the axis X Y, it is clear that starting from a position of the rectangle, in which its plane is parallel to that of the flux lines, during one quarter of its complete rotation lines of flux are being put into the circuit, and during the other quarter of a revolution are being withdrawn from it. The circuit gradually increases in apparent area, as seen from the direction in which the magnetic flux proceeds, and as it opens out it includes more lines of flux; then it diminishes again in apparent area, and reduces the number of flux lines, or total amount of magnetic flux linked with it. This process is repeated during the other half revolution. A little consideration will show the reader, that during one half of its journey round there must be an electromotive force acting in one direction round the circuit, and in the remaining half an oppositely directed electromotive force. It must be noticed that the insertion of magnetic flux into a circuit produces exactly the same electromotive effect, as regards direction, as the withdrawal of oppositely directed magnetic flux. Therefore, putting in positively directed flux during one quarter of a turn creates an induced current in the same direction as withdrawing negatively directed flux.

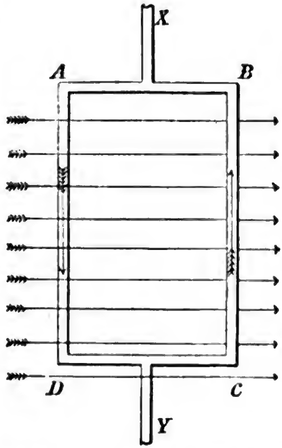

Fig 46.—Rotation of a Conducting Rectangle in a Uniform Field of Magnetic Flux.

Hence the rapid rotation of the ring or circuit in the uniform field of flux, will cause it to be traversed by a current of electricity which is first in one direction and then in the other. Such a current is called an *alternating current*, and the properties of these alternating currents we shall study in another chapter.

CHAPTER VII.

ELECTROMAGNETS.

§ 1. **Magnetisation Curves.**—We have in previous chapters defined magnetic force, magnetic flux density, or induction, reluctivity and permeability, and we must, in the next place, study somewhat more carefully the manner in which these quantities are related to each other in the case of ferromagnetic materials, such as iron, when forming a magnetic circuit. For this purpose, it is best to begin by examining in detail the magnetic changes taking place in iron when a circular iron ring is magnetised by an electric current. Let an iron ring be covered over with a magnetising spiral or solenoid, and also be surrounded at one place with a secondary coil which is connected with a ballistic galvanometer. We have already explained in Chapter VI. the manner in which the ballistic galvanometer is employed to measure the magnetic flux linked with a conducting circuit. If, then, a certain measured electric current is passed through the magnetising spiral of the ring magnet, and if we know the number of turns on this spiral, we can calculate the magnetising force acting on the iron, since this force is numerically equal to the number of ampere-turns per half inch of the mean perimeter of the ring. This magnetising force creates a certain magnetic flux in the interior of the iron ring, and the value of this flux can be obtained by reversing suddenly the direction of the magnetising current, and noting the swing of the calibrated ballistic galvanometer connected to a secondary circuit wound on the ring. In this way, knowing the sectional area on the ring, we can determine the magnetic

flux density in the iron corresponding to various magnetising forces. The results are then set out in a curve, called a *Magnetisation Curve* (see Fig. 47), as follows:—

On a diagram, take horizontal distances to represent the magnetising forces acting on the iron ring, reckoned

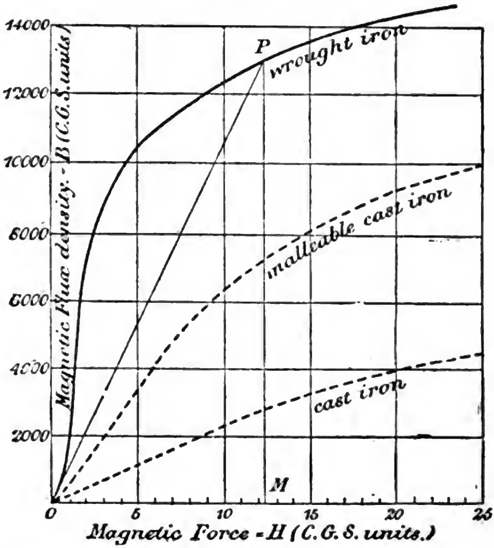

Fig. 47.—Magnetisation Curve (firm line) for Annealed Swedish Iron Ring. The dotted curves are the Magnetisation Curves for Cast Iron and Malleable Cast Iron.

in *ampere-turns per centimetre or per half inch*, or in absolute C.G.S. units (0·8 of an ampere-turn per centimetre = 1 C.G.S. unit), and vertical distances to represent resulting magnetic flux density, measured either in C.G.S. units or in microwebers per square centimetre (100 C.G.S. units = 1 microweber). The curve so obtained rises somewhat steeply at one place, and then becomes more nearly flat.

In the following table are given the results of such an experiment on a soft iron ring, all measurements being in C.G.S. units :—

MAGNETISATION CURVE OF AN ANNEALED SWEDISH IRON RING AT 15° C.

Magnetising Force = H.	Magnetic Flux Density = B.	Magnetic Permeability $= \mu = \frac{B}{H}$.
0·725	1,000	1379
0·971	2,000	2060
1·174	3,000	2555
1·378	4,000	2903
1·595	5,000	3135
1·840	6,000	3261
2·10	7,000	3333
2·58	8,000	3101
3·35	9,000	2687
4·47	10,000	2237
6·27	11,000	1754
8·99	12,000	1335
12·35	13,000	1053
17·22	14,000	813
22·1	14,400	652

The results of the above observations are set out in the curve in Fig. 47.

The magnetisation curves of cast iron, steel and wrought iron, differ in form from one another in such a manner as to show that the production of a given flux density in wrought iron, or in certain kinds of steel may be attained with a far less magnetising force than is necessary in the case of cast iron or hardened high carbon steel. The purest Swedish wrought iron is characterised by having a magnetisation curve, which rises up very steeply to a "knee" in such a manner as to show that a high value of the magnetic flux density is attained with

a comparatively small expenditure of magnetising force. The same magnetising force would not produce a magnetic flux density of anything like the same magnitude if acting upon a cast-iron ring. Thus a magnetising force of 20 C.G.S. units will produce in Swedish wrought iron, or mild cast steel, a flux density of 14,000 or 15,000 C.G.S. units. The same force would not produce a flux density of more than 4000 or 5000 in cast iron. Certain kinds of cast steel having small percentages of carbon in them may, however, under high magnetising forces, have produced in them an even larger flux density than would be the case if the same force acted on wrought iron. This is shown by the following figures :—

MAGNETISATION CURVES OF MILD ANNEALED STEEL AND SOFT SWEDISH IRON IN THE FORM OF RINGS TAKEN AT 15° C.

Flux Density B.	Magnetising Force H.		Permeability μ.	
	Annealed Cast Sheet Steel.	Annealed Soft Iron.	Annealed Cast Sheet Steel.	Annealed Soft Iron.
2,000	1·2	0·8	1800	2675
4,000	1·7	1·17	2400	3375
6,000	2·5	1·73	2425	3500
8,000	3·7	2·55	2150	3150
10,000	5·55	3·95	1800	2550
12,000	8·95	6·45	1375	1850
14,000	15	12	950	1050
16,000	29	54	550	325
18,000	90	126	200	125
20,000	224	324	87	55

It will be seen that, for the small forces, this particular steel has smaller permeability than the iron, but for the large forces greater permeability than the iron. This, however, is not generally true of all steels. The permeability of steel decreases as the percentage of carbon

increases. As a matter of fact, the chemical composition of the steel and iron in the above table is not very different, and the terms steel and iron refer rather to the mode of production of the metal than to the precise chemical composition.

The terms wrought iron or iron, steel, and cast iron, are applied to different, more or less variable, compounds or mixtures of chemically pure iron with other substances. Probably no one has yet seen absolute pure iron in the sense of iron 100 per cent. fine, or if produced it is but a chemical rarity. It cannot be obtained in commerce. The usual impurities present in iron are carbon combined with iron in the form of various carbides of iron, carbon free in the form of graphite, silicon, phosphorus, sulphur and manganese. In the purest Swedish charcoal iron these impurities may be present to the extent of only 0.1 or even 0·07 per cent. Cast iron contains all the elements which the crude iron brings in from the ore and the fluxes used in reduction, and is characterised by the presence of an excess of carbon, most of it in the uncombined form as graphite. Steel contains carbon almost wholly in the combined form, and is spoken of as low carbon, or good, or mild steel, when the carbon is less than ·25 per cent., and as high carbon steel if carbon is present in greater percentages. As an illustration of the compositions of various irons and steels the following analyses are given, merely as typical:—

—	Charcoal or Swedish Wrought Iron.	Cast Steel.		Cast Iron.
		Magnetically good.	Magnetically inferior.	
Carbon combined .	0·05	0·08	0·36	0·70
Carbon as graphite	—	—	—	3·30
Silicon . . .	0·05	0·01	0·47	2·00
Sulphur . .	0·03	0·05	0·04	0·04
Phosphorus . .	0·08	0·08	0·15	0·85
Manganese . .	0·06	0·04	0·62	0·38
Iron . . .	99·73	99·74	98·36	92·73
	100·00	100·00	100·00	100·00

In Fig. 48 the above magnetisation curves for mild steel and for soft iron are delineated. When a magnetic flux density of about 18,000 C.G.S. units, or 180 microwebers per square centimetre, has been reached, the increase of flux density in wrought iron proceeds very slowly compared with the increase in magnetising force. At this stage the iron or steel is said in popular language to be "saturated" magnetically. It has, however, been

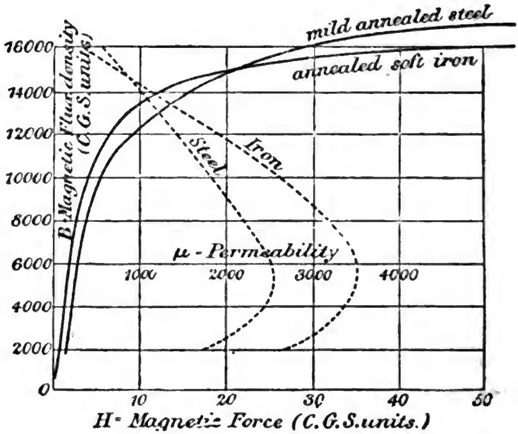

Fig 48.—Magnetisation Curves (firm lines) of Annealed Mild Steel and Swedish Iron. The dotted curves are the corresponding Permeability Curves.

shown that there is no definite limit to the extent to which the magnetic flux density in the iron can be increased, and, by the use of a sufficiently powerful magnetising force, the flux density in iron has been pushed up, by Ewing and others, to 40,000 C.G.S units or more (400 microwebers per square centimetre). The form of the magnetisation curve shows, however, that there are three well marked stages in the process of magnetising

iron. First, a stage when the flux density increases almost proportionately to the magnetising force, and rather slowly. This is called the *initial stage* of magnetisation. In the next place, at some stage in the process corresponding to a small increase in the force, there is a relatively rapid increase in the magnetic flux density, and the magnetisation curve rises very steeply. In the third place, there is a slow final stage, beyond what is called the "knee" of the curve, in which immense increase in the magnetising force makes very little corresponding increase in the flux density or induction. We shall examine presently the probable physical explanation of these facts. The form of the magnetisation curve is very characteristic of different classes of iron and steel. In Fig. 47 are shown in dotted lines the magnetisation curves for cast iron and malleable cast iron, and it will be seen how greatly they differ from that of wrought iron. We have already pointed out that, in the case of certain varieties of steel, although the flux density produced by a small magnetising force is considerably less than that produced by the same force on soft or pure iron, yet for large magnetising forces the reverse is the case, and a given large magnetising force may produce a greater flux density in mild steel than in iron. Hence it follows that the magnetisation curve of such steel crosses that of iron, and whilst below it for small forces is above it for great ones. It is for this reason that cast steel having a small percentage of carbon in it is now much used in dynamo building. The dynamo builder is only concerned in getting the greatest flux density possible in the field magnets of his dynamos, for a large magnetising force, and he can do this to a slightly greater degree by the use of certain qualities of steel than by iron, and in addition he has in the steel a material with better mechanical qualities for working.

The ratio between the magnetic flux density, or the induction, and the magnetising force is called the *permeability* (μ) of the material. If, therefore, from any

point P, on the magnetisation curve (see Fig. 47) a straight line is drawn to the origin O, the ratio of the length of the perpendicular P M to that of the base M O, or the tangent of the slope of the line P O gives us the *permeability* of the material corresponding to the magnetic flux density represented by P M. It is usual to plot out a curve of permeability in terms of the magnetic flux density or induction, and such curves are called *Permeability Curves.* The permeability curves are delineated in Fig. 48 for annealed Swedish iron, and for mild annealed steel by the dotted lines. It will be seen that the permeability curve has a maximum value for a certain flux density, and this maximum value is an important characteristic magnetic constant of the iron or steel. This maximum permeability may rise to 4000 units or more for certain special kinds of Swedish iron, but is more generally a number in the neighbourhood of 2000 or 2500 for most common classes of iron. Its value greatly depends on the manner in which the iron has been annealed. The reciprocal of this number is the *reluctivity* of the iron for that particular flux density. The magnetisation curve for air or non-magnetic materials is a straight line, so drawn that the ordinate or height representing the flux density corresponding to any magnetising force has a numerical value equal to that of the base length denoting the force. Hence, on a diagram representing a magnetisation curve for iron, it is hardly possible to represent an air magnetisation curve to the same scale of flux density, since it would be hardly distinguishable from the base line itself. The magnetic flux density is always denoted by the letter B, and magnetising force by the letter H ; and hence, since the magnetisation curves show the relation of B to H, they are called B-H curves. The permeability is denoted by the Greek letter μ, and hence permeability curves are called B-μ curves.

§ 2. **Hysteresis Curves.**—If, instead of measuring the total change of magnetic flux produced by revers-

ing the magnetising force, we proceed to increase and diminish the magnetising force in a cyclic or periodic manner, step by step, we can delineate another curve, called the *Hysteresis Curve*, of the iron which has very important properties. Suppose that an iron ring is taken in a perfectly unmagnetised condition, and starting from this state, we apply a magnetising force represented by + 1 (see Fig. 49), and measure the magnetic flux density produced by it in the iron. Next, let the magnetising force be increased to + 2, and the *increment* or increase of flux density B, caused by this increase of H from + 1 to + 2, be measured. Proceeding in this manner, let us suppose the force increased to + 4, and then diminished again to zero. It will be found that the values of the magnetic flux density for given magnetising forces, during the period of decrease of the force, do not agree in value with those for the corresponding values of the force during the period of increase. In other words, the flux density is not the same on the way down as on the way up, but *lags behind*. The term *hysteresis* denotes a lagging behind or want of correspondence between two things.

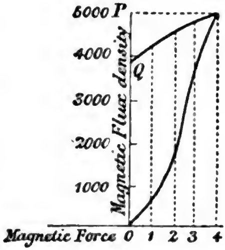

Fig. 49.—Portion of Magnetisation Curve of Iron Ring showing Hysteresis.

The firm line curve in Fig. 49 delineates the relation of magnetic flux density to magnetic force during these operations. It will be seen that, after the magnetic force has been applied and removed, magnetic flux still lingers in the iron or is retained, and this property of the iron is called its *Retentivity*. The ordinate O Q represents the magnetic flux density remaining after the force has been again reduced to zero. The ordinate O P represents the maximum value of the flux density attained, and the ratio of O Q to O P (expressed as a

percentage) is the measure of the retentivity of the iron under those conditions. It may even reach 90 per cent. In the next place, after reducing the magnetising force to zero let us suppose it is reversed, and a force of -1 applied, and then -2, and so up to -4; after which, let the force be again increased and brought back to zero. In this way a *complete cycle of magnetising force* is applied, ranging from 0 to $+4$, from $+4$ to 0, from 0 to -4, and from -4 back to 0 again. Corresponding to this, we find that the magnetic flux in the iron goes through a cycle or range of periodic values, and that the flux and force are related to one another, as shown by the ordinates and abscissæ of the closed curve in Fig. 50. This curve is called a *hysteresis diagram or loop*. The ordinate O P, represents the maximum value of the magnetic flux density during the cycle. The ordinate O Q represents the magnetic flux density retained in the iron after the force is withdrawn, and represents, therefore, the *retentivity* of the iron. The intercept or distance O R, represents the negative or reverse magnetic force which has to be applied to reduce the magnetic flux to zero, after applying a positive force, and it is therefore the negative magnetising force required to annul the result of applying a positive magnetising force O M. This force O R is called the *Coercive force* required, and the length O R is a measure of the *coercivity* of the iron.

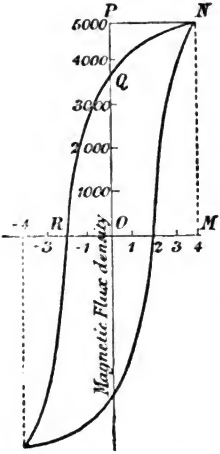

Fig. 50.—Complete Hysteresis Loop for Soft Iron Ring.

This hysteresis diagram or loop, therefore, shows us at a glance the magnetic qualities of the material, and

it has another important property as well. In the case of a steam-engine indicator diagram, the area of the diagram represents the *work done* in the engine cylinder by the steam at each stroke. In the same way when iron is magnetised, the work done per unit of volume of the metal, in increasing its magnetic flux, is measured by the product of the increment of the flux density and the force. Owing to the unfortunate fact that the unit-magnetic pole is defined by the mechanical force between it and another unit pole, instead of being defined as a pole from which proceeds a unit of magnetic flux, we have a numerical constant introduced into the product expressing the work done in making an increase in the magnetic flux density in iron. Properly speaking, the work done in making a small increase in the magnetic flux density in iron should be numerically expressed by the product of the mean magnetising force and the small increase in magnetic flux density made by it. Owing, however, to the use of an irrational system of units, we have, in fact, to introduce a multiplier $\frac{1}{4\pi}$ where π is the ratio of the circumference of a circle to its diameter, viz. $3 \cdot 1416$, or nearly $\frac{22}{7}$. Hence, since $\frac{1}{4\pi}$ is very nearly $\frac{7}{88}$, we have, under the present system of irrational magnetic units, to remember that the work done in increasing, by a *very small amount*, the magnetic flux in a cubic centimetre of iron is measured in ergs per cubic centimetre of the iron by $\frac{7}{88}$ times the product of the mean impressed magnetising force and the small corresponding change or increase of the magnetic flux density produced by the increment of the force. The magnetic flux density and magnetising force being measured in C.G.S. units. If the magnetic flux density is measured in *webers per square centimetre*, and the mag-

netising force in *ampere-turns per linear centimetre*, then the work done in taking a cubic centimetre of iron through a magnetic cycle is measured in *joules*. In any case, the *whole area of the hysteresis loop represents the total work done* in taking a cubic centimetre of the iron through the magnetic cycle represented by that loop, and is generally expressed in ergs.

The reader may be assisted to understand the above mentioned statements by considering the analogy between *magnetic work* and *mechanical work*. Imagine a cylinder with a tightly fitting but frictionless piston in it, and let the cylinder be filled with a volume of air equal to v cubic centimetres, and under a pressure of p dynes per square centimetre. If the piston is pushed in *a little way*, so as to cause a *small decrease* in the volume of the air by an amount represented by dv, then *work* has to be done to overcome the resistance of the air to compression, and it is easy to see that the work done in making this small change in volume in the air is equal to $p\,d\,v$ ergs.

In the same way if a cubic centimetre of a magnetic substance has in it a magnetic flux density B, and if it is under a magnetising force H, and if the force is increased so as to cause a *small increase* in the magnetic flux by an amount represented by dB, the *work done* in effecting the change is equal to H dB ergs, *if the magnetic units are rational units*. Unfortunately, the C.G.S. units are not rational units, and hence the intrusive 4π comes in, and we have the value of the small amount of work done in increasing the magnetic flux density by a small amount dB, the mean magnetising force being H, given by the expression $\frac{1}{4\pi}$ H dB. Hence it follows, that the work done (reckoned in ergs) in taking one cubic centimetre of iron through one complete magnetic cycle, is equal to $\frac{1}{4\pi}\left(=\frac{1}{12\cdot 76}\right)$ of the area of the hysteresis diagram, reckoned in terms of a rectangular unit of area, one side of which is the length taken as a unit of magnetic force, and the other the length taken as a unit of magnetic flux density.

§ 3. **Steinmetz's Law.**—If we take one and the same iron ring through a series of magnetic cycles,

increasing at each cycle the maximum value of the magnetic flux density, we can delineate a series of hysteresis loops as in Fig. 51, each one of which corresponds to a definite maximum value of the magnetic flux density during the cycle. If the areas of these loops are measured with a planimeter, and the areas set out as distances on a vertical scale, with the corresponding maximum values of the magnetic flux density as horizontal distances, we obtain a curve (see Fig. 52) called a Steinmetz curve.

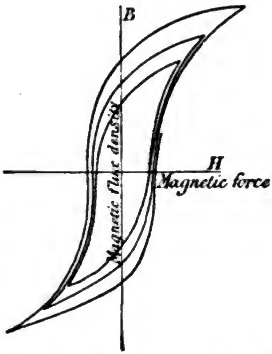

Fig. 51.—Series of Hysteresis Loops for Wrought Iron Ring for progressive Maximum Flux densities.

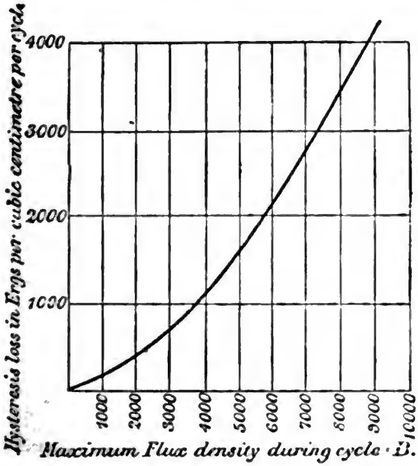

Fig. 52.—Steinmetz Curve.

This curve shows us the relation between the work done in taking the iron through a magnetic cycle, and the maximum value of the induction density during that cycle. It is found that the hysteresis loss in work done in performing a magnetic cycle, varies as the 1·6 power of the maximum magnetic flux density reached during the cycle. If B denote this maximum value, represented by the line N M in Fig. 50, then the work done in carrying the iron round the magnetic cycle is measured by the product of a certain constant called the *Hysteretic Constant*, and the value of $B^{1·6}$.

Generally speaking, for most varieties of pure iron, the work W, measured in ergs, spent in taking one cubic centimetre of iron through one complete magnetic cycle, is nearly given by the equation

$$W = ·002\ B^{1·6}.$$

The arithmetic constant ·002 is called the *hysteretic constant*, and the values of the hysteretic constants for different kinds of iron and steel are given in the table below.

TABLE OF HYSTERETIC CONSTANTS.

Metal.	Hysteretic Constant.
Swedish wrought iron, well annealed	·0010 to ·0017
Annealed cast steel of good quality, small percentage of carbon	·0017 to ·0029
Cast Siemens-Martin steel	·0019 to ·0028
Cast ingot iron	·0021 to ·0026
Cast steel, with higher percentages of carbon, or inferior qualities of wrought iron	·0031 to ·0054

The effect of annealing on the value of the hysteretic constant is very marked; thus, we find that Swedish cast steel (annealed) has a hysteretic constant of ·0015, but

Swedish cast steel (unannealed) has a hysteretic constant of ·0029; also German cast steel (annealed) has a hysteretic constant of ·0017, but German cast steel (annealed) has a hysteretic constant ·0033.

In order to calculate the hysteresis loss in ergs per cubic centimetre, for any value of the maximum magnetic flux density or induction, we require a table giving the values of $B^{1.6}$ for different values of B as follows:—

TABLE GIVING THE VALUES OF $B^{1.6}$ FOR VARIOUS VALUES OF B.

Maximum Value of the Magnetic Flux Density or Induction B.	Value of $B^{1.6}$.	Value of $\cdot 002\, B^{1.6}$.
1,000	63,100	126
2,000	191,300	383
3,000	365,900	732
4,000	580,000	1160
5,000	828,800	1658
6,000	1,111,000	2222
7,000	1,420,000	2840
8,000	1,758,000	3516
9,000	2,122,000	4244
10,000	2,511,000	5022

The last column in the above table gives the hysteresis energy loss in ergs per cubic centimetre of soft iron, produced by one complete cycle of magnetisation, in which the maximum value of the flux density or induction during the cycle has the value given in the first column of the table.* It is easy to see from the above figures, that if a cubic foot of soft iron is carried through a complete cycle of magnetism, in which the maximum flux density

* The figures in the third column of the table in question are merely typical of ordinary good iron, and must not be considered as the absolute values for any iron whatever.

is 10,000 C.G.S. units, that this operation requires the expenditure of about 10 foot-pounds of energy. To do this 100 times a second, therefore involves doing 1000 foot-pounds of work. It is obvious, therefore, that magnetising and demagnetising iron quickly, is an operation which absorbs a considerable amount of power. The power absorbed is dissipated as heat in the iron. The energy dissipation increases at a much more rapid rate than the maximum flux density. Hence, whenever iron has to be magnetised and demagnetised quickly or frequently, the flux density should be kept low, if it is desired to economise power.

The hysteresis diagram of an iron is therefore capable of affording us a large amount of information as to the magnetic utility of the metal, and its suitability for various purposes. Thus, for instance, the maker of a dynamo requires a quality of iron or steel for the magnets of his dynamo magnets which shall have very high magnetic flux density for small magnetic forces, or large permeability. He is not, however, very particular in requiring small hysteresis loss. He therefore selects an iron or steel which has a hysteresis diagram somewhat like the curve in Fig. 53.

The loop is very upright, because the magnetic flux density is relatively large for small magnetising forces; that is to say, the permeability is high. The area of the loop is moderately small, and hence the iron has a fairly small hysteretic constant.

The maker of an alternating current transformer is not nearly so particular about the permeability of the iron or steel he uses, but he is very desirous of obtaining the smallest possible hysteretic constant. He therefore employs a mild steel, or pure Swedish iron, having a hysteresis diagram like the curve in Fig. 54. This curve has a very small area, thus indicating the small hysteretic value of the metal.

The maker of a permanent steel magnet does not care at all about the hysteretic constant or permeability

of the steel he employs, but he does require a steel which shall have large *retentivity* and large *coercivity*. He therefore selects a steel such as glass-hardened tungsten steel, which has a hysteresis diagram like the curve in Fig. 55. In this diagram the intercept O Q (representing the retentivity) is large, and also the intercept O R (representing the coercivity). It will be seen, therefore that it is impossible to have large retentivity and great

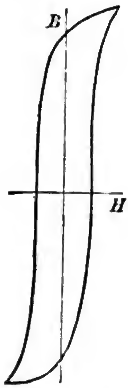

Fig. 53.—Type of Hysteresis Loop required for Dynamo Steel.

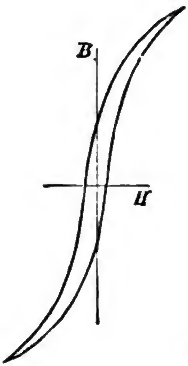

Fig. 54.—Type of Hysteresis Loop required for Transformer Steel.

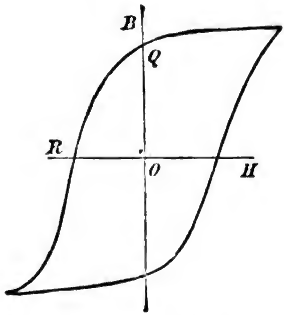

Fig. 55.—Type of Hysteresis Loop required for Permanent Magnet Steel.

coercivity in a steel without at the same time having large hysteretic constant. Hence it follows that all varieties of magnet steel which are useful for making good permanent magnets, are very bad qualities of steel to employ for the cores of transformers, or for field magnets and armatures of dynamos. Fortunately it happens that we can adjust the quality of the metal for the purpose for which it is to be applied.

§ 4. **Electromagnets.**—An electromagnet consists of an iron or steel core, called the magnet body, which is surrounded by coils of insulated wire, called the magnet coils, through which a magnetising current is passed. If the iron is a completely closed iron ring, the magnet is called a *poleless electromagnet*. If, as is more usually the case, the iron does not form a complete circuit, then it has one or more air spaces in the magnetic circuit, called the *air gap or gaps*. The terminations of the iron circuit which bound the air gaps are called the *pole pieces* or pole faces. Very often the magnet has the form of upright round iron legs, on which the coils are wound, united by a cross piece, called the *yoke*.

For experimental purposes, the poles are often fitted with removable *pole pieces*.

The material most commonly used for the cores of electromagnets is soft Swedish iron, on account of its high permeability. Soft Siemens-Martin steel is, however, more easily obtained now, and for most purposes, where rapid changes of magnetism are not required, is quite as effective. The steel will not generally fall below the iron in permeability by more than 5 or 6 per cent. at high magnetisations, even if not superior to it, whilst the permanent magnetisation of the steel, as compared with the iron, after removal of the magnetising force, will not generally be greater by more than 10 per cent.

The legs and yoke of the magnet may be either of solid iron or steel, or else built up of wire or plates, in which case they are said to be *laminated*.

In the construction of an electromagnet, the object, generally speaking, is to procure the strongest possible magnetic flux density in the interpolar air gap or gaps, and to obtain this by the least possible *excitation* or ampere-turns on the magnetising coils.

In the construction of an electromagnet it is important, therefore, to be able to predetermine the magnetising force in ampere-turns per centimetre required to produce a given magnetic flux density. In certain cases

this can be done without difficulty. Let us consider a few of the simpler instances.

Take first the case of a circular iron ring. When this is magnetised by an endless solenoid wound on it, it forms a poleless magnet. Suppose that the dimensions of the ring are given, and that it is required to predetermine the magnetising force in ampere-turns necessary to produce practical magnetic saturation in this iron. This involves creating in the iron a flux density of, say, 16,000 to 18,000 C.G.S. units or 160 to 180 microwebers per square centimetre.

To determine the windings to be put upon the iron ring, we must consult a magnetisation curve for a complete iron circuit, and from that curve (see Fig. 47, p. 178) we see that, to produce a magnetic flux density of 18,000 C.G.S. units in an iron ring, requires the application of a magnetising force of about 100 C.G.S. units, or of 100 ampere-turns per half inch of length of the mean perimeter of the ring. If, therefore, the mean perimeter of the ring is known, we can at once deduce the total ampere-turns to be put upon the ring.

Let us suppose the mean perimeter of the ring is 12 inches, and the diameter of cross-section 1 centimetre. We then require 2400 ampere-turns on the ring to magnetise it practically to saturation. This alone does not tell us how many turns of wire to put upon the ring.

Let us suppose we have at disposal some No. 18 cotton-covered copper wire. It will be found possible, on a ring of the above dimensions, to wind on about five or six layers of No. 18 wire, each layer having 100 turns. For a short time this wire would carry a current of about 4 amperes without overheating, and thus the necessary magnetising force, consisting of 4 amperes flowing 600 times round the ring, or 2400 ampere-turns, could be obtained.

There is no absolute rule for fixing the size and length of wire to be employed for the magnetising coil; it has to be determined by various conditions, such as

the electromotive force at disposal for producing the current, and the amount of power which it is decided shall be dissipated in this coil as heat.

In the next place, let us consider a slightly less simple case, viz. that of an iron ring with a narrow cut or air gap in it. If a magnetising coil is to be wound on this ring, to produce a magnetising force creating a stated magnetic flux, the predetermination of the ampere-turns to be put upon the ring is based upon the principle that the total magnetomotive force or ampere-turns to be applied to the ring, to produce a required magnetic flux round the circuit and across the air gap, may be divided into two parts, viz. one part which is required to overcome the reluctance of the iron itself, and the other part to overcome the reluctance of the air gap. It is to be noted that magnetomotive forces are additive, that is to say, we obtain the total magnetomotive force required if we estimate separately the magnetomotive forces (in ampere-turns) required to force the required induction, or magnetic flux, respectively through the iron and through the air gap, and then add the values of these magnetomotive forces together to obtain the nett or resultant magnetomotive force.

For the sake of illustration let us suppose the ring has a cross-sectional area of 2 square centimetres, and a mean perimeter of 30 centimetres, and let the air gap be very narrow, and be only 1 millimetre in width. Let us further suppose that we require to produce a magnetic flux density of 10,000 C.G.S. units across the air gap. In order to reduce the difficulties of the problem, we must, in the present instance, suppose that the magnetic flux is wholly confined to the iron, and jumps across the air gap without spreading, so that the air space which is magnetised is 1 millimetre or 0·1 centimetre in length, and 2 square centimetres in section. Hence, since the reluctivity of air is taken as unity, the reluctance of the air gap is equal to $\frac{\text{length}}{\text{section}}$ of the air gap, or is represented

by the number $\frac{1}{20}$. The above statement is not strictly true. As a matter of fact, the magnetic flux in crossing the air gap spreads or swells out, so that it is somewhat more difficult to calculate the actual reluctance of the air gap. If the magnetic flux density is to be 10,000 C.G.S. units per square centimetre, and if the section of the ring is 2 square centimetres, it is obvious that we have to produce in the ring a total magnetic flux of 20,000 C.G.S. units or 200 microwebers.

Bearing in mind that the quotient of the numerical value of the magnetomotive force by that of the reluctance gives us the value of the total magnetic flux, and also that the magnetomotive force in C.G.S. units is equal to $1\frac{1}{4}$ times the ampere-turns, it is easy to show that the ampere-turns to be put upon the circuit to overcome any given air reluctance is given by the rule—

$$\text{Ampere-turns} = 0{\cdot}8 \times \text{the total magnetic flux} \times \text{the air gap reluctance.}$$

The coefficient $0{\cdot}8$ being the reciprocal of $1\frac{1}{4}$ or of $\frac{4\pi}{10}$.

Hence to overcome the reluctance of the air gap represented by $\frac{1}{20}$, and force through it a total magnetic flux of 20,000 C.G.S. units, requires

$$0{\cdot}8 \times 20{,}000 \times \frac{1}{20} = 800 \text{ ampere-turns.}$$

To overcome the reluctance of the air gap, having a width of 1 millimetre, requires therefore 800 ampere-turns. Next, as regards the iron. On consulting the magnetisation curve of iron (see Fig. 47, p. 178) we see that to produce a flux density of 10,000 C.G.S. units in soft wrought iron, requires approximately a magnetising force of 4 C.G.S. units, or of 4 ampere-turns per half-inch of the iron circuit. Since the iron circuit in this case is

30 centimetres, or nearly 24 half-inches, in length, we shall require 96 ampere-turns on the ring to overcome the reluctance of the iron. The total ampere-turns required is then 896, made up of 800 ampere-turns required to force the magnetic flux across the 1 millimetre air gap, and 96 ampere-turns to force it through the 300 millimetres of iron. This fact will bring forcibly to the student's notice the effect of an air gap, however narrow, in increasing the total magnetomotive force required to produce a given total magnetic flux in that magnetic circuit. When the air gap is large, the problem of determining the ampere-turns to produce a given flux density is much more difficult, and its solution is beyond the limits of this treatise.

§ 5. **The Lifting Power of Electromagnets.**—If a piece of soft iron is held against the pole of a magnet, or if a *keeper* is placed across the poles of a horseshoe-shaped electromagnet, it requires a certain pull or force to detach the iron or keeper. The weight in grams or pounds required to effect this detachment is called the *lifting power* of the magnet.

The laws governing the tractive power of magnets are best examined by considering, in the first place, the force required to separate two equal magnetic poles, with perfectly flat surfaces, placed against each other.

Consider the case of two uniformly and equally magnetised magnets with perfectly flat ends, having their opposite poles pressed against each other. Let the strength of each of these poles be m units (ordinary or C.G.S. system), and let the magnetic flux density over the contact surface (which is really a very narrow air space) separating the poles be denoted as usual by B. Then we have seen in Chapter III. that the total magnetic flux coming out of the pole of strength m is $4\pi m$ units, and hence if the section of each magnet is s square centimetres, and we suppose the whole flux to pass across the surface of contact, we have the relation

$$B s = 4 \pi m.$$

This equation expresses the fact that the total flux out of the pole is equal to 4π times the pole strength.

The total flux across the very narrow air gap separating the attracting poles may be considered to be made up of two parts, one half being as it were a flux belonging to, and coming out of one pole, and the other half being a flux *in the same direction*, proceeding into, and belonging to, the adjacent opposite pole. Hence we may regard the attraction of the poles as being due to the tendency of the one pole of strength m to move in a magnetic field of strength $\frac{B}{2}$ belonging to the other pole, since the field strength of either of the magnets just outside the surface of its flat pole is numerically equal to the flux density of the flux attached to, and existing in the body of that magnet.

The mechanical force which acts upon a magnetic pole placed in a magnetic field, not its own, is numerically equal to the product of the pole strength and the field strength. Hence the mechanical force pressing the poles together, that is to say the total pressure between them reckoned in dynes, must be numerically equal to the product $m \times \frac{B}{2}$. We have already shown that in the C.G.S. system the total magnetic flux coming out of a pole of strength m is $4\pi m$ units. If the flux density at the polar surface of the magnet is denoted by B, and if the polar surface has an area of s square centimetres, the total flux out of the pole is $B\,s$ units. Hence the relation between these quantities is expressed by the equation

$$4\pi m = B\,s,$$

or

$$m = \frac{B\,s}{4\pi}.$$

It follows, then, that the mechanical force between the polar surfaces of area s square centimetres is equal to

$$m \times \frac{B}{2} \text{ dynes},$$

or to

$$\frac{B\,s}{4\pi} \times \frac{B}{2} \text{ dynes},$$

or to

$$\frac{B^2}{8\pi} s \text{ dynes.}$$

Hence the "pull" or attraction between the poles per square centimetre of surface is

$$\frac{B^2}{8\pi} \text{ dynes.}$$

If a weight of W grammes is required to be applied to one magnet to detach it from the other, then, since this weight W is equivalent to a mechanical force of 981 W dynes, we have the equation

$$981\,W = \frac{B^2}{8\pi} s,$$

or

$$157\sqrt{\frac{W}{s}} = B.$$

The above equation tells us that the *flux density across the surface of contact of the poles is equal to* 157 *times the square root of the grammes weight per centimetre of total surface required to detach the poles.* The above rule enables us to calculate the value of B when we have the observed value of $\frac{W}{s}$.

It will be seen, therefore, that the measurement of the tractive force of a magnet affords a means of measuring magnetic flux density; because, if in the above equation the value of s and W as obtained by observation is inserted, the value of B can be calculated.

Experiments to determine the lifting power of electromagnets have been carried out by Mr. Shelford Bidwell and others. The following is the description of an experiment made by Mr. Bidwell :—

Two pieces of apparatus were prepared. The first consisted of a rod of iron hooked at each end and divided transversely in the middle, together with a long solenoid, inside which the divided rod could be placed. The second was an iron ring cut into two equal parts, each of

which was encircled with a coil of insulated copper wire. In both cases the construction was such that an intense magnetic force could be produced with comparatively small battery power. The divided ring could be used either as a semicircular electromagnet with a semicircular armature, or, if the current were passed through both coils, as two semicircular electromagnets.

Merely to test the suggestions of Joule and Rowland, that a limit to the lifting power of an electromagnet was a tractive force of 200 lb. per square inch, two or three determinations were made of the weight which could be sustained when the current was caused to circulate around one only of the semicircles, the other being used as an armature. With a current of 4·3 amperes the weight supported was 13,100 grams per square centimetre of surface; with a current of 6·2 amperes the weight supported was 14,200 grams per square centimetre. In the latter case, therefore, the lifting power exceeded that which both Joule and Rowland considered the greatest that could be imparted to a magnet by an infinite current. Had it been worth while to incur the risk of injury to the insulation of the coil, there is no doubt whatever that, by applying stronger currents, the lifting power might have been carried still further—for there was no indication that a limit was being approached. But it was of greater interest to study the effects produced when both portions of the ring or of the rod were under the direct influence of the magnetising coil.

The first experiment was made with the divided rod. One portion was supported by means of its hook in a vertical position; a scale-pan was attached to the hooked end of the other portion, and the flat ends of the two were brought into contact and surrounded by the solenoid. Currents of gradually increasing strength were then caused to pass through the solenoid, and note was taken of the greatest weight which could in each case be placed in the scale-pan without tearing asunder the ends of the two rods. The general results are briefly as

follows. When the magnetising force (H) due to the solenoid had reached about 50 C.G.S. units, the weight supported was nearly 7000 grammes per square centimetre of the section of the rod. After this value was exceeded, it became quite evident that the weight which could be sustained was increasing more slowly than the magnetising current, and the proportionate increase became rapidly smaller as the current was made stronger. This state of things continued until the magnetising force was about 270 units, when the weight supported amounted to 10,800 grammes per square centimetre of section. But from this point onwards *the magnetising force and the weight that could be carried increased in exactly the same proportion.* The rate of increase of the load was, indeed, comparatively small, but it was perfectly constant, and continued so until the field had attained the high intensity of 1074 C.G.S. units. Here the experiment was stopped, the greatest weight supported having been 15,100 grams per square centimetre.

On account of some uncertainty as to the possible influence of the external ends of the divided rod, it was thought desirable to make the experiment with the divided ring, the current being caused to pass in the same direction through the coils surrounding both portions. The general character of the results was the same as before, but the weight supported per unit of area was from first to last somewhat greater. The falling off in the rate of increase of the lifting power was well marked when the magnetic force had reached 50 C.G.S. units, at which point the weight sustained was about 10,000 grams per square centimetre; and it continued to diminish until the magnetic force was 250 units and the weight supported 14,000 grammes. From this point the increments of lifting power and of magnetic force appeared to be exactly proportional, and continued to be so until the magnetic force had been carried up to 585 units, when the limit of the battery power was reached and the experiment stopped, the maximum

weight supported having been 15,905 grammes per square centimetre, or 229·3 lbs. per square inch.

Detailed results of the experiment with the divided ring are given in the first and second columns of the table below. A curve, plotted with the magnetic forces as abscissæ and the weights lifted as ordinates, becomes sensibly a straight line inclined to the horizontal axis for the values of the magnetic force greater than 240 units.

LIFTING POWER OF ELECTROMAGNETS.

H = magnetising force.
I = magnetisation.
κ = susceptibility.
W = weight in grms. per sq. cm.
B = magnetic flux density.
μ = permeability.

H	W	I	κ	μ	B
3·9	2210	587	151·0	1899·1	7390
5·7	3460	735	128·9	1621·3	9240
10·3	5400	918	89·1	1121·4	11550
17·7	7530	1083	61·2	770·2	13630
22·2	8440	1147	51·7	650·9	14450
30·2	9215	1197	39·7	500·0	15100
40	9680	1226	30·7	386·4	15460
78	11550	1337	17·1	216·5	16880
115	12170	1370	11·9	150·7	17330
145	12800	1403	9·7	122·6	17770
208	13810	1452	7·0	88·8	18470
293	14350	1474	5·0	64·2	18820
362	14740	1489	4·1	52·7	19080
427	15130	1504	3·5	45·3	19330
465	15275	1508	3·2	41·8	19470
503	15365	1510	3·0	38·7	19480
557	15600	1517	2·7	35·2	19630
585	15905	1530	2·6	33·9	19820

The above table shows us that, as the magnetising force is gradually increased, the magnetic flux density, magnetisation, and tractive power all increase also. There is no indication, however, that any absolute limit can be reached for the tractive power. By the use of a very powerful magnet, Mr. Wilde has produced a tractive force of 29,676 grammes per square centimetre, or 422 lbs. per square inch between magnetised surfaces of soft iron —thus far exceeding the amount of 200 lbs. per square inch, which was once considered to be a limiting value. There is also no indication that the flux density would approach a limit. As previously mentioned, the flux density in iron has been raised to a value of 40,000 C.G.S. units or more. There are, however, indications that the magnetisation, or intensity of magnetisation (I), has a limiting value for each magnetic metal, and, in the case of iron, cannot be raised beyond a value of about 1600 or 1700 C.G.S. units.

The traction method is occasionally useful as a workshop method for determining the permeability of iron samples. Special forms of apparatus for doing this —called *permeameters*—have been invented by Prof. S. P. Thompson, Mr. Kapp, Dr. du Bois, Prof. Ewing and others. As an illustration of the method, the following description of an experiment made by the Author may be given :—

Two small horseshoe-shaped or U-shaped electro-magnets were made, and the polar end surfaces accurately faced. The legs of these magnets had a diameter of 0·476 centimetres, or a cross-sectional area of 0·177 square centimetres, and the mean length of each magnet was 8·75 centimetres. One magnet was suspended, and the other placed with its poles in contact with those of the first, so that the two magnets together formed a closed iron magnet circuit. A measured current was then sent through the wire coils, all joined in series in such a manner as to cause the poles in contact to attract each other. The weight in grammes W required to pull

the magnets apart was then determined. The number of turns on each leg of each magnet was 385, hence in all there were 4×385 or 1540 turns of wire. The mean length of the whole magnetic circuit was $8 \cdot 75 \times 2$ or $17 \cdot 5$ centimetres. Therefore, the magnetising force H being applied to the iron by a known current of A amperes passing through the coils, and which is equal to $1\frac{1}{4}$ times the ampere-turns per centimetre, is given by the equation—

$$H = 1\tfrac{1}{4} \times \frac{1540}{17 \cdot 5} A = 110 A.$$

The magnetic flux density B across the polar surfaces having a *total area of contact* of s square centimetres, is related to the tractive force W by the equation—

$$157 \sqrt{\frac{W}{s}} = B.$$

Hence, in this case, since s (the total opposed polar surface) is equal to $0 \cdot 354$ square centimetres, we have—

$$B = 157 \times \frac{\sqrt{W}}{\sqrt{0 \cdot 354}} = 264 \sqrt{W}.$$

The measurement, therefore, of the current in amperes A which had to be passed through the coils so as just to hold up a total weight of W grammes, gives us the means of determining the value of B and also of H, viz. the magnetic flux density and the magnetising force, and hence the ratio of B to H, which is the permeability (μ) required. In one particular case it was found that a weight of 64 grammes just sufficed to pull off the lower magnet when a current of $\cdot 01$ ampere was flowing round the coils. Hence we have—

$$H = 1 \cdot 1$$
$$B = 2112;$$

therefore the permeability μ, which is the quotient of B by H, is equal to $\frac{2112}{1 \cdot 1} = 1920$.

In this way two simple measurements, which can be made in the workshop without difficulty, suffice to determine the permeability of a sample of iron.

The student should make, or get made for him by a blacksmith, two semicircular rings of soft iron. These may be made of iron rod one quarter of an inch in diameter. The ends must be carefully squared and polished, and made to fit exactly, so that when the two halves are put together they make a complete ring. Each half ring must then be wound over with several layers of No. 18 double cotton covered wire, and before so doing it is best to wind on the bare iron a layer or two of silk tape. One of these electromagnets is then to be hung up with poles downwards to a fixed support. The other must have a scale pan attached to it. Weigh the last electromagnet and scale pan, and reckon this total weight as a fixed quantity to be added to the weight put in the pan.

Then pass a measured electric current of A amperes through both coils, joined up in series, and determine the total weight in grammes (W) required to detach the lower magnet. Knowing the total number of turns of wire (N) in both magnets, and the cross sectional area s of both pole surfaces taken together, calculate by the formula

$$H = \frac{4\pi}{10} \frac{N}{L} A = \text{(magnetising force)},$$

$$B = 157\sqrt{\frac{W}{s}} = \text{(magnetic flux density)},$$

where L is the mean length of the total magnetic circuit. Then from the values of B and H determine the value of (μ) the permeability of the iron for the flux density of (B) employed.

When a piece of iron is held at a distance from the pole of an electromagnet it is pulled or attracted towards it. The reason for this is because the iron, when held in a non-uniform field, is acted upon by a mechanical force

urging it from places where the field is weak to places where it is strong. This mechanical force is proportional to the product of three factors, viz. the magnetic susceptibility (κ) of the body, the strength of the field, and the rate at which the strength of the field varies in the direction of the movement. Hence there is no tendency for a piece of iron to move bodily in a *uniform* magnetic field. If an electromagnet has a pointed pole, then near

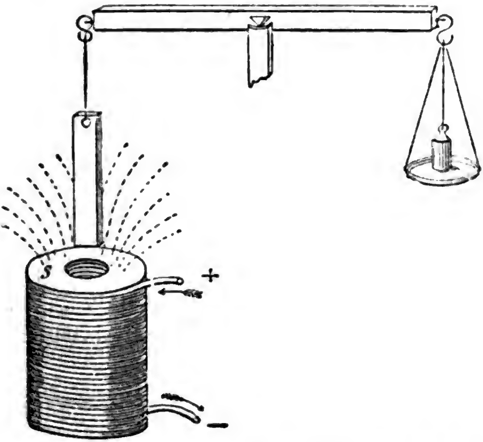

Fig. 56.—Attraction of a Soft Iron Rod into a Helix traversed by a Current.

that pole the field strength varies very rapidly, and hence there is a strong force causing a small mass of iron held near it to move up against the pole. In the same way, if a rod of iron is held suspended just near the mouth of a solenoid, when a current is passed through the solenoid the iron will be sucked into the coil. It is drawn in because it tends to move into the stronger field in the centre of the solenoid.

Advantage of these facts is taken in the construction of many electrical instruments and machines.

It should be noted that a piece of iron, as a whole, has no tendency to move along the lines of a uniform field, but if one end is in a place of stronger field than the other, the iron will tend to move until equilibrium is established between the mechanical forces acting on the different parts of the bar. If an iron rod is free to move,

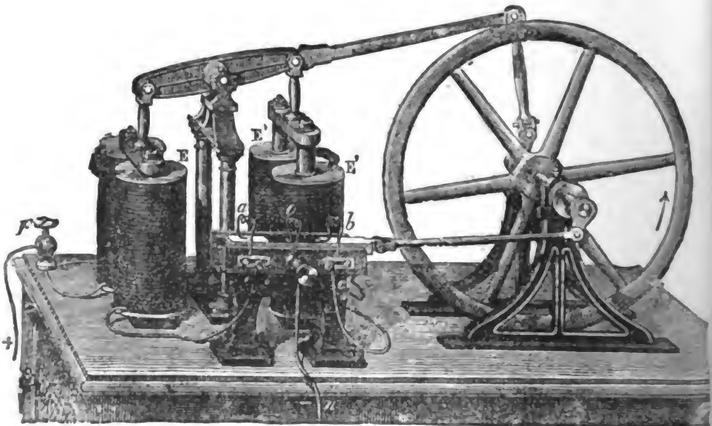

Fig. 57.—Bourbouze's Electric Motor.

and is held near the mouth of a solenoid traversed by a current (see Fig. 56), it will be sucked into the solenoid until it is symmetrically placed with respect to it; that is, until the forces on each end of the bar are in equilibrium.

It is hardly necessary to point out that if, instead of suspending a bar of soft iron from the scale beam, we had employed a permanent magnet, then we should have had attraction or repulsion, according as the end of the

magnet presented to the solenoid is an opposite or a similar pole.

The action of a solenoid on an iron rod is applied in the construction of the magneto-electric motor, called after its inventor Bourbouze's motor, in which motion is communicated to a flywheel by pistons of soft iron, which are alternately sucked into cylindrical coils of wire, current being distributed to each in turn by a sort of slide-valve arrangement. (See Fig. 57.) The alternate pulls of pairs of solenoids, operating on plungers of soft iron attached to opposite ends of the beam, are made to produce oscillatory motion, which is converted into circular motion by means of a crank and connecting rod.

§ 6. **Construction of Electromagnets for various purposes.**—The form which an electromagnet must take, will depend upon the nature of the operations it is to perform, or the purpose to which it is to be put. A design of electromagnet which is very suitable for some purposes, may be very unsuited for others. Supposing it is desired to make an electromagnet which shall be capable of rapid changes of strength, or possess small residual magnetism, it should be made of very pure Swedish iron well annealed, and have the form of a short stout bar, rather than a long thin one. The reason for this is that the ends or poles of a magnet exert a demagnetising action upon the mass of the interior of the magnet. If the iron bar has the form of a long thin bar, or a wire whose length is, say, 300 times as great as its diameter, then the poles or ends are very far removed from the middle, and the demagnetising action is feeble; and such a long thin magnet, even though made of very soft iron, will retain a good deal of magnetism after the magnetising force is removed. On the other hand, a short thick bar quickly demagnetises itself, even without the assistance of shakes or twists. For the same reason, a soft iron ring magnet, with no free poles, retains magnetism to a very large extent after the magnetising force is removed. When, as in many telegraphic instruments, a

piece of soft iron, called an armature, is to be attracted to the poles of a horseshoe-shaped electromagnet, this armature should be prevented from quite touching the polar faces of the magnet, either by the interposition of paper or a brass stud. If the soft iron mass does quite touch the poles, then it completes the magnetic circuit, and abolishes the free poles, and the magnet is deprived to a very great extent of its self-demagnetising power. This is the explanation of the well-known fact, that after magnetising an electromagnet and then stopping the current, it still requires a good pull to detach the "keeper," but when once the keeper has been detached, the iron exhibits comparatively small magnetic qualities. If the use to which the electromagnet is to be applied is that of attracting a soft iron keeper or armature, then its form will depend upon whether that attraction or pull has to be exerted over a large or small distance. In the case of ordinary horseshoe electromagnets with flat poles, the strength of the magnetic field diminishes very rapidly as we recede from them, and accordingly such magnets, though attracting with considerable power when the armature is very near the poles, exercise but little force on the armature when it is a short way removed from the poles. It was this fact which rendered the early efforts to construct electromagnetic engines so fruitless. If it is desired to construct a magnet which shall exercise a strong pull upon a keeper at a very short distance, then the magnet should be of horseshoe shape, and have broad flat ends kept far apart, and the keeper to be attracted should also have large surfaces opposed to the polar ends, and its cross-section should not be less than the least cross-section of the iron of the electromagnet. In the case where a bar electromagnet is used, increased effect is obtained by surrounding the bar with a tube of soft iron, attached at one end to the base plate which carries the bar, and having the other edge level or flush with the polar end. (See Fig. 58.) The outer iron case serves to decrease the reluctance of the circuit, and

to strengthen the field just above the polar surface of the magnet. If the electromagnet is employed to produce a pull over a great distance, means must be adopted to prevent a very rapid rate of diminution of the field in receding from its poles. An ingenious device for doing this is adopted in the Thomson-Houston dynamo, and is also applied in the arc lamps of the same inventors. The pole of the electromagnet is prolonged into a sort of nose (see A, Fig. 59), and the armature to be attracted is pierced with an aperture through which this nose protrudes. The effect of this contrivance is to render the pull of the magnet on its armature more uniform and operative over a greater distance. At one time, Mr. Joule occupied himself a

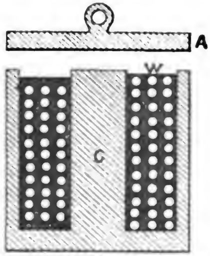

Fig. 58.—Tubular Electromagnet.

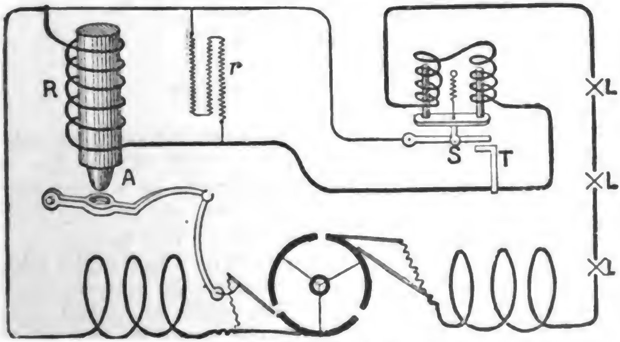

Fig. 59.—A, Long-pull Electromagnet in Thomson-Houston Dynamo Regulator.

good deal with the question of the best method of producing electromagnets which could support great

weights, or which could exercise immense attractive power over armatures in contact with their poles. After many experiments he discovered that the most effective form was obtained by taking a thick cylinder of soft iron, boring a hole lengthwise through it, planing over one side of the cylinder so as to expose the longitudinal hole, and providing the horseshoe sectioned bar with a long segment-shaped piece of soft iron, as a keeper. (See Fig. 60.) The iron cylinder was then wound over lengthwise with strands of insulated wire, and the magnet and keeper provided with means for supporting and attaching to them weights respectively.

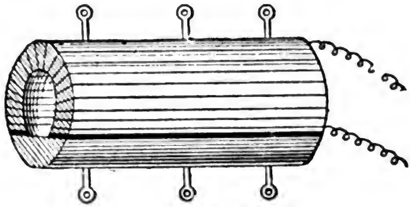

Fig. 60.—Joule's Electromagnet.

Mr. Joule in this way constructed a magnet weighing only 15 lbs., but which could support a weight of 2090 lbs.

A magnet, devised by Mr. Currie as a long-pull electromagnet, for working railway signals at a distance, is constructed as shown in Fig. 61. The magnet is a tubular magnet, or solenoid, wound on a brass tube with an outer iron sheath. The armature is a mushroom-shaped piece of soft iron. The stalk is conical, and projects into the solenoid. The action of the magnet is as follows. The first operation is the attraction of the stalk into the core, then as it enters the core the mechanical force on it gets less, but the mushroom head now is approximated to the polar surfaces of the outer

iron sheath, and it is in turn attracted; the joint effect being to give a considerable pull over a large range.

The larger uses of electromagnets in dynamo-electric machinery are confined generally to the production of a magnetic field as intense as possible. The electromagnets in a dynamo machine, whose function it is to produce the magnetic field in which the armature coil revolves, are called the field magnets. Field magnets

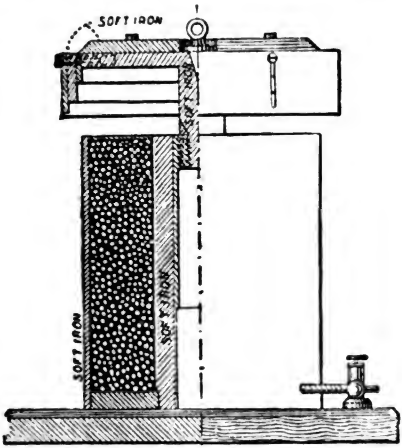

Fig. 61.—Currie's Long-pull Electromagnet.

are made in many different forms. In some, such as the Edison dynamo, they take the form of round bars or legs, united by a square yoke, and having at the bottom massive pole pieces. (See Fig. 62.) In the earlier Edison machines, these legs were made rather long and thin, and the magnetic leakage of flux from leg to leg was large. By adopting shorter and thicker legs, Dr. Hopkinson improved the machine, and obtained a greater

inter-polar magnetic flux density for the same or less magnetising force.

Electromagnets of a great variety of forms, as shown

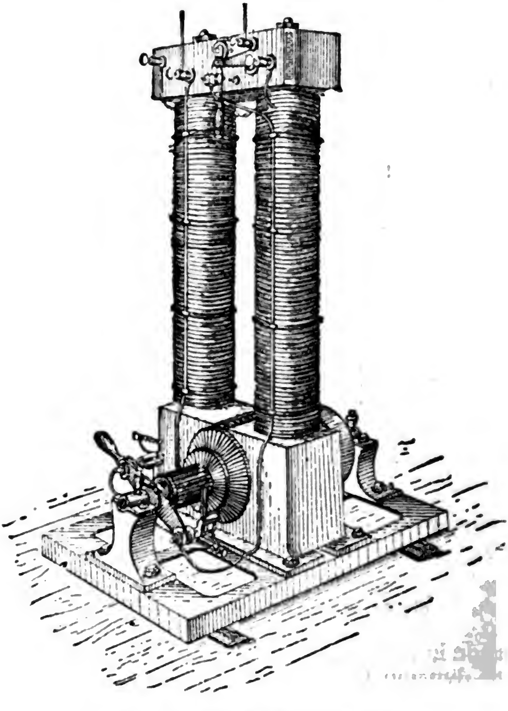

Fig. 62.—Edison Dynamo (old form).

in Fig. 63, are employed for the field magnets of dynamos. In all these cases the object aimed at is to obtain the required field with the least expenditure of magnet-

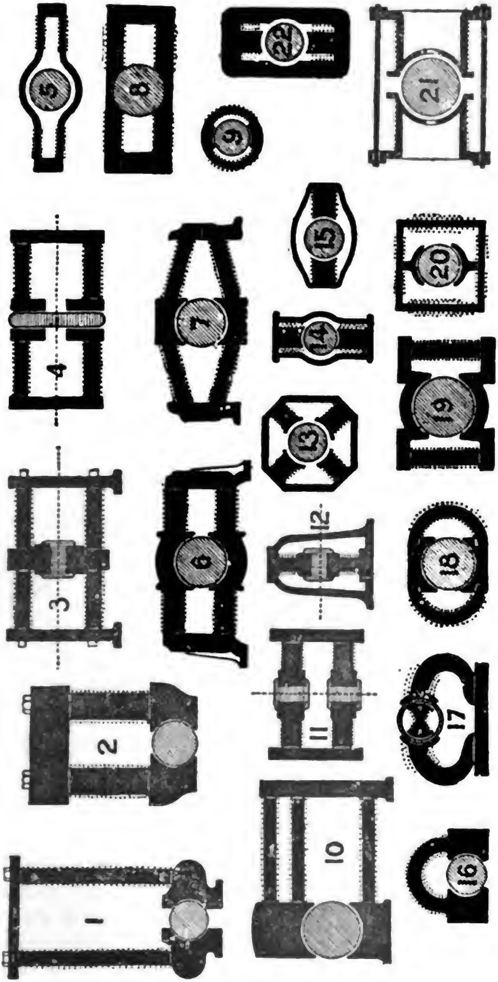

Fig. 63.—Various forms of Field Magnets of Dynamos

ising force and material. If the poles of the field magnet are placed downwards on a cast-iron bed-plate, then it is necessary to interpose a thick plate of zinc or gunmetal to prevent contact between the polar ends and the iron bed-plate, otherwise a magnetic short circuiting would take place which would reduce the strength of the inter-polar field. In the form of field magnets shown in 6 and 7 in Fig. 63, there is a pair of horseshoe magnets placed with poles against one another, and there is therefore no such difficulty. In other cases, single U-shaped magnets are placed with poles uppermost, with the object of altogether removing the pole pieces from the neighbourhood of the bed-plate. (17, Fig. 63.) In some cases, electromagnets have been constructed with cores of cast iron, chiefly with the object of reducing expense.

The magnetic permeability of cast iron is, however, very much less than that of wrought iron, and hence a given magnetising force, consisting of a certain number of ampere-turns per centimetre of length, is less effectual in producing magnetic flux. In modern dynamo machines, generally nothing but the best cast steel, or wrought iron of the highest permeability, is used for field magnet construction.

If the coil of an electromagnet is traversed by alternating or intermittent electric currents, then, at every variation in the current strength, induced currents will be generated, which will circulate in the mass of the metal. The path of these currents is in directions parallel to the coils of the exciting helix. These induction currents dissipate energy in producing heat in the iron core. This heat represents so much energy abstracted from the magnetising current. If the continuity of the iron core is interrupted, by making it of iron wire or thin sheets of iron, so as to cut it up in such a direction that these currents cannot be formed in it, then there will not be this waste of energy. These currents which are thus produced in the iron core are called the *eddy currents* in the core. Foucault first gave an instance of their

formation by rotating a copper disc rapidly between the poles of an electromagnet. The disc became very hot. If a penny be suspended between the poles of a powerful electromagnet by means of a twisted thread, when it is released it commences to spin rapidly. If the electromagnet be excited so that the penny is revolving in a strong magnetic field, it rapidly comes to rest. The reason for this is that the motion of the penny generates in its mass, under the influence of the magnetic field, eddy currents, which are in such a direction as to oppose the motion. If the penny be forcibly twisted against this resistance, then the energy so expended has its equivalent in heat produced in the metal by these eddy currents.

Foucault showed that the forcible rotation of a highly conducting disc in a strong field can generate in it heat sufficient to bring it to a very high temperature. In all dynamo machines the armature, or revolving bobbin of wire, by which the current is generated, consists essentially of an iron core, wound over with covered copper wire or with copper bars, and it revolves in a strong magnetic field. Such a core, if made of solid iron, would be almost immediately rendered hot enough to destroy the insulation of wire wound over it. It is necessary to construct this core in such a manner as to prevent the formation of these eddy currents. This is done by making the core of discs or sheets of thin iron, separated from each other by a layer of varnish or thin paper, or some nonconducting material. It may be also accomplished by constructing the core of iron wire rolled up, but in any case the planes or lines of division must be parallel to the direction of the magnetic field, because the induction tends to create eddy currents in planes at right angles to the field; and hence the subdivision of the iron must be so arranged as to defeat this, and render impossible any electric flow in the direction at right angles to the direction of the field.

In the instrument commonly called an induction coil,

we have an arrangement which consists essentially of an electromagnet, the wire of which (called the primary) is traversed by an intermittent or an alternate current. In order to avoid the production of eddy currents, the core, either straight or annular, is constructed of iron wires or thin iron plates. These are oxidised or rusted on the surface by exposure to the fire, and this film of oxide is sufficient to form an obstacle to the production cf currents across from wire to wire, whilst, at the same time, the continuity of the iron is preserved in the direction in which it is essential it should have the greatest possible magnetic permeability.

§ 7. **Measurement of the Field Strength of a Electromagnet.**—There are many ways in which the strength of the magnetic field, or the magnetic flux density in the air at any point near an electromagnet, may be measured. Of these the most simple and practical are: (1) the measurement of the field by comparison with another known field; (2) the measurement of the field by the ballistic galvanometer; and (3) the measurement of the field by means of the increase in electrical resistance of a pure bismuth wire placed transversely in that field.

The first method is only applicable in the case of the measurement of rather weak fields. The field due to a magnet at any point in its neighbourhood may be measured by comparing it with the known magnetic field of the Earth. The Earth is a great magnet, and at every point on the Earth's surface the terrestrial magnetic force or flux density has a certain direction and a certain magnitude. The magnetic force due to the Earth, estimated in a horizontal direction at any place, is called the Earth's horizontal magnetic force at that spot. In England that magnetic force has a magnitude of about 0·18 of a unit C.G.S. A compass needle sets itself, if undisturbed, in the direction of that horizontal force.

If, then, a small compass needle or exploring needle is placed at any point in the field of an electromagnet

(see Fig. 64), and if the electromagnet is so arranged that the direction of its own field at the place where the compass needle is situated is at right angles to the direction

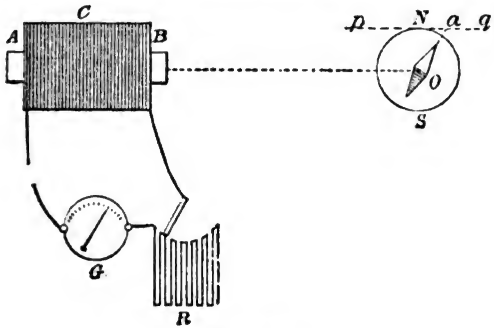

Fig. 64.

tion of the Earth's horizontal force at that point, then when the two fields, the Earth's field and the magnet's field, act together on the compass needle, each pole of the needle is influenced by two magnetic forces which are at right angles to each other.

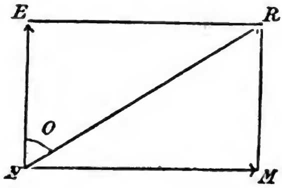

Fig. 65.–Measurement of Magnetic Field Strength of an Electromagnet at a Point on the Axis.

Let the line N E (see Fig. 65) stand for the direction and magnitude of the Earth's horizontal force on the North pole of the compass needle, and N M stand for the magnetic force due to the electromagnet on the same pole. Then the resultant of these forces is N R, and the direction in which the compass needle will stand when acted upon by both fields is N R. Hence the needle has been caused by the magnet to deviate by an angle

E N R = θ from its original undisturbed position. The ratio of the lengths N M = E R to E N is called the tangent of the angle E N R. Hence, if this angle is observed, and its tangent taken from the tables, we have the relation

$$\text{N M} = \text{E N} \tan \text{E N R},$$

or the magnetic field due to the electromagnet is numerically equal to the value of the field due to the Earth multiplied by the tangent of the angle of deviation of the needle. But the line E N represents a magnetic force of 0·18 of a unit. Hence we have the rule—*

$$\left.\begin{array}{l}\text{Magnetic field due to}\\ \quad\text{the electromagnet at}\\ \quad\text{the point N}\end{array}\right\} = 0{\cdot}18 \left\{\begin{array}{l}\text{Tangent of angle of}\\ \quad\text{deviation of compass}\\ \quad\text{needle.}\end{array}\right.$$

The student should in this manner measure the magnetic field due to a small straight short magnet at various points in the *axial* and *equatorial* lines,† and plot out in a curve the variation of this field with distance from the electromagnet. In the case of a *short* magnet, the field parallel to the magnet at various points along the *equatorial* line varies inversely as the cube of the distance from the magnet. For the method of making these measurements the reader may consult the author's 'Electrical Laboratory Notes and Forms,' Elementary Form No. 1.

In the case of stronger magnetic fields, the ballistic galvanometer may be employed to make a measurement of the strength of the field at any point. For this purpose a small coil of very thin insulated wire is prepared. This coil may be wound in one layer on a box-

* Provided the experiment is conducted at a place where there is no other field than the normal field due to the Earth. This, however, is far from being the case. Iron hot-water pipes in a room greatly disturb the direction and strength of the terrestrial field.

† In the case of a *short* magnet, the field at any point on the axial line some way from the magnet is *twice* that on the equatorial line at a point the same distance from the centre of the magnet. From this fact it can be shown that the magnetic force due to a magnetic pole varies inversely as the square of the distance.

wood bobbin. The number of turns of wire on this coil must be known.

The coil is connected by long wires to a ballistic galvanometer. This galvanometer must be previously calibrated, in the manner described on p. 157, so that any deflection or "throw" of the needle being observed, the quantity of electricity measured in microcoulombs which has produced that "throw" is known.

Let the exploring coil be held anywhere in the field of an electromagnet, so that the plane of its coil is at right angles to the direction of the field at that point, then the turns of the coil are perforated by or linked with a stream of magnetic flux. If this coil is suddenly snatched away, the whole of this flux is removed from the coil circuit, and sets up in it an induced electromotive force. If the resistance in ohms of the galvanometer and coil circuit is measured, we then have, by the rule given in Chapter VI., a relation between the number of turns on the coil (the linkages), the resistance (in ohms) of the coil and galvanometer circuit, the quantity (in microcoulombs) of electricity set flowing through the galvanometer (known from the deflection or "throw") by the removal of the flux (measured in microwebers) from the coil circuit, as follows :—

$$\textit{Microwebers} \times \textit{linkages} = \textit{microcoulombs} \times \textit{ohms}.$$

Hence we can calculate the flux in microwebers penetrating the coil aperture. If the area of the aperture is measured, we then know the flux per unit of area or flux density, reckoned in microwebers per square centimetre, at the centre of the coil, and this figure is the numerical value of the magnetic field at the centre of the coil.

The above method is very useful for measuring the interpolar field of strong electromagnets.

If, however, the place in which we wish to measure the field is very narrow, or not otherwise suitable for the employment of the ballistic coil, we can then measure

the field strength at that point by the use of a bismuth wire. Pure metallic bismuth has the remarkable property that its electrical resistance is immensely increased by a transverse magnetic field. If a wire of the metal is placed across the lines of a field, and its resistance measured (as described in a later chapter) when the field is "on" and when the field is "off," we find the bismuth wire has a far greater resistance in the former case than the latter.

Some notion of the kind of increase in resistivity of bismuth, when a wire of it is transversely magnetised, can be obtained from the following figures :—

INCREASE IN RESISTIVITY OF PURE BISMUTH WHEN TRANSVERSELY MAGNETISED IN A MAGNETIC FIELD.

Strength of the Field in C.G.S. Units.	Electrical Resistivity of the Bismuth in C.G.S. Units.
0	116,200
1,375	118,200
2,750	123,000
8,800	149,200
14,150	186,200
21,800	257,000

Hence it is seen that a field of 20,000 C.G.S. units more than doubles the resistance of the bismuth.

A convenient form of instrument to use for taking the field strength in the narrow air gap of a dynamo, is the bismuth spiral as constructed by Messrs. Hartmann and Braun. In this instrument, a small flat spiral of pure bismuth wire is attached to a suitable handle, and the spiral can be introduced into any narrow gap, and the resistance of the bismuth measured when it is held there and traversed by a magnetic field. From this measurement in the field, and another similar one made out of the field, the field strength can be determined at once from a calibration curve which accompanies each instrument.

§ 8. **Effect of Heat on Magnetic Properties.**— Rise of temperature has a marked effect upon the magnetic properties of the ferromagnetic bodies, iron, nickel and cobalt. For each one of these substances there is a temperature called the *Critical Temperature*, beyond which it is not more magnetic than most paramagnetic substances. That is to say, its permeability sinks to nearly unity, and it becomes in popular language non-magnetic. For iron, this temperature is about a good red heat, or lies somewhere between 690° C. and 870° C. If a small fragment of stout iron wire is suspended by a fine platinum wire, so that it can be heated by a spirit lamp or gas flame red hot, it is easy to show the above fact with an ordinary horseshoe magnet. Suspend the iron wire from a stand, and fix a horseshoe magnet at such a distance that it will attract the iron to its poles. Then holding the iron away from the pole, heat it to a bright red heat, and leave it suspended in front of the magnet poles. It will be found not to be attracted. As it cools, a temperature is reached at which it recovers ferromagnetic properties, and suddenly flies to the pole of the magnet. The critical temperature varies for different specimens of iron. It lies, however, between 690° C. and 870° C. The critical temperature of nickel is between 300° C. and 400° C., but that of cobalt apparently much higher.

Beyond these temperatures all ferromagnetic bodies become changed into paramagnetic bodies, with a constant permeability independent of the flux density. The magnetic permeability of iron undergoes remarkable changes as the temperature rises. If magnetisation curves are drawn for iron at several different temperatures, the curves cross each other at certain places. The curves shown in Fig. 66 are magnetisation curves for soft iron at three different temperatures. If from these magnetisation curves a series of permeability curves are drawn, indicating the permeability for different temperatures and for different magnetic forces, it has been

found that the permeability under *large* magnetic forces *decreases* steadily as the temperature rises. The permeability for *small* magnetic forces, however, steadily increases (with the exception of a curious temporary drop at about 550° C) until the critical temperature is nearly reached, and at this point the permeability may have the enormous value of 8000 to 10,000. Immediately the critical temperature is reached, the permeability falls with great rapidity to quite a small value, as shown in Fig. 67.

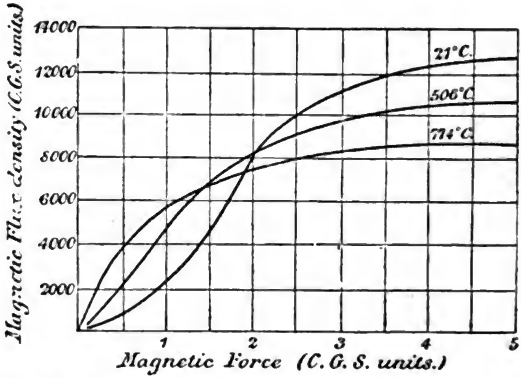

Fig. 66.—Magnetisation Curves of Iron at different Temperatures. (D. K. Morris.)

In the same way the hysteresis loss in the iron gradually diminishes as the temperature rises, as shown by the ordinates of the curve in Fig. 68. At the critical temperature the hysteresis disappears, and no work is then done in carrying the iron round a magnetic cycle. If a series of hysteresis loops are drawn for different temperatures, it is found that they gradually close up and diminish in area as the temperature rises.

At the critical temperature, the electrical resistivity

of iron also undergoes a remarkable change. Up to that point the curve of resistivity of iron in terms of

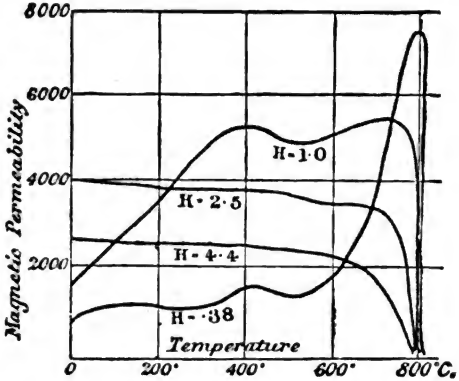

Fig. 67.—Variation of Magnetic Permeability with Temperature. (D. K. Morris.)

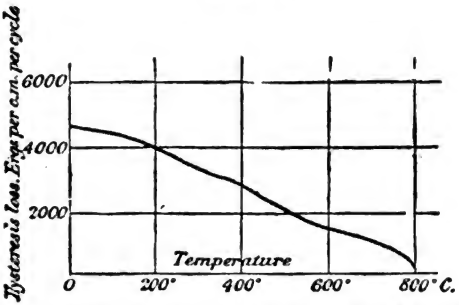

Fig. 68.—Variation of Hysteresis Loss in Iron with Temperature. Iron annealed at 1050° C. (D. K. Morris.)

the temperature is a curve which is concave upwards At the critical temperature the curve changes its direction of curvature and becomes concave downwards

There is, therefore, a point of *contrary flexure* at the critical temperature. In fact, at the critical temperature, the whole of the physical properties of the substance we call iron are greatly changed, and it passes into a so-called *allotropic form.* Its magnetic, electric, and thermo-electric properties are entirely altered at a temperature near 800° C.

§ 9. **Effects of Magnetisation upon the Length and Physical State of Iron.**—It has been known for a long time that when an iron bar is strongly magnetised a sound, or "tick," is heard proceeding from it. This is called the *Magnetic Tick.* If an electromagnet is placed in one room, and a current sent to it by long wires, the circuit being closed in an adjacent room, an observer placing his ear to the iron will hear a ticking sound each time the iron is magnetised or demagnetised. If the magnetisation and demagnetisation proceed very rapidly, the ticks run together into a musical note or hum. This can best be heard by placing a wooden stick against the iron core of a transformer and pressing the ear against the other end. This humming sound indicates a molecular disturbance in the iron at the moment of reversing the magnetisation.

When an iron bar is magnetised it also undergoes changes in dimension. This was first investigated by Joule, who came to the conclusion that the bar is always lengthened by magnetisation. The subject of late years has been carefully investigated by Bidwell, and he has shown that, in the case of iron bars which are unannealed or not very well annealed, the result of magnetisation is to *lengthen* the bar, if the magnetising force has less than a certain value. For very strong magnetising forces the bar actually contracts or *shortens* in length.

If, however, the bar is of exceedingly soft iron, very well annealed, then magnetisation will produce a contraction in length, no matter how small the magnetising force. In the case of nickel and cobalt, magnetisation by any force produces a contraction in length. There

appears to be also evidence of a change in volume on magnetisation.

§ 10. **Molecular Theory of Magnetism.**—On reviewing the whole of the known facts concerning the magnetisation of ferromagnetic bodies, they lead to the conclusion that magnetisation, in the case of ferromagnetic bodies at least, consists in *arranging in the same direction* or *colineating* a number of small particles or portions of the body which are already and always small magnets. It is no explanation of the ultimate nature of magnetisation merely to postulate that a magnet of sensible size consists of small portions, each of which is a magnet, but at the same time it is in itself an important fact.

It is now believed that a mass of iron is made up of what are called *molecular magnets.* These may be single molecules or groups of molecules. There are, however, facts which point to the conclusion that it is not the *atom* of iron which is magnetic but a certain group of atoms arranged in a particular way.

These molecular magnets are, for the most part, quite free to turn round their centres like little compass-needles. In a mass of iron which is, in the ordinary sense of the word, non-magnetic, these molecular magnets must be assumed to group themselves in such a way that they have no external magnetic moment. For this purpose closed chains or loops of molecular magnets must be formed, each molecular North pole neutralising another molecular South pole.

When an external magnetising force is applied to the iron it breaks up these closed rings and more or less *colineates* the molecular magnets, or makes them stand in one direction. To do this the mutual attraction of opposite poles has to be overcome; hence *work* is done in magnetisation in pulling apart the opposite poles of the molecular magnets. If we apply a very small magnetising force, the result is merely to strain the molecular magnets from their initial position. If that force is

withdrawn, the molecular magnets drop back into their old position. This constitutes the initial stage of magnetisation. If, however, the force is increased beyond a certain point, there is a more or less rapid commotion amongst the magnetic molecules causing them to fall over into a new position of equibrium, in which they are much more colineated. This is the second stage of magnetisation, in which the magnetisation curve rises rapidly. When once the molecular magnets have fallen over into the second stable position, the result of a further application of magnetising force is to make but little further increased colineation of the molecular magnets. Hence, in the third stage of magnetisation the magnetisation curve rises very slowly. There are therefore three stages in the magnetisation of iron :—

(i) An elastic stage, in which only temporary displacement of the molecular magnets takes place under small forces.

(ii) A catastrophic stage, in which a complete change of position takes place within a narrow range of increased force.

(iii) A final stage, in which small increased displacement accompanies great increase in the force.

If we attempt to reverse the process and reduce the magnetisation, we cannot do it by simply withdrawing the force. We have to apply an augmented negative or reversed force to bring the molecular magnets again into a position in which they fall over into more or less closed chains. Hence both operations require work to be done against the mutual attraction of the poles of the molecular magnets. The work so expended is frittered away into heat in setting up vibrations in the molecular magnets. This is the energy loss by hysteresis. The increased permeability under small forces due to heating the iron must arise from the greater freedom which the heating of the metal bestows upon the molecular magnets; a freedom which involves a greater ease of colineation of the molecules under a given force. In order,

however, to explain the diminished permeability under large forces, we must suppose that rise of temperature has two actions. On the one hand, it reduces the magnetic moment of all the molecular magnets, or at least of some of them, and hence, on that account alone, the magnetic result of colineation is less marked. On the other hand, it renders it easier for the molecules to be colineated, gives them, so to speak, more elbow-room; and, under the action of weak forces, the gain from the increased freedom of movement for the moment more than counterbalances the decreasing actual average magnetic moment of each magnetic molecule. Ewing has shown that we need not assume anything more than mutual magnetic attraction between the poles of molecular magnets to explain all the phenomena of magnetic hysteresis.

A very instructive model can be made, as first shown by Ewing, by placing a large number of small compass needles together so as to influence each other's action. Procure a couple of dozen of the small cheap compass *charms*, and take out the little compass needles from them. These needles should be about three-eighths of an inch or half an inch long. On a sheet of glass fix 16 small drawing pins, sticking the heads of the pins to the glass by isinglass cement. The pins should be placed at regular intervals, and so far apart that when the small compass needles are placed upon them the needle poles do not quite touch. It is convenient to arrange 16 or 25 in a square form. Then mount over this another sheet of glass so that the needles cannot fall off the pivots. If these little magnets are all stirred up they will generally arrange themselves in an irregular manner. In this condition the little magnets will hardly affect a single compass needle placed a little way off. The group has no external magnetic moment. These little magnets represent then the condition of the molecular magnets forming a ferromagnetic body when that body is not magnetised.

Bring up then, gently, to the group, the two opposite poles of two bar magnets, and notice what happens. The little compass needles will be *slightly* displaced, but will go back to their old positions if the bar magnets are withdrawn again. If

however, the bar magnets are brought up closer, then the little compass needles will tumble over into a new position, in which they will be more nearly in one direction. If the bar magnets are then withdrawn, they will fall back a little, but not much, from this new position. The model therefore imitates the first and second stages of magnetisation, and also the residual magnetisation or phenomenon of *magnetic retentivity* in iron. If the bar magnets are brought up closer still, the result is merely to effect a little more colineation of the small needles, and thus an imitation of the third stage of magnetisation. The model therefore represents the effect of retentivity, magnetisation and hysteresis, and can be shown also to imitate the action of high and low temperature on permeability. A careful examination of the behaviour of such a model, formed of a large group of small compass needles, will enable the student to see that there are good grounds for believing that it is a true representation of what really occurs during the magnetisation of iron.

It will be seen to be an immediate consequence of this theory that, if a mass of laminated iron is revolved in a very strong fixed magnetic field, or if a very strong magnetic field is made to revolve round a mass of laminated iron, there should be no hysteresis loss. For, under these conditions, the magnetic molecules will be held so firmly in the grasp of the magnetic field that they will not be able to execute free vibrations in falling from one position of magnetic equilibrium to another, and hence there will be no dissipation of molecular energy into heat. This has been found to be the case. If a thin disc of iron is rotated in a strong magnetic field, with its plane parallel to the lines of flux, it is found that no energy is dissipated in hysteresis if the field is very strong.* If the field is not strong, then energy is dissipated by hysteresis during each revolution of the disc. This fact affords support to the theory of hysteresis, which consists in regarding the energy losses during

* This was first proved by Mr. F. G. Baily, and confirmed by experiments made by Messrs. Beattie and Clinker.

magnetic reversals as due to the irreversible work done in displacing the magnetic molecules against their mutual attraction from one position to another, this work being frittered away into heat by molecular vibrations set up in consequence of the magnetic molecules springing from one condition of magnetic stability to another.

CHAPTER VIII.

ALTERNATING OR PERIODIC CURRENTS.

§ 1. **Alternating Currents.**—We have hitherto limited our attention to the properties and measurement of continuous currents of electricity. Modern applications of electric currents involve, however, to a very large degree, the utilisation of electric currents which do not continuously flow in one direction, but which change or alternate in direction many times in a second. Such currents are called *Alternating Currents*, and their study introduces the student to a new class of difficulties and ideas. In order to gain a clear notion of the nature of these currents, let the reader think of a river like the Thames at Oxford. The water in this river always flows in one direction, and it would, in electrical language, be called a *continuous* river. Consider, in the next place, the same river at London Bridge. At that place the water in the river sometimes flows down to the sea, and sometimes flows up in the opposite direction, owing to the action of the tidal wave. Hence the water-flow is not continuous, but at regular intervals reverses the direction of its flow, and goes through a *cycle of motions*. The flow is said to be *alternating*. In the same way, if a current of electricity in a conductor flows first in one direction and then in the opposite, as determined by the direction of the embracing magnetic flux, and if it repeats this reversal of direction at regular intervals, the current is called an *alternating current*. The time in which an entire cycle of operations is completed is called the *Periodic Time*. The number of periods completed in a second is called the *frequency*, and denoted by the sign ∿. Hence 100 ∿ means a frequency of 100, or a

periodic time of ·01 of a second. The frequencies of alternating currents mostly used in practice lie between the limits of 30 ∿ and 150 ∿. A very useful frequency is 100 ∿.

It is necessary to have some method of delineating the manner in which the current or electromotive force is changing during the period, in the case of alternating currents and electromotive forces, and this is done in one of three ways by diagrams, called respectively—*wave diagrams*, *polar diagrams*, and *clock diagrams.*

§ 2. **Graphical Representation of Alternating Currents.**—The simplest method of delineating the changes that take place during the period in the case of

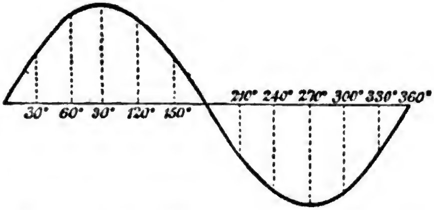

Fig. 69.—Wave Diagram of an Alternating Current.

alternating currents is by a *wave diagram.* Let a horizontal line be taken (see Fig. 69), the length of which represents to a convenient scale the periodic time of the current. Divide this line into any number of parts, say 12; at each point of division let a vertical line be erected, the length of which represents to some suitable scale the strength of the current in the conductor at that instant during the period, and let these vertical lines be drawn *above* the horizontal line if the current they represent is in one direction in the conductor, and *below* the horizontal line if the current is in the opposite direction. If, then, the extremities of these vertical lines are joined by a curve, we obtain a wavy line which represents the ebb and flow of the current in strength during the period.

The current flows first one way with increasing strength, then reaches a maximum, and then dies down to zero; then reverses in direction, and increases to a maximum in the opposite direction, and then gradually becomes zero again. The wavy line represents this periodic process. Reckoning time from the instant when the current has a zero value, we may express any fraction of the periodic time by stating it in *degrees*, the whole period being considered to contain 360°. The *phase* of the current at any instant is the fraction of the whole period which has elapsed up to that instant, expressed in degrees, the whole period being 360°. Two currents may *differ in phase*, in which case one is said to *lag behind* the other, or to be out of step with it, and when this is the case both currents do not come to their zero value at the same instant. This can be represented by drawing two wave diagrams on the same horizontal or time line. Alternating currents may also differ in their maximum value during the period, and hence to define them we require to know not only their periodic time, but their *amplitude* or *maximum value* during the period.

Alternating currents may differ also in *wave form* or in the mode of variation of the current during the period; and the wave form, amplitude, and wave length or periodic time being given, we can define exactly the alternating current in question.

The second graphical method is by a *polar diagram*. Imagine a straight line O P to revolve with uniform speed round one of its extremities O, and that it performs one revolution in the periodic time of the current. Then suppose that on this line, at each instant, a length is cut off or set off proportional to the strength of the current at that instant. This is called the instantaneous value of the current. The line or path described by the end of this revolving line is called a *Polar Curve* (see Fig. 70), and the radii of this polar curve represent the varying values of the fluctuating current during the period. This

polar curve will always consist of two closed loops if it represents the complete wave of an alternating current.

The third method of representing alternating currents is by a *clock diagram*.

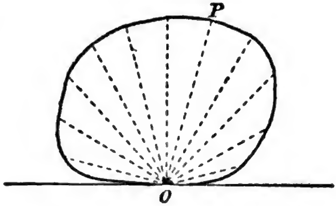

Fig. 70.—Polar Diagram of an Alternating Current.

Let a straight line O P be taken, the length of which represents to some scale the amplitude or maximum value of an alternating current. Let this line revolve (see Fig. 71) round one end O like the hand of a clock. Through the centre of revolution O draw a vertical line X Y, and at any instant *project* the length of the revolving line O P on this vertical line. Then it is easy to see that the projection O *p* will grow and shrink, first increasing to a maximum, and then waning to zero; and if distances projected on this vertical line above the centre are considered as positive, and distances below as negative, then the length of the projection becomes alternately positive and negative, and it can represent the varying or fluctuating values of the current during the complete phase. Each of these diagrammatic methods has some particular advantage of its own.

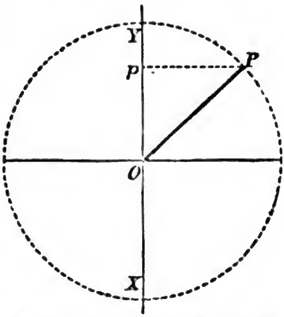

Fig. 71.—Clock Diagram for an Alternating Current.

§ 3. **Root-Mean-Square Value of an Alternating Current.**—Since the strength of an alternating current during the period runs through a cycle of values, we

cannot define it by one number unless we express its value by a mean value of some kind. When we have a number of values of different kinds, such as the runs in cricket matches made by a man during the season, if we add all these values together and divide the sum by the number of the separate values, we obtain the *true arithmetic mean* or average, as we call it, of these values. In the same way, if we measure the true value of an electric current in amperes at many equidistant intervals of time during the period, and take their average, we obtain a measure of the *true mean value* (T.M.) of the current during the phase. As a matter of fact we hardly ever require to know this T.M. value. The electric measuring instruments which we use to measure alternating currents do not give us the T.M. value of the current, but a more complicated sort of average called the *root-mean-square* (R.M.S.) *value.*

Suppose that at equidistant intervals of time during the period we measure the strength of the alternating current and square this value, we obtain the *square of the instantaneous current strength.* If, then, we take the mean of all these squared values, we obtain the *mean square value* of the current; and, finally, if we take the square root of the mean of the squares of these equidistant instantaneous values, we arrive at the *root-mean-square value* of the current. Most of the instruments we use for measuring alternating currents and voltages give us the root-mean-square (R.M.S.) value of the current or electromotive force.

The question then arises, what do we mean by an alternating current of one ampere, or an alternating electromotive force of one volt? The answer is, an alternating current is said to have a value of one ampere if the root-mean-square value of the current measured in amperes at the different instants is unity. Hence there may be many different kinds of one-ampere alternating currents depending on the wave forms, but they will be all alike in producing the same heating effect in the

same resistance. An alternating current is, therefore, said to have a value of one ampere if it produces the same amount of heat in a given conductor as does a continuous current of one ampere in the same time. The same applies to the alternating voltage. An alternating electromotive force is said to have a value of one volt if the R.M.S. value of the voltage is unity during the period; instantaneous values being measured in volts.

§ 4. **Form Factor of a Wave.**—In some questions we have to consider the ratio between the root-mean-square value of a periodic quantity and the true mean value, and this ratio is called the *Form Factor* of the wave. The more peaky the form of the wave the larger will be the form factor. Otherwise, the form factor may be defined to be the number by which we must multiply the true mean value of the ordinate of a periodic curve to obtain the root-mean-square value of the ordinate during the period.

§ 5. **Sine Curves, or Simple Harmonic Curves.**—If we take a table of sines of angles (see Appendix) and set out on a horizontal line, at equal intervals, vertical lines representing on some convenient scale the *sines* of all angles from 0° to 360°, taking say the sine of every 10°, and setting out sine 10°, sine 20°, &c., we shall find that the extremities of these lines define a wavy curve called *a Sine Curve.* In setting out this curve the lines must be drawn above the horizontal if the sine is positive, and below the horizontal if the sine is negative.

The student should plot out on squared paper three curves, one representing the variation of the sines of all angles from 0° to 360°, the other the cosines, and the third the tangents. He will obtain three lines like Fig. 72. He should then plot out a curve representing the increase or decrease of the sines *per degree*, taken at intervals of 10°; that is to say, plot out a curve whose ordinates represent the value of sine 1° − sin 0°, sine 10° − sin 9°, sine 20° − sin 19°, &c.; and he will find that this curve when plotted can be made, by plotting to a suitable scale, to fit exactly over the cosine curve. Hence he will learn that

the *rate of change of the sine*, or the change or increment of the sine per degree of angle, is represented by the cosine curve. In other words, the cosine of an angle expresses to a certain scale the rate at which the sine of that angle is waxing or waning.

A very large class of alternating currents are properly represented by a sine curve, and are thus called *Sine Curve Currents*, or simple harmonic or simple

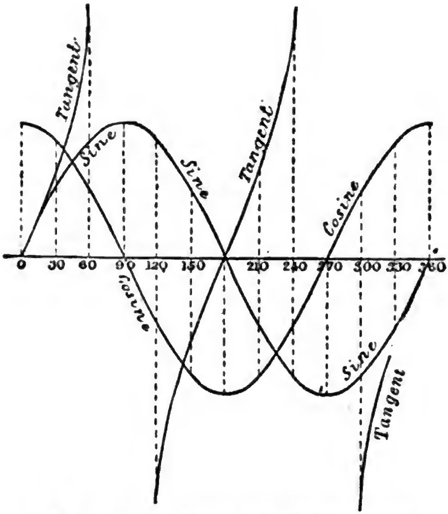

Fig. 72.—Sine, Cosine, and Tangent Curves.

periodic currents. One important property of these currents is, that if we suppose a wire simultaneously traversed by a number of simple harmonic currents of wave-lengths or periodic times, in the ratio of 1 : 3 : 5 : 7, &c., and if we suppose these currents can be varied in relative phase and in amplitude, then we can imitate any kind or shape of alternating current by suitably adjusting the relative phase and amplitude of these component

sine waves. Sine curve currents of the above kind are therefore like bricks, out of which we can build a wall having any outline we please by suitably piling the bricks one upon the other.

Since many machines for producing alternating currents generate a current which is closely similar to a sine curve current, we shall, in the first place, study the properties of these currents, and then, subsequently, modify these notions to include other forms.

§ 6. **Addition of Alternating Currents.**—When we desire to *add together* two things, the result at which we wish to arrive is their *joint effect.* If the things to be added are such things as masses, energies or volumes, all that is necessary is to arithmetically add together their numerical values. Thus, a mass of 8 lbs. added to a mass of 5 lbs. is equivalent to a mass of 13 lbs. Such quantities are called *Scalar Quantities*, because they are added together like lengths on a scale. There are other quantities, which, however, cannot be completely described without stating two things about them, viz. their *direction* as well as their *magnitude.* Thus forces cannot be added together simply by adding arithmetically their numerical values, unless we know that they act in the same direction. If they act in different directions, then they have to be added by the *parallelogram method.* In the case of these last quantities, called *Vector Quantities*, which have direction as well as magnitude, we can always represent them by straight lines, so drawn that the lengths and directions of the line denote the magnitude and direction of the vector quantity in question. If, then, the line O A represents one force acting on a body, and O B represents another force acting on it at the same instant, the joint effect of these two forces O A, O B is as if a force O C, represented by the diagonal of the parallelogram formed on O A, O B, had acted instead. O C is called the *resultant* of O A and O B.

Electric currents belong to the category of vector quantities. Hence, if two alternating currents of different

amplitudes and differing in phase flow at the same time in one conductor, they give rise to a resultant current, the amplitude and phase of which is found from those of the two component currents by the parallelogram law.

Suppose, then, that the line O P (see Fig. 73) revolving round O represents in a clock diagram the amplitude or maximum value of an alternating current, and hence the fluctuating value of the projection of O P, as it revolves, on the line X Y represents the various instantaneous values of the alternating current in question.

Then let the line O Q represent another alternating current existing at the same time in the same circuit, and differing from O P in phase and amplitude. The angle P O Q represents the difference in phase between the two currents. To obtain the joint effect of the currents O P and O Q coexisting in the same wire, we simply complete the parallelogram on O P, O Q, and draw the diagonal O R. This diagonal represents the maximum value or amplitude of the resultant of the two currents O P, O Q, and also its relative phase with respect to them.

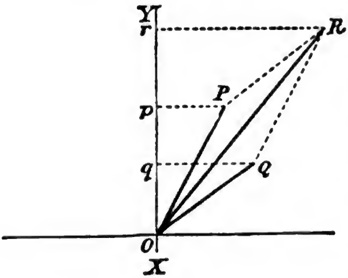

Fig. 73.

If, then, we suppose the lines O P, O R, O Q to revolve round O, all rigidly fixed together like the hands of a toy clock, the fluctuating projections O p, O q, O r of these lines on the line X Y will represent the instantaneous values of the components and resultant alternating currents. In clock diagrams it is convenient to speak of an alternating current merely by its maximum value. Hence an alternating current O P means one whose maximum value or amplitude is O P.

With regard to the projections of the lines O P, O R, O Q, on X Y, one important fact must be noticed. This is, that at all moments and positions of the parallelogram R P O Q, the projection O r of the resultant O R is equal in length to the arithmetic sum of the projections O p of O P and O q of O Q. The student will have no difficulty in seeing this, if he notices that the length of the projection of the line P R on X Y is equal to that of the projection of an equal and parallel line O Q on the same line X Y. Hence, if O P, O Q represent the maximum values of two alternating currents existing at the same time in the same conductor, their corresponding instantaneous values O p, O q added together give the value at the same instant, viz. O r, of the resultant current O R. This is of course obvious, as the instantaneous value of the resultant current is simply the arithmetic sum of the instantaneous values of the component currents existing at the same moment in the same direction. All that has been said above as to the process of addition of alternating currents equally applies to the addition of alternating electromotive forces, and to the representation of these last by straight lines on a clock diagram.

More than two alternating currents or electromotive forces can be added together if co-existing at the same time and place, by adding them in pairs until a final resultant is reached, or by a method similar to that called the *Polygon of Forces* in mechanics. If a number of alternating currents, represented as regards maximum values by straight lines drawn in different directions, to represent their proper relative phases, flow simultaneously in a conductor, we can obtain their joint effect by drawing a polygon, the sides of which are equal and parallel to the current lines, and the line requisite to close the polygon represents in magnitude and direction the resultant of these currents.

§ 7. **Inductance.**—Before we can deal at greater length with the properties of alternating currents, we

must refer again to the quality of electric circuits, called their *inductance*, an exceedingly important one in connection with periodic currents. We have already mentioned that electric circuits possess a quality similar to the *inertia of matter*, and in virtue of which time is required to produce in them a current under the influence of electromotive force. We know perfectly well that if we apply a steady pull to a heavy body, say a garden roller, we do not at once produce in it the full velocity due to that pull. Time is required to get up the speed. Moreover, when the speed has been accumulated a reversed force will not at once bring the heavy body to rest, and time is required also to destroy the motion. This is due to the inertia of the garden roller. There is an exactly similar *inertia-effect* in connection with conducting circuits. If a steady electromotive force is applied to the ends of a conductor, it does not at once produce in the conductor the full possible current strength in accordance with Ohm's law. There is a *variable* period during which the current is gradually rising to its *Ohm-law value*, or to its *Ohmic value*. Again, if the electromotive force is removed the current does not at once vanish, but goes on flowing with gradually diminishing strength until it dies away. The quality of the circuit in virtue of which this effect takes place is called the *inductance*. Circuits are said to have large inductance when the current under a given steady impressed electromotive force would be a long time, relatively speaking, in rising to its Ohmic value. They are called *non-inductive*, or negligibly inductive, when the inductance is too small to be of practical consequence. No circuit can be truly non-inductive, just as no material body can have absolutely no mass, or inertia.

The inductance of a coil or bobbin of wire of many turns is greatly increased by putting an iron core into it. Hence the circuits of large electromagnets have great inductance.

Inductance is measured in terms of the unit called the

Henry, and the inductance of a circuit may be measured in *henrys* if large, or in *milli-henrys* if small.

Two illustrations may be given of the above facts. Let a loop of copper wire have its ends bent round nearly to meet (see Fig. 74), and let the distance separating the points *a* and *b* on the wire be about one-sixteenth of an inch. If, then, a charged Leyden jar be discharged through this wire a spark will be seen at *a b*. The greater part of the current from the jar prefers to jump across the air space *a b*, even although of exceedingly high resistance, rather than flow round the copper spiral of very low resistance. The reason for this is because the application of the charged surfaces of the jar to the ends of the copper wire, is an application of a high electromotive force very suddenly to the ends of an inductive circuit. The inductance of the conductor opposes such an obstacle to the immediate production of the current in it, that part of the discharge finds it easier to break down the resistance of the air gap, and pass across *a b*.

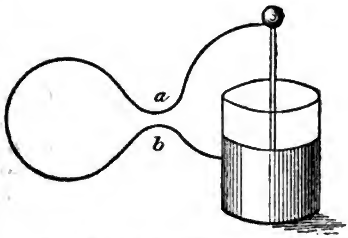

Fig. 74.—Experiment to show the Inductance of a Looped Circuit.

The action is very much like that of the sudden explosion of guncotton. A pad of guncotton placed on a stone slab and gently ignited burns away quietly. If, however, it is detonated by a fuse, the gases suddenly evolved cannot displace the air instantly, and hence the detonated guncotton breaks the stone slab. The superincumbent atmosphere can easily be pushed out of the way gently and slowly, but to a very sudden blow the air opposes, in virtue of its inertia, an immense resistance.

The second experiment illustrating inductance is conducted with an electromagnet. An incandescence lamp is joined across the terminals of an electromagnet (see Fig. 75), and an electromotive force or voltage is put on the terminals of the magnet, just sufficient to bring the lamp filament to a dull red heat. On breaking the circuit of the battery, the lamp flashes up brilliantly for one instant. This is due to the fact that the current running through the electromagnet coils cannot, in virtue of inductance, at once be stopped, and hence, when the battery is withdrawn the current in the magnet coils runs on, and flows back through the lamp and causes it to brighten up. This current is for the moment a stronger current than that which flows through the lamp when the battery is steadily applied.

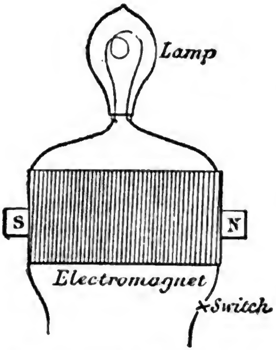

Fig. 75.—Experiment to show the Inductance of an Electromagnet.

There are two ways in which we may regard this effect of inductance. In the first place, following Faraday, it may be looked upon as the result of an electromotive force, due to what is called the self-induction of the circuit, which comes into play to retard the current when it is beginning and help it when it is ending. We have seen that when a coil of wire is traversed by a current, it is surrounded by and linked with a magnetic flux of its own making. We have also seen that the insertion or withdrawal of a magnetic flux into a circuit gives rise to electromotive force. If, then, the student considers a simple circular current (see Fig. 76) and its associated magnetic flux, he will see that the introduction of this self-made flux into the circuit must create an electromotive force, which acts to *oppose* the

external electromotive force driving the current. In the same way the withdrawal of the flux acts to create an electromotive force of self-induction, which tends to make the current run on a little in the circuit, in the same direction in which it was going, after the external electromotive force is withdrawn. Very often this electromotive force of self-induction may be many times greater than the external impressed steady electromotive force. Hence, when a current is being started in a wire or

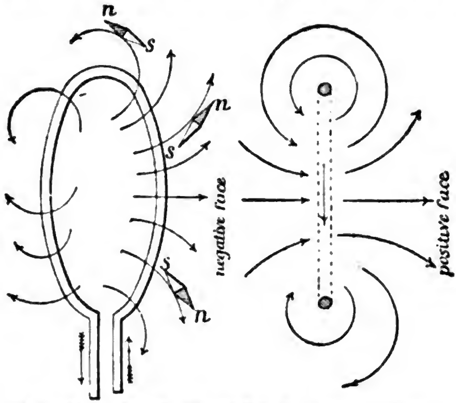

Fig. 76.—Magnetic Flux round a Circular Current.

circuit, the external electromotive force, called *the impressed electromotive force*, has to do two things: first, to overcome the resistance of the circuit, and second, to overcome the opposing electromotive force of self-induction. When a current is flowing through a circuit, the magnetic flux due to the current in that circuit, which is linked with the circuit itself, is proportional to the current strength, and to a quantity called the *inductance of the circuit*, which is a constant quantity for all values of the current if the circuit of the magnetic flux is wholly an air circuit or circuit of unit permeability. It is not however constant, if the magnetic circuit passes

through iron, that is, if the coil is wound on an iron core. The electromotive force of self-induction at any instant is measured by the *rate at which the total flux linked with the circuit is changing*, hence, in the case of circuits of *constant inductance*, the electromotive force of self-induction must be measured by the product of this inductance (or, as it is sometimes called, the coefficient of self-induction), and the number which expresses the rate at which the current is changing in strength at that instant.

If there is at any instant in such a circuit a current, one part of the voltage which is being applied to the circuit is employed in maintaining this current, and the numerical value of this part is obtained by multiplying the resistance of the circuit reckoned in ohms by the current flowing in it measured in amperes. In other words, if the resistance of the circuit is R ohms, and the current in it at any instant is A amperes, then R times A volts is the voltage employed in keeping the current flowing, and may be called the *Ohmic Voltage*, since these volts, amperes and ohms are related to one another by Ohm's law. This, however, is not the whole of the story. If the current is increasing in strength, then an additional voltage has to be applied to the circuit, reckoned by the product of the inductance of the circuit (call it L), and the rate at which the current is increasing. This additional voltage represents the electromotive force being employed in increasing the current. Hence we have the following equation for the total or impressed electromotive force :—

The total or impressed electromotive force, reckoned in volts, acting on a circuit at any instant.	=	The product of the resistance (R) of the circuit measured in ohms, and the value of the current in amperes at that instant.	+	The product of the inductance (L) of the circuit reckoned in henrys, and the value of the rate of change (increase) of the current at that instant.

This equation is called the *Current Equation*, and it holds good for the circuit at every instant, and gives us a means of connecting the instantaneous value of the current in the circuit with the instantaneous value of the electromotive force in that circuit. It will be seen that Ohm's law is, so to speak, a piece or fragment of the above more general current equation, and Ohm's law merely expresses the state of affairs in the circuit, *after* the current has become steady and has ceased to vary.

§ 8. **Relation of Current and Electromotive Force in Alternating Current Flow.**—It is evident, therefore, that whenever a current in a circuit is constantly changing its strength, as is the case with alternating currents, the simple law of Ohm no longer expresses the relation between the current and the electromotive force or difference of potential. The relation is more complicated, and involves the inductance as well as the resistance of the circuit.

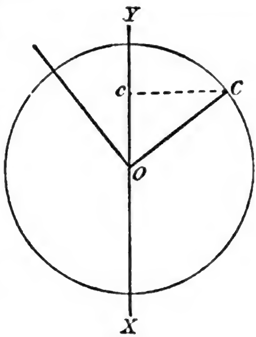

Fig. 77.

If, for the sake of simplicity, we first confine our attention to simple harmonic currents and electromotive forces in circuits of constant inductance the relation between them can easily be established.

Let the line O C in a clock diagram (see Fig. 77) represent the maximum value of an alternating current flowing in a circuit. We have then, in the first place, to consider how to draw in the same clock diagram a line which shall represent *the maximum value of the rate of change* of the current represented by O C.

If we project O C on the line X Y, then the length of the projection O *c* represents, as O C revolves, the actual current in the circuit at the instant considered. The rate of change of this current is obviously denoted

by the velocity with which the point c is moving, because that expresses the rate at which the length O c is changing in value. The point c will have its maximum velocity when c is just passing the centre O. But when c is passing the centre O, it has the same velocity as the point C of which it is the projection.

Let us suppose that the line O C revolves round O with a uniform speed, and with an angular velocity denoted by p, that is to say, p is the angle turned through per second. Then the linear velocity of the point C on its circular orbit is equal to the product of the length of O C and the angular velocity p.*

It is obvious, therefore, that the maximum velocity of the point c, and therefore the maximum rate of change of O c, is represented by the product of the length O C and the angular velocity of O C. Accordingly, if O C represents the current in a circuit, p times O C will represent the magnitude of *its maximum rate of change.* Again, it is easily seen that if we wish to draw on the clock diagram a line whose projection shall always represent the rate of change of the projection of O C, then that line must be drawn at right angles to O C, and be equal in length to O C multiplied by the angular velocity of O C. Let then a line be drawn at right angles to O C and equal in length to p times O C. This line represents the maximum value of the rate of change of O C.

It will easily be seen, therefore, that if we draw in a clock diagram a line O C (see Fig. 78), which is taken to some scale to represent R times I, where R is the resistance (in ohms) of a circuit, and I is the maximum value

* If the length of O C is denoted by r then the circumference of the circle described by the point C is equal to $2 \pi r$. If this circumference is travelled over in a time T, then $\frac{2 \pi r}{T}$ is the *linear velocity* of the point C. But the *angular velocity* of the radius O C is represented by $\frac{2 \pi}{T} = p$. Hence the linear velocity of C is equal to the angular velocity of O C multiplied by r or by the length O C.

(in amperes) of an alternating current flowing in it; and if we draw a line O D at right angles to this, and equal in length on the same scale to the product of L, p, and I, where L is the inductance (in Henrys), and p is equal to the angular velocity of the current, viz. 2π times the frequency n of the current;* then it is clear that the varying length of the projection of the line O D will always represent at each instant L times the rate of change of the current; and, similarly, the projection of O C will always represent R times the current strength. Hence, on the same scale on which O C represents in magnitude and position the ohmic voltage in the circuit, O D will represent in magnitude and proper relative phase the counter electromotive force of self-induction in that circuit, maximum values being understood. The counter electromotive force of self-induction always differs 90° in phase from the ohmic electromotive force, or, as it is often called, the effective electromotive force, and lags behind it.

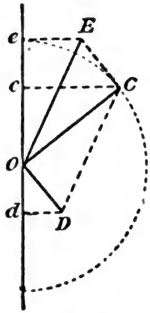

Fig. 78.

The total impressed electromotive force is always equal to the sum of the two above electromotive forces. Hence, if we call E the impressed electromotive force, we have at once the means of obtaining the value of E, by drawing the side O E of the parallelogram O D C E parallel to the diagonal C D, drawn connecting the ends of O C and O D. The line O E represents in magnitude and direction the maximum value of the *impressed electromotive force.* Since D O C is a right angle, therefore

$$(O\,E)^2 = (O\,C)^2 + (O\,D)^2,$$

* The frequency n is inversely proportional to the time T of one cycle. Hence if $\frac{2\pi}{T}$ is represented by $p =$ the angular velocity, we have $2\pi n = p$ in all alternating current formulæ.

and therefore, since O E = E, O C = R I, and O D = L p I we have

$$E^2 = (R\,I)^2 + (L\,p\,I)^2,$$

or

$$E^2 = (R^2 + p^2 L^2) I^2,$$

or

$$I = \frac{E}{\sqrt{R^2 + p^2 L^2}},$$

The above equation expresses the fact that the maximum value or amplitude of a simple periodic alternating current in a circuit, reckoned in amperes, is obtained by dividing the maximum value of the external or impressed electromotive force, measured in volts, by the value of the quantity $\sqrt{R^2 + p^2 L^2}$. This quantity is called the *Impedance of the Circuit*, and is equal to the square root of the sum of the squares of the resistance R of the circuit and p times the inductance L of the circuit. The quantity p L is called the *Reactance* of the circuit.

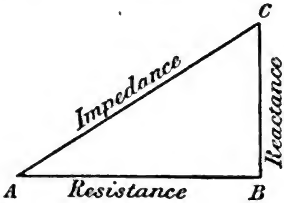

Fig. 79.

We can therefore represent by the sides of a right-angled triangle A B C (Fig. 79) the relation of the *resistance*, *reactance* and *impedance* of an alternating current circuit of constant inductance. In all cases, however, the quantity called the impedance stands to the current and electromotive force, in the case of alternating currents, in the same relation that the true resistance does to the same quantities in the case of continuous currents. In the case of continuous currents we obtain the value of the current in amperes by dividing the value of the impressed electromotive force in volts by the resistance of the circuit in ohms ; or,

$$\text{Current in amperes} = \left\{\frac{\text{Electromotive force in volts}}{\text{Resistance in ohms}}\right\} \text{(Continuous currents).}$$

In the case of periodic or alternating currents, we obtain the R.M.S. value of the current in amperes by dividing the R.M.S. value of the impressed electromotive force in volts by the impedance of the circuit measured in ohms, or

$$\begin{matrix}\text{Current in}\\ \text{amperes}\\ \text{(R.M.S. value)}\end{matrix} = \left\{\frac{\text{Electromotive force in volts (R. M. S. value)}}{\text{Impedance in ohms}}\right\} \text{(Alternating currents).}$$

The student will therefore see at once that if the inductance of a circuit is large, the value of its impedance may greatly exceed the value of its resistance, since $\sqrt{R^2+p^2L^2}$ will then be greater than R. Hence the alternating current obtained by a given alternating electromotive force may be very much smaller than would be the case if they were continuous.

For example, if a continuous electromotive force of 100 volts be applied to a circuit having a resistance of one ohm, the current in it will be 100 amperes. If, however, an alternating electromotive force, having an R.M.S. value of 100 volts, be applied to an inductive circuit of the same resistance, but of large inductance, the actual R M.S. value of the current produced may be very small, perhaps only a fraction of an ampere.

The following experiment may be tried with an alternating current. Let a coil of wire, consisting of a large number of turns, but of low resistance, be placed in series with an incandescent lamp and subjected to an alternating electromotive force. Let the voltage be sufficient to bring the lamp to normal brightness. Next let an iron bar be inserted in the aperture of this coil. The inductance of the coil is thereby largely increased. The current is therefore diminished, because the impedance of the circuit is raised, and hence the brilliancy of the lamp is greatly reduced.

§ 9. **Lag of current behind Electromotive Force in Inductive Circuits.**—The reader must note that in an inductive circuit the actual current which flows at any

instant is produced by the operation of a voltage acting on the resistance of the circuit, which voltage is the algebraic sum of the external or impressed voltage and the electromotive force of self-induction, also sometimes called the back electromotive force.

Hence, in a clock diagram which is intended to represent correctly the relative phase as well as relative magnitude of these three electromotive forces, we must pay attention to this fact. The electromotive force of self-induction is, as we have seen, always represented by a line at right angles to the line representing the current or the ohmic voltage in a clock diagram.

Let us suppose the lines in such a diagram to revolve in the same direction as the hands of a watch, then a little consideration will make it clear that the line representing the inductive electromotive force must be drawn 90° *behind* the line representing the ohmic or effective electromotive force, in order that the projection of the former may be opposed to the projection of the latter in sign or direction as the current is increasing. We have then to draw another line, which will represent the impressed or external electromotive force, and it is clear that if O C is to be the resultant of O D and O E, that O E must be drawn in advance of O C (as shown in Fig. 78). Hence the line representing the ohmic voltage or effective electromotive force will lag behind the line representing the impressed voltage by an angle, the tangent of which is represented by Lp divided by R. The current in the circuit is always in step with the ohmic or effective voltage, and hence in an inductive circuit the current is never in step with the impressed alternating electromotive force, but always lags behind it.

When an alternating electromotive force is producing an alternating current in an inductive circuit, the current does not have its greatest value at the moment when the electromotive force or electrical pressure has *its* greatest value, but happens somewhat later. We have already referred to that view of the nature of induction which

makes it depend upon the presence of an opposing or assisting electromotive force of self-induction ; but there is another mode of regarding the facts which is probably more nearly akin to the actual processes at work. When an electric current is started in a wire, it is believeve—dwith good reason—that the process begins at the surface of the wire, and consists in the establishment of a magnetic field *inside the wire* as well as outside it. The process of establishing the field inside the wire goes on, however, very much more slowly than the process of creating it outside the wire. During the time the field is being created, the current is said to be gradually rising in strength, and is not constant even under a constant electromotive force until this process is complete. The inertia which has to be overcome in effecting this is not in the "current," but in the surrounding medium or *ether*, the motion of which constitutes the magnetic field. If the wire is in the form of a thin flat strip, the establishment of the interior field takes place more rapidly than if the same quantity of metal is disposed in a round sectioned wire, and hence the inductance of the circuit is lessened.

If the current is an alternating current, and the conductor a thick copper wire, the field may not be propagated very far into the wire before it is (so to speak) recalled. In the case of conductors above a certain diameter conveying alternating currents, it is only a certain thickness or "skin" of the conductor which is utilised. There is not the least use in making round conductors for the conveyance of alternating currents having a frequency of 100 $\sim$ of greater diameter than about half an inch. Beyond this thickness the interior metal is not useful for conductance. Hence, concentric metallic tubes or twisted concentric cables are used for the conveyance of alternating currents. This particular fact is called the "skin effect."

§ 10. **Alternating Current Transformers.** — The principles above explained find a practical application

and illustration in the construction of alternating current transformers. Consider an iron ring wound over with two circuits of insulated wire, called respectively the primary and secondary circuits (see Fig. 80). Let an alternating current, called the primary current, be sent through the primary circuit, by applying to the ends of that circuit an alternating primary electromotive force. This circuit, being wrapped round an iron core, will have considerable inductance, and hence, under these circumstances, the primary current produced will in general be a very small current. This alternating current produces in the iron core an alternating magnetic flux; and it will also generate in the core wasteful *eddy currents* of electricity, unless the iron core is laminated or made up of iron wire, so divided that the conductivity of the iron is very much reduced in a direction parallel to the wire windings of the primary circuit, but yet its magnetic permeability not much reduced parallel to the magnetic axis of the primary coil. As a matter of fact, the iron core is always laminated or divided as described. The alternating magnetic flux in the iron core is linked with the turns of the secondary circuit, and hence generates in them an alternating secondary electromotive force. The arrangement has, therefore, this advantage that, by suitably choosing the number of turns of wire for the primary and secondary circuits, we can make any given primary electromotive force produce any required secondary electromotive force.

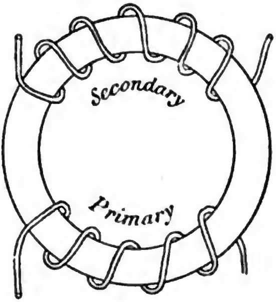

Fig. 80.—A Closed Iron Circuit Transformer.

As a first approximate rule, the student may take it that in modern well-designed transformers the ratio of

these electromotive forces is in the ratio of the number of turns in the two circuits; that is to say, if there are 2000 turns in the primary circuit and 200 in the secondary, then the application of 1000 volts to the primary circuit will result in the generation of 100 volts in the secondary circuit. On the other hand, the secondary current is larger than the primary current, and the appliance does not create any energy; it merely serves to transform electric energy from one type to another. The secondary circuit can be made to yield an electric current which is larger than the primary current if the secondary voltage is smaller than the primary voltage.

The exact state of affairs is as follows:—

The electric power supplied to the primary circuit is partly dissipated in heating the primary circuit. Another part of the power is absorbed in hysteresis loss in the iron, caused by the reversals of magnetism that take place; and also in making some small eddy-current loss in the iron, due to electric currents set up by the changing magnetism in the iron strip, plate, or wire, of which the core is made. The rest of the power appears in the secondary circuit, and is partly used up in heating the internal secondary circuit of the transformer and partly available for use in an external secondary circuit. The difference, however, between the electric power *put into* the primary and that *taken out* of the secondary circuit is, that one may be in the form of a large current produced by a relatively small electromotive force, and the other may be in the form of a relatively small current produced by a high electromotive force. The transformer can, however, no more create power than can a lever, or wheel and axle, or other machine. All it can do is to change the form of the power, and it does this at the expense of, or by the dissipation of, some power wasted as heat in its interior. It acts, in fact, like a simple machine with friction. If the transformer is used to raise electric pressure, it is called a *step-up* transformer.

If it is used to lower electric pressure it is called a *step-down* transformer.

The ratio between the power taken out of the transformer at any load and that put into it is called the *efficiency* of the transformer at that load. The efficiency is always expressed as a percentage. The power dissipated in the iron core is called the *core loss*. The power wasted in heating the coils is called the *copper loss*. The greatest output in power which the transformer can normally make on the external secondary circuit is called the *full secondary output*, or secondary watts of the transformer, and transformers are denominated by their kilowatt output. Thus, a 30 kilowatt (or 30 K.W.) transformer is one which can safely yield 30,000 watts in the external secondary circuit. The core loss is always stated as a percentage of the full secondary output. Thus, a transformer is said to have a core loss of 1·3 per cent. of its full load. If this transformer were a 10 K.W. transformer, this would mean that the iron core loss in this transformer was 130 watts. It has been shown by careful experiment by the Author and others, that the iron core loss is constant at all loads; on the other hand, the copper loss increases with the currents.

If the resistance of the primary circuit is P ohms when warm from use, and if the full-load primary current is A amperes, then the primary copper loss is PA^2 watts. The secondary current at full load can always be approximately calculated by taking it as equal to the primary full-load current, multiplied by the ratio of the number of turns on the primary coil to those on the secondary coil. Hence, if S is the resistance of the secondary circuit, and the number of primary turns is N_1, and of secondary N_2, the secondary full-load current is nearly equal to $\frac{N_1}{N_2}A$ amperes. Hence the copper loss in the secondary circuit is equal to S times $\left(\frac{N_1}{N_2}\right)^2 A^2$ watts.

An illustration will make this clear. Suppose it be desired to calculate the full load copper losses in a 30 K.W. transformer, and that the *transformation ratio* is 20 to 1, that is, the transformer transforms an electromotive force of 2000 down to 100 volts. The secondary voltage is then 100 volts, and at full load the secondary current will be 300 amperes, because $300 \times 100 = 30{,}000$ watts. Here the primary full load current is $\frac{300}{20} = 15$ amperes.

If, then, the resistance of the secondary circuit is ·002 of an ohm, and that of the primary is ·5 of an ohm, the copper losses in the primary and secondary circuits are respectively $(15)^2 \times 0·5 = 112$ watts, and $(300)^2 \times ·002 = 180$ watts, at full load. Hence, under full load, the internal losses in the transformer are

Iron core loss, 1·3 per cent.	=	130 watts
Primary copper loss	=	112 ,,
Secondary ,,	=	180 ,,
Total loss	=	422 watts

Hence, to get 30,000 watts out of the transformer, a power equal to 30,422 watts has to be put into it, and the efficiency at full load will be

$$\frac{30{,}000}{30{,}422} = 98 \text{ per cent.}$$

As the load on the transformer decreases, the efficiency estimated as a percentage will decrease also, and it is generally represented in terms of the secondary output by a curve called an *efficiency curve* (see Fig. 81).

The student must particularly notice one property of the transformer on which its industrial use essentially depends. It can be shown by experimental means,*

* For further and fuller details on the theory of the transformer the reader is referred to the author's treatise on 'The Alternate Current Transformer in Theory and Practice,' vol. i.

that when a transformer of the kind we are considering has its primary circuit traversed by an alternating current, and its secondary circuit also closed and traversed by a secondary induced current, that at full load, or any moderate load, the phase of the primary current is exactly opposite to that of the secondary current, or differs from it by 180°. In other words, the primary and secondary currents come to their maximum values at the same instant, but they are flowing in opposite directions round the core. Hence, if the primary current is magnetising the iron core one way, or creating magnetic flux in it in one direction, the secondary current is opposing it and

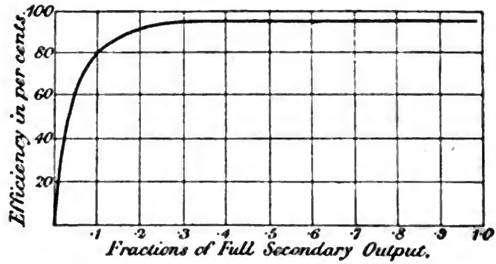

Fig. 81.—Efficiency Curve of an Alternating Current Transformer.

creating magnetic flux in the opposite direction. We have already shown that the inductance of a circuit may be regarded as due to an electromotive force set up in it by the thrusting in, or withdrawal of, its own magnetic flux from the circuit itself. Hence, if the self-linked flux due to the primary current is annulled by the simultaneous creation of an oppositely directed flux in the core, the inductance of the primary circuit must thereby be diminished or destroyed. Accordingly, the transformer possesses the following property. If the secondary circuit is open, the primary circuit possesses large inductance, and a very small current will flow through it,

even under a large primary electromotive force. If the secondary circuit is closed by a gradual diminution of the external secondary circuit resistance, then the gradually increasing secondary current, little by little, annuls the inductance of the primary circuit, and lets more primary current flow through the primary circuit. The transformer thus feeds itself with primary current as it requires it, to meet the demand for secondary current. When secondary current is not being made at all, the impedance of the primary circuit blocks the way, and prevents any but a small current flowing through the primary circuit.

An intelligent grasp of the above fact is necessary in order to understand the method of working transformers in practice.

§ 11. **Alternate Current Transformer Distribution.**—The service which the transformer renders us in electrical engineering is, that it enables us to convey large amounts of electric power in the form of relatively small electric currents produced by high electromotive force, or in the form of what are called high tension currents.

Beyond a certain distance from the source of supply, the transmission of a given amount of electric power costs less when this is conducted in the form of a small current flowing under a high electromotive force, than when it is in the form of a large current under low electromotive force. The size of the conductor, for one thing, can be less; hence less copper is required; but, as a set-off against this saving, a higher and more costly insulation has generally to be employed, and at the same time the additional cost of the transformer is incurred.

Hence the economy is not proportional to the diminution of the current, but beyond a certain distance an economy *is* found to be effected by the use of the high tension current. High pressure currents, however, are not directly applicable for electric lighting and power purposes, and the necessity exists for transforming the

energy into a form in which it presents itself as a large current flowing under lower electromotive force. The alternate current transformer enables us to effect this transformation in the case of power supplied by alternating currents. If a high pressure current, representing a certain power, is passed through the primary coil of a transformer, we can take off from the secondary coil almost the same amount of energy, but in the form of a

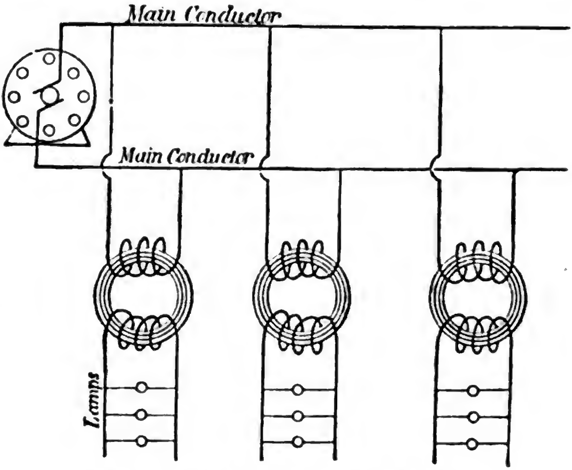

Fig. 82.—Transformers arranged in Parallel.

low pressure current. Transformers for electric distribution on the alternating current system are arranged as in Fig. 82. From a machine for generating alternating currents proceed two *primary mains*, and between these mains is maintained a large potential difference, say, of 2000 or 10,000 volts. At the places where the transformation of energy is to be made, alternate current transformers are placed, the primary coils of these

transformers being connected across between the two primary circuits. These transformers, taking primary current from these mains at a high pressure, transform down the pressure in a certain ratio, say from 2000 to 100 volts; and from their secondary terminals proceed two conductors, called secondary leads, to which are attached the lamps or motors, or other devices for utilising the current. In this system there is a loss of power due to the heating of the primary mains, and also a loss due to the internal losses in the transformers; but under proper conditions, and beyond a certain distance, these two sets of losses may be made to involve less total cost than would be the case under the direct transmission of the same amount of electric power at a lower pressure. In the early days of electric lighting attempts were made to arrange the transformers with their primary coils in series, as in Fig. 83, but the fact that the impedance of the primary coil of a transformer is diminished by closing the secondary circuit was not then fully appreciated.

On arranging transformers with their primary coils in series, it was found that, on switching on incandescent electric lamps on the secondary circuit of one transformer station—that is to say, on closing the secondary circuit of that transformer—the common primary current was increased through the primary coil of every transformer, and hence caused an increase of secondary electromotive force in every other transformer. In other words, transformers arranged with *primary coils in series* are not independent of each other, whereas transformers arranged with *primary coils in parallel* between two constant potential primary mains are independent of each other. In this latter case the reduction in the resistance of the secondary circuit of one transformer, made by adding lamps in parallel to that circuit, merely decreases the impedance of the primary circuit of that transformer alone, and makes no difference to any other transformer.

When incandescent lamps are operated from the secondary circuit of a transformer, the lamp circuits being all arranged *in parallel*, that is to say like the rungs of a ladder between the two secondary mains, then an essential condition for successful working is, that whatever the number of lamps within the limits of the normal load of the transformer, the difference of potential

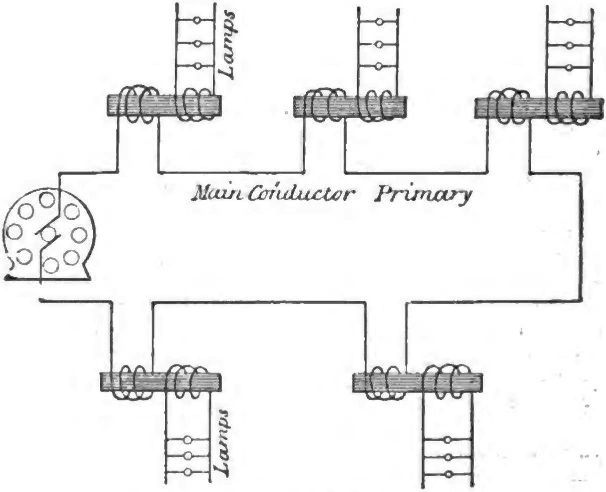

Fig. 83.—Transformers arranged in Series.

between the two secondary mains shall always be preserved as nearly as possible constant. It is not possible to do this exactly, one reason being that the increased call for secondary current causes a greater potential drop or volt-fall down the internal or transformer part of the secondary circuit, and so reduces the available potential difference between the external secondary leads. The total reduction of secondary terminal voltage is called

the *secondary drop* of the transformer. The secondary drop of a well-designed transformer should not exceed 2 per cent. between full secondary load and no load, reckoned on the voltage at no load. Hence, if the potential difference or secondary volts of the transformer on open secondary circuit is 100 volts, then that voltage should not decrease below 98 volts when the transformer is fully loaded. This secondary drop is not altogether due merely to the resistance of the secondary circuit. It is partly caused by so-called *magnetic leakage* in the transformer. The very act of making a secondary current in the transformer causes some of the magnetic flux created in the core by the primary current to be rejected or thrust out of the core so as to escape linkage with the secondary circuit. Hence a reduction in secondary electromotive force takes place, which is called the *leakage drop*.

The design of a good transformer is a matter of compromise; the following points, however, have to be considered :—

1. The iron core loss for small transformers should not exceed 3 or 4 per cent. of the full load output in watts. For transformers larger than 10 kilowatts size, it can be kept down to 1·3 or 1 per cent., or less.

2. The secondary drop should not exceed 2 per cent. of the normal secondary voltage, and the secondary drop must not be made small by making the iron core loss large.

3. The transformer must have cooling surface enough to dissipate all the core losses without rising above the temperature of boiling water. To do this at least 3 or 4 square inches of radiating surface must be provided for every watt dissipated in the core.

4. The insulation must be strong enough to withstand double the working pressure without failure.

5. In large transformers some mechanical ventilation must be provided to keep down the temperature of the core.

§ 12. **Construction of Alternating Current Transformers.**—In the construction of transformers the first question is the structure of the core.

The core is now always made of iron or steel plates or strips about ·014 of an inch in thickness. These plates are varnished or painted to give them an insulating

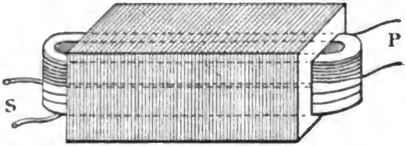

Fig. 84.—A Mordey Transformer.

coating. The plates or strips are then so arranged as to form a core, on which are wound two insulated copper circuits. The core generally takes one of two forms, either (1) the core forms a closed ring or circuit and the two coils are wound upon it, or (2) the two circuits are laid together and embraced by the core.

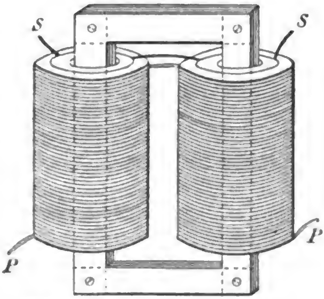

Fig. 85.—A Kapp Transformer.

An example of the first type is the Mordey transformer (see Fig. 84), and of the second type the Kapp transformer (see Fig. 85).

It will be seen that whichever form of core is adopted the effect is the same, viz. to provide a closed iron path for the magnetic flux. The core plates having been stamped out to shape and arranged, the coils are then wound on *formers*. These are made of high-conductivity copper covered with cotton and well varnished. The primary and secondary coils are

separated by ebonite or mica sheets to insulate them perfectly.

The circuit generally called the secondary circuit is in large transformers formed of thick copper strip, band, or rod. The primary circuit is of a much longer and finer wire. Either circuit may, however, be the primary or secondary circuit, depending on the way in which the transformer is used, that is, whether employed to raise or to lower potential, or as a *step-up* or *step-down* transformer. In any case the final result of the construction is to give us two insulated circuits, one having generally a few turns of thick wire and the other many more turns of thinner wire, and both of these wound round an iron ring, or enclosed in iron in such fashion that the magnetic flux produced by the passage of a current through the coils is wholly confined to the iron. A transformer of this sort is called a *closed iron circuit transformer*. It is usual to split up the primary coil into two or more coils, and to sandwich these in between the secondary coils, or to wind the secondary coil over the primary coil, the object of these arrangements being to reduce the magnetic leakage.

When a constant primary electromotive force is applied to the terminals of the primary coil, the result is to create a secondary electromotive in the other circuit, which is reduced or increased in R.M.S. value according as the second circuit has fewer or more turns than the first. Moreover, whatever may be the *wave form* of the primary electromotive force, the secondary electromotive force wave has exactly the same form to a reduced or increased scale.

The transformer is therefore a sort of electrical pantograph, which copies exactly to a reduced or enlarged scale varying potential difference applied to one pair of terminals.

The object of laminating the iron core is to prevent the production of eddy electric currents in the core, which would be, if permitted, a source of loss of power. No

amount of lamination, however, will prevent the hysteresis loss.

§ 13. **The Induction Coil.** — Although with alternating currents the above described closed circuit transformer is always used, yet, when employing intermittent or interrupted continuous currents, the transformer generally takes a form in which it is called an *Induction Coil* or *Open Circuit Transformer*. In this case the iron core is simply a bundle of iron wires or iron strips, and the primary and secondary coils are wound over it in sections. This form of transformer is used to produce intermittent high pressure currents of short durations, called secondary discharges, from an intermittent or interrupted primary current. In this use of the transformer, the core is magnetised and demagnetised each time as the primary current is stopped and started, but the magnetic flux in it is always in one direction. The secondary electromotive force is produced by the insertion or withdrawal of this magnetic flux into or from the secondary circuit. Since the secondary electromotive force depends on the *rate at which* the magnetic flux is created or destroyed, it is obvious that to obtain high electromotive force in the secondary circuit it is necessary to magnetise and demagnetise the core very suddenly. The arrangements for doing this are shown in Fig. 86. Near the end of the iron core is held a small block of iron H, called the hammer, supported on an elastic shaft *h*. The primary current passes from one terminal of the coil to a fixed support which carries a platinum-tipped screw *o* at one end. The current passes through this screw into the point, and then passes into a small platinum point fastened at the back of the hammer head or to the spring shaft. It then passes down the elastic spring to the end of the primary coil *q*, and so through the coil and back to the other terminal *l*. If, then, the screw is so arranged as just to touch the contact pin in the spring shaft, the result of this arrangement is that as soon as the current magnetises the iron core I of the induction coil, this last,

acting as an electromagnet, attracts the hammer head H. This movement breaks the contact at the platinum surfaces, and so interrupts the current. The hammer head then flies back again, and, making contact with the screw, again re-starts the current. The arrangement is, therefore, a self-acting contact breaker, which continually starts and stops the primary current, and hence magnetises and demagnetises the iron core. This alone would not be sufficient to effect the magnetic changes in the core

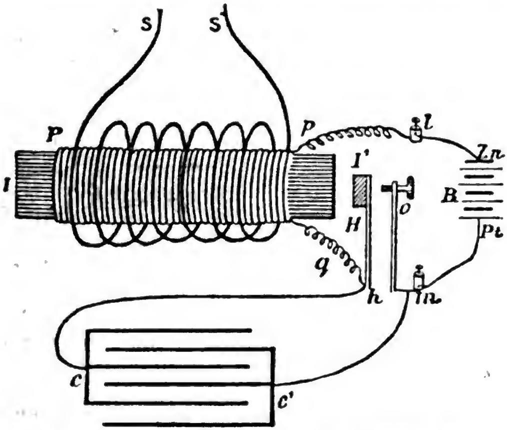

Fig. 86.—Diagram of Induction Coil Connections.

sufficiently quickly without the addition of a piece of apparatus called the *Condenser*. The condenser cc' consists of a number of sheets of tinfoil, which are sandwiched in between sheets of paraffined paper or mica. Alternate sheets of tinfoil are connected together, and the two sets of connected tinfoils with the paper between them virtually form a Leyden jar. The terminals of this condenser are connected to the platinum terminals, between which the "break" of the primary circuit occurs.

The action of the condenser appears to be somewhat as follows. When the platinum terminals separate, the core of the induction coil is at that instant strongly magnetised in one direction. The separation of the platinum contacts throws the condenser into series with the primary circuit. Under these conditions the primary current, instead of continuing to flow and making a spark across the contact tips, in virtue of the high electromotive force of self-induction set up at the "break" of the circuit, expends itself in setting up what are called electrical oscillations in the primary circuit. The current surges backwards and forwards in the circuit, but dying away in strength. These electrical oscillations or surgings take place with extreme rapidity, and they rapidly demagnetise the core. Hence, instead of the magnetic flux in the core diminishing slowly, and so producing a small secondary electromotive force, it is very rapidly destroyed, and therefore a much larger electromotive force is set up in the secondary circuit. The condenser is, therefore, an essential adjunct in a coil intended to give long sparks from the secondary circuit when used with an interrupted primary current, but it is of no value if the induction coil is used with alternating currents; that is, if an alternating current is passed through the primary circuit and generates an alternating current in the secondary circuit. The condenser only fulfils this function at the "break" of the primary circuit, and hence in an ordinary "spark coil" the electric discharges in the form of sparks from the secondary are all in one direction. The electromotive force set up in the secondary at "make" or starting of the primary is relatively very small, and the whole of the secondary sparks are due to the "break" or stoppage of the primary current.*

The maximum value of the electromotive force set up in the secondary circuit by the interruption or reversal

* For a fuller discussion of the action of the condenser, and for a description of the mode of constructing induction coils, the reader desirous of more advanced information may be referred to the Author's book on 'The Alternate Current Transformer,' vol. ii.

of the primary circuit can be roughly estimated from the length of the secondary spark.

If the secondary terminals are fitted with metallic balls one centimetre in diameter, the maximum secondary voltage can be approximately inferred from the following table of sparking distances, by which it will be seen that a spark about an inch in length requires for its production a voltage of about 30,000.

Sparking Distance in Centimetres between metallic balls 1 centimetre in diameter.	Maximum Potential Difference between the balls in Volts.
0·1	4,765
0·2	8,140
0·3	11,307
0·4	14,119
0·5	16,664
0·6	19,210
0·7	21,823
0·8	22,792
0·9	24,153
1·0	25,071
1·1	26,255
1·2	27,024
1·3	27,765
1·4	28,359
1·5	28,949
1·6	29,363
1·7	29,837
1·8	30,133
1·9	30,547
2·0	30,932
2·1	31,198
2·2	31,494

Many circumstances, such as the form of the sparking surfaces or points, the pressure and nature of the surrounding atmosphere, the nature of the light falling on the terminals, affect the sparking distance for given voltages, and the above table must not be taken as giving more than approximately the sparking distance for given maximum potential differences expressed in volts.

CHAPTER IX.

ELECTRIC MEASURING INSTRUMENTS.

§ 1. **Classification of Measuring Instruments.**—The next subject which must engage attention is that of the practical measurement of electric currents. We have already shown how a simple tangent galvanometer can be constructed and calibrated, and it is necessary to supplement this elementary information by some further knowledge of electrical testing instruments. We shall not attempt to make even a mention of all the numerous processes and instruments employed in electrical testing, but confine our attention to a few of the most practically useful appliances and methods. Electrical instruments are classified according to the purpose for which they are to be used and the scientific principle involved in their construction. We have thus one classification as follows :—

Electric measuring instruments may be—

1. *Amperemeters*, or *Ammeters*, for measuring the strength of electric currents.
2. *Voltmeters*, for measuring electric pressure, potential difference, or electromotive force.
3. *Ohm-meters*, for measuring electrical resistance.
4. *Watt-meters*, for measuring electrical power.
5. *Coulomb-meters*, or *Ampere-hour meters*, for measuring electric quantity.
6. *Joulemeters*, or *Watt-hour meters*, for measuring electric energy in joules or Board of Trade units.

They may also be classified according to the physical principles involved in their construction, as—

1. *Electrodynamic* instruments.

2. *Electrostatic* instruments.
3. *Electrochemical* „
4. *Electrothermal* „
5. *Electromagnetic* „
6. *Electro-optical* „

It will be convenient to consider, in the first place, the methods of making certain general electrical measurements which are fundamental.

§ 2. **The Potentiometer.**—One of the most practically useful instruments in making electrical measurements with continuous currents is the arrangement called

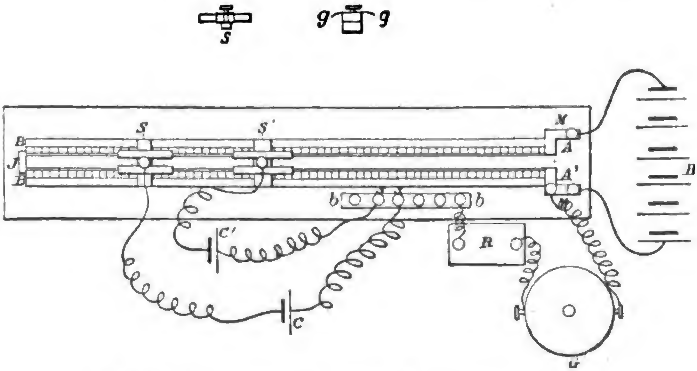

Fig. 87.—Potentiometer for comparing Electromotive Forces.

the *Potentiometer*. The construction of this instrument is as follows :—

Let A B B′ A′ (see Fig. 87) be a long fine bare wire of platinoid or German silver, or some material having a small resistance temperature co-efficient. Let this wire be stretched over a scale 2 metres long, divided into 2000 parts.

The ends of this wire are connected, through an adjustable resistance, with a secondary battery B, generally

of two cells, having a very constant electromotive force. The resistance of the wire A B B′ A′ should be at least 40 ohms, in order that the current flowing through it under a pressure of 4 volts may be only the fraction of an ampere. The wire must be of such uniform diameter and structure that the fall in potential down the wire per centimetre of length is the same at all parts of the wire. The current through it is then to be adjusted to a standard value in which the fall in potential down the 2000 divisions of length of the wire is exactly 2 volts. This is done by means of a resistance and a Clark standard cell. A Clark standard cell C, supposed to be at a temperature of 15° C., has its positive pole connected, through a sensitive galvanometer G, with that end of the divided wire in connection with the positive pole of the working battery B. The other pole of the cell is connected to a slider S, which makes contact with the wire at any desired point. The slider is first moved to touch the wire at 1434 divisions from the positive end, and a resistance in series with the potentiometer wire and battery (not shown in Fig. 87) is then varied until the galvanometer shows no current through it. When this is the case we know that the fall of potential down the 1434 divisions of the wire, due to the working battery, must be equal to the electromotive force of the Clark cell, which is 1·434 volts at 15° C. If the cell is not at 15° C., then, looking out in the table on p. 99 the electromotive force of the cell at the temperature at which it actually is at the time being, we set the slider to read that corresponding number on the scale. Thus, if the E.M.F. of the cell is 1·440 volts at the time, let the slider be set to make contact at 1440 divisions on the scale. This being done, we now know that, the fall in potential or voltage drop down 1434 divisions of the wire being 1·434 volts, the voltage drop down 2000 divisions must be two volts, and the voltage drop down 1 division must be $\frac{1}{2000}$th of a volt. The potentiometer is then said to be set for use. Suppose, then, that we desire to measure the electromotive

force of any other voltaic cell C′. We substitute this cell for the Clark cell and find the new position S′, to which the slider must be moved, so that the galvanometer G indicates no current. Suppose that it is at 1920 divisions on the scale, then we know that the electromotive force of the cell C′ is 1·920 volts.

Again, suppose that we require to measure a continuous electromotive force or potential difference of 100 volts, or some such value. We connect across the terminals or ends of the circuit, between which exists the potential difference to be measured, a wire of considerable resistance, say of 10,100 ohms. A connection is made to the ends of a fraction of this which may be, for instance, 100 ohms, or one-hundredth of the larger resistance. The wire is contained in a box, usually called a

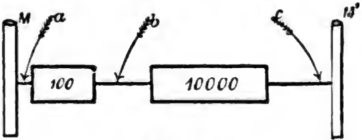

Fig. 88.

Volt-box (see Fig. 88). Thus, if there exists a potential difference of 101 volts at the extremities *a c* of the whole wire of 10,100 ohms resistance, the potential drop down the 100 ohms *a b*, in series with the resistance of 10,000 ohms, will be one volt. If we measure, as above described, by means of the potentiometer when set for use, the drop in potential down this one-hundredth of the volt-box resistance, we know that the whole potential difference between the ends of the volt-box wire must be just 101 times as great.

The divided resistance, therefore, enables us to measure on the potentiometer any voltage by measuring a known fraction of it as above described. By a suitable divided resistance any continuous potential difference or electromotive force may be measured on the potentio-

meter. In using the potentiometer two or three standard Clark cells should be carefully checked against each other, to ensure accuracy in setting the potentiometer. The potentiometer is also used to measure continuous electric currents in the following manner. A series of low resistances are provided, made of strips of platinoid or manganin, or some material not changing its resistance much with temperature. To these resistances terminals are fixed for sending through them large electric currents, and smaller wires, called potential wires, are soldered to the strips at such places that the resistance between the potential wires is 0·1 ohm, 0·01 ohm, or 0·001 ohm, as the case may be. These low resistances must be accurately adjusted, and have a cooling surface so great that they do not become sensibly heated when 10, 100 or 1000 amperes are sent through them respectively.

Suppose, then, that a current of 10 amperes is sent through the one-tenth of an ohm, and potential wires are taken from its potential terminals to the potentiometer. It is clear that there will be a fall of potential of one volt down this resistance, and this voltage may be measured on the potentiometer just as if it were the electromotive force of a cell. If, then, an unknown current is sent through the strip, and we find the terminal voltage of the strip to be ·951 volt, we know the current is 9·51 amperes. In this manner, by the use of two or three different low resistance strips, we can make measurements of continuous currents over an enormous range of values, and measure, by means of a Clark cell and a potentiometer, any continuous current or any continuous voltage.

In using the potentiometer in a workshop, it is desirable to employ a galvanometer of the type called a movable-coil galvanometer, because this is not disturbed by other electric currents in its neighbourhood, as is the fixed-coil type of galvanometer. The potentiometer is an exceedingly useful instrument for calibrating other

direct-reading instruments, or instruments which show directly on a scale current values or voltage values.

§ 3. **Galvanometers.**—In many electrical measurements it is necessary to be able to detect the presence or prove the absence of an electric current in a circuit, and if it is present to be able to measure it. Generally speaking, instruments for the measurement of electric currents which are not so constructed as to show at once or directly the value of the current in amperes passing through them, are called *galvanometers*. If, however, they have scales so constructed that a needle or indicating device shows at once the current passing through them reckoned in amperes, then they are called *ampere meters* or *ammeters*.

The majority of the most generally used galvanometers are electromagnetic instruments, and depend for their operation on the fact that an electric current passing through a coil creates round it a magnetic flux, and gives it, as we have seen, all the properties of a magnet. This flux, other things being equal, is proportional in strength, at any point in the field of the coil, to the current passing through the coil. If the coil is fixed, and if we create another fixed magnetic field, called *the controlling field*, in a direction at right angles to the flux due to the coil along its axis, then, as we have seen, we can use a very small suspended magnet to tell us the ratio between the strength of the field due to the coil and the strength of field due to the fixed controlling magnet. An arrangement of this kind is called a *fixed coil* or movable needle galvanometer. In Fig. 89 is shown a Kelvin Mirror galvanometer of the above type. In this instrument there are two pairs of coils placed one above the other; and also a curved controlling magnet sliding on a vertical rod, by means of which the controlling field is produced. The indicating needles form an *astatic system* —that is to say, they consist of two groups of little magnets fixed to a stem, one group set with their North poles in one direction, and the other with their North

poles in the opposite direction. The stem also carries a concave mirror having a radius of curvature of one metre.

This astatic system of needles is suspended by a cocoon silk fibre, so that the upper set of needles hangs

Fig. 89.—Kelvin Mirror Galvanometer.

in between the two upper coils, and the lower set hangs in the aperture of the two lower coils; and the four coils are so connected electrically, that a current flowing through them would act on both sets of needles so as to

tend to turn the plane of all the needles parallel to the axis of the coils.

The object of employing the astatic system of needles is to get greater sensitiveness, by rendering the system of needles free from the control due to the Earth's magnetic field. In setting the instrument so as to get the greatest sensibility, it is placed with the plane of the coils perpendicular to the magnetic meridian, or with the side of the coils towards the magnetic North. The needles, being never absolutely astatic, will then place their axes in the direction of the axes of the coils. The controlling magnet is then lowered into such a position that it exactly reverses the position of the needles, and then, without turning it, the controlling magnet is raised up parallel to itself. A position will then be found in which the astatic system of needles will stand parallel to the plane of the coils. Any small electric current, if then sent through the coils, will cause the needles to turn so as to set more or less at an angle to the plane of the coils. It is not desirable that the Earth's field should be *quite* annulled by the controlling magnet, or else there will be no restoring force tending to bring the needles back to their original position after the current is stopped. It is advisable to lower and turn round the controlling magnet very slightly, so as to produce a resultant controlling field which is very feeble, but which is in a direction parallel to the plane of the coils.

The *sensibility* of such a galvanometer in any condition is measured by the current which will deflect the needle through an angle of nearly one-thirtieth of a degree. This is measured as follows. An incandescent electric lamp is placed at a distance of about one metre from the galvanometer, and the light from the lamp-filament is allowed to fall on the concave mirror attached to the needle stem. The lamp should be covered with a metal or asbestos hood having a slit in it, so that only the light from one half of the filament (which should be

a horseshoe-shaped filament) can fall on the galvanometer mirror. The mirror reflects the light back, and forms a sharp image of the filament on a semi-transparent scale of celluloid or glass, which must be placed at a distance of one metre from the galvanometer. The scale is divided into millimetres. If, then, the image of the filament is displaced by one millimetre on the scale, we know that to do this the ray of light coming from the galvanometer mirror a metre distant must have been turned through an angle whose tangent is $\frac{1}{1000} = \cdot 001$; and hence, by optical principles, the mirror itself must have turned through an angle of half that value, viz. an angle whose tangent is $\frac{1}{2000} = \cdot 0005$. This angle is an angle of about $1\frac{3}{4}$ minutes of arc, or somewhat less than $\frac{1}{30}$th of a degree. The sensibility of galvanometers can be compared by taking note of the current required to produce the above deflection. It must, however, be observed that this comparison must be subject to the condition that the galvanometer needle returns to a fixed zero when the current is arrested. It is very easy to obtain a spurious sensibility which is of no practical utility.

The drawbacks to the use of a movable needle galvanometer of the above type are, that it is disturbed by the stray fields produced by currents in neighbouring wires; and, when once the needle has been disturbed, it does not regain its zero position without time-wasting vibrations. On this account it has been largely replaced of late years in electrical laboratories by the *movable coil* galvanometer.

In this latter instrument, the coil which is traversed by the current to be detected or measured is a small light coil of silk-covered copper wire wound on an ivory or silver frame, and having a rectangular or circular form. This coil is suspended by a pair of very fine flattened phosphor-bronze wires, either attached to the top and bottom of the coil, or arranged parallel to one another

to form a *bifilar* suspension. These fine suspending wires are attached to the ends of the coil wire, and serve to bring in and take off the current.

The coil is suspended (see Fig. 90) in the field of a fixed permanent magnet, so arranged that the normal position of the coil is with its plane parallel to the direction of the fixed magnetic field. If, then, a current is passed through the coil, the coil turns round so as to place its magnetic axis more or less in the direction of the fixed

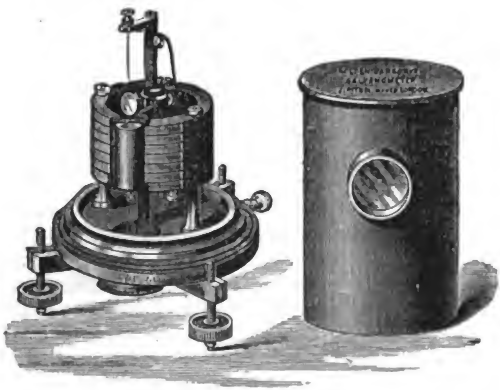

Fig. 90.—Holden d'Arsonval Movable Coil Galvanometer.

field. The torsional elasticity of the phosphor-bronze suspending wires affords the necessary *control*, and brings the coil back to its original position when the current ceases.

The coil carries attached to it a concave silvered mirror, which, by means of a lamp and scale, is made to indicate the smallest movement of the coil. The coil is hardly, if at all, disturbed by the field due to currents in neighbouring conductors. Moreover, it is very *dead-*

beat—that is, does not execute many vibrations before it comes to rest.

A galvanometer of the above kind can be so calibrated as to enable us at once to determine the value in *milli-amperes* of a small current passing through it. This is done as follows. Measure on the potentiometer the electromotive force of a secondary battery cell. Connect this battery cell in series with the movable coil galvanometer, through a very high resistance, and observe the scale deflection of the image of the filament of the galvanometer lamp. Thus, suppose the cell had an electromotive force of 2·110 volts, and that it produced a scale deflection of 100 millimetres, when connected with the galvanometer, through a resistance of 10,000 ohms. Suppose the resistance of the galvanometer to be 500 ohms. Then an electromotive force of 2·11 volts, acting on a resistance of 10,500 ohms, produces a current of $\frac{2\cdot 11}{10500}$ amperes = 0·2 milliampere nearly. Hence, a deflection of one millimetre on the scale corresponds very nearly to a current of ·002 milliampere through the galvanometer.

§ 4. **Measurement of Electrical Resistance.**—The most direct method of measuring electrical resistance is by measuring the voltage required to produce a known current. Suppose, for instance, it is required to measure the insulation resistance of the electric light wiring of a house. A movable coil mirror galvanometer is set up and standardised, as described in the last section, so as to determine the current through it reckoned in fractions of an ampere corresponding to a certain scale deflection. A battery of small cells is then provided sufficient to give an electromotive force of, say, 100 volts. The electromotive force of the battery can be accurately measured by the potentiometer. This battery, having a known electromotive force is then connected to the galvanometer already standardised, through the insulation resistance. This is done by connecting the copper circuit of the house wiring with one pole of the galvanometer,

which must be carefully insulated, and the other pole of the galvanometer to one terminal of the battery, which must also be insulated. The remaining battery terminal is connected to a "good earth," by attaching it to the gas or water pipes. The galvanometer will then show a deflection, and the current passing through it, reckoned in fractions of an ampere, is known from this deflection. Hence we know the current in amperes through the conductor produced by a given electromotive force in volts; and the resistance in ohms is obtained by dividing the numerical value of the volts by the ampere value of the current. The above method is a good one to adopt in the case of fairly high resistances.

We can also use a movable coil galvanometer to measure low resistances, such as the resistance of the secondary coils of transformers or armatures of dynamos. For this purpose a set of standard resistances must be provided, constructed in the manner described in § 6. At the present moment we will assume that resistance standards of 1 ohm, 0·1 ohm, and 0·01 ohm, are available. Having such standard resistances at hand suitable for carrying currents of one or two amperes, the value of an unknown low resistance, like a dynamo armature, can best be measured by what is called the *fall of potential method.* This method depends on the fact that when a continuous and constant electric current is flowing through a circuit, the fall of potential or drop in volts down any portion of that circuit is proportional to the resistance of that part of the circuit.

The meaning of the phrase, fall of potential, can be made clear by an hydraulic illustration. If water is flowing along a horizontal pipe, and if gauge glasses or stand pipes are inserted at intervals (see Fig. 91), then when the water is not flowing we should find the water standing up at the same level or pressure, or potential, in each gauge glass. If, however, the water in the pipe is allowed to flow, then there would be a regular diminution in pressure all along the pipe, as shown by the

height of the water in the stand pipes. This is called the *hydraulic gradient*, or fall in pressure along the pipe. Selecting any two points on the pipe, we should find that there is a difference in pressure at the two points; the water pressure being higher at that point from which the water flows, and the difference in pressure between the two points being the force driving the water along

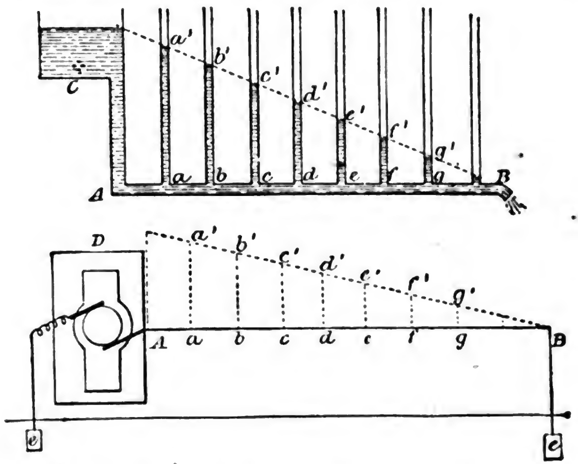

Fig. 91.—Analogy between Hydraulic Gradient and fall in Electric Potential.

the intermediate piece of pipe. In the same manner, when an electric current is flowing along a conductor from a dynamo or battery, there is a difference of electric pressure or fall of pressure or potential between points taken on the conductor. By Ohm's law the current in amperes is measured by the quotient obtained by dividing the difference of pressure in volts between any two points by the resistance measured in ohms. Hence, if the current is the same through two sections or portions

of a circuit, the fall in volts down the sections must be proportional to their resistances.

In order, therefore, to make use of this principle, we join up the unknown low resistance, such as the armature, in series with a known low resistance, say of 0·1 of an ohm, and with a battery capable of giving a constant current through this resistance. A movable coil galvanometer of high resistance, or with a high resistance inserted in its circuit, then has its terminal wires applied first to the ends of the known low resistance, and next to the ends of the unknown low resistance, and in each case the galvanometer deflection is noted. This galvanometer deflection is proportional to the difference of potential between the ends of the conductors tested. If then we find a deflection of 25 scale divisions of the galvanometer spot of light, when the ends of the galvanometer wires are attached to the terminals of the armature, and a deflection of 5 scale divisions when they are attached to the terminals of the 0·1 ohm resistance, we should know that the armature resistance was to 0·1 ohm in the ratio of as 25 to 5, or 5 to 1. Hence, this particular armature resistance is 0·5 of an ohm. The galvanometer deflections should be observed several times, and for several different currents, and the mean of their ratio taken as the proper value.

For the measurement of resistances varying in magnitude from 1 ohm to 10,000 or 20,000 ohms, the most convenient method to employ is that known as the *Wheatstone's Bridge Method.** In this appliance the unknown resistance to be measured is joined up with three other known resistances, so as to form a quadrilateral, or set of resistances, joining four points arranged as in Fig. 92. Let P, Q and W be the known resistances, and R the unknown resistance to be determined. Then let a battery B be joined to the points *a* and *b*, and a gal-

* The instrument always known as the "Wheatstone's Bridge" was not invented by Wheatstone, as generally supposed, but was described first by Mr. Hunter Christie, in 1833, to the Royal Society.

vanometer G to the points *c* and *d*. Under these conditions it is always possible to make a selection of the resistances P, Q and W, such that when the galvanometer is joined in at *c d*, no current flows through it. When this is the case the points *c* and *d* must be at the same

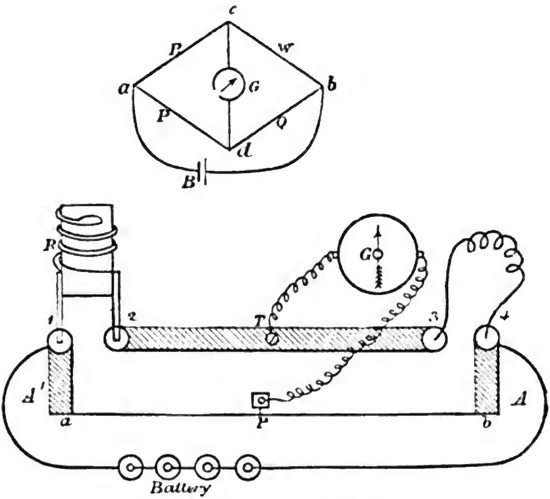

Fig. 92.—Wheatstone's Bridge.

potential or pressure. Hence the fall of potential down the resistance P must be equal to that down the resistance R, and similarly the fall of potential down the resistance Q must be equal to that down the resistance W. Hence it is easy to see that the following proportion holds good :—

$$\frac{\text{Fall of potential down resistance P}}{\text{Fall of potential down resistance R}} = \frac{\text{Fall of potential down resistance Q}}{\text{Fall of potential down resistance W.}}$$

Also, since no current flows through the galvanometer, it follows that the current through P is the same as that through Q, and the current through R is the same as that through W. Hence, by the principles previously explained, the resistances of P, Q, R and W stand to each other in the proportion of these potential falls, or when the galvanometer current is zero

$$\frac{\text{Resistance of P}}{\text{Resistance of Q}} = \frac{\text{Resistance of R}}{\text{Resistance of W.}}$$

If, therefore, three of these resistances are known, the fourth can be calculated.

The branch in which the galvanometer is placed is called the *Bridge* circuit, and the bridge is said to be balanced when the four resistances have the above-named relation.

In actual practice, the Wheatstone's bridge takes one of two forms, called respectively the *Divided Wire* or *Metre Bridge*, or the *Post-Office Pattern Plug Bridge.*

The divided wire bridge is made as follows :—

A stout mahogany board is prepared, about 4 feet long and 6 inches wide. It should be stiffened with cross pieces like a drawing board, to keep it from warping out of shape. On this board is glued a scale of one metre in length, divided into millimetres. Over this scale is stretched a uniform wire *a b* (see Fig. 92), of platinoid, German silver, or platinum-iridium or other alloy, having a small resistance temperature coefficient. The ends of this wire are attached to thick strips of copper, A A′, and brought to two terminals 1 and 4 (see Fig. 92), between which is a third insulated terminal, T. A slider P moves over the wire by means of which contact can be made with it at any desired point. A plug resistance box must then be provided. This is constructed in the following manner. On an ebonite slab forming the top of a small box, are arranged a series of pieces of brass (see Fig. 93), which can be connected by fitting metal plugs into the space between the

blocks. These plugs are ground to fit into conical recesses in the ends of the blocks. In the interior of the box are coils of insulated wire, the ends of which are soldered to the brass blocks. When a plug is put into a hole so as to connect two blocks, it *short circuits* one particular coil connected to these blocks, or throws it out of action. The resistances of these coils may be of various magnitudes, such as 1 ohm, 2 ohms, 5 ohms, &c. Hence, if all the plugs are in their places there is practically no resistance interposed between the terminals of the box, except that of the plugs and blocks,

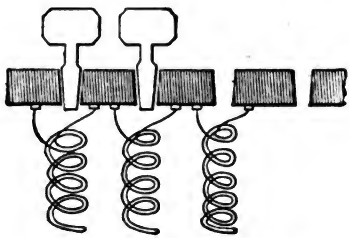

Fig. 93.—Plug Resistance Coils.

which is negligible. If a plug is withdrawn it puts the resistance of the coil belonging to it into series, and by suitably withdrawing plugs we can make any resistance we please. The resistances in a full-sized resistance box are generally arranged to be respectively 1, 2, 3, 4, 10, 20, 30, 40, 100, 200, 300, 400, 1000, 2000, 3000, 4000 ohms. The whole of the plugs being withdrawn give a total resistance of 11,110 ohms. In using the box, it is necessary to see that the plugs are well fitted to their holes, and in putting in a plug the user should give it a slight twist to make it make good contact, but not force it in too tightly, or else in getting it out again the ebonite head may be wrenched off. Never lay the plugs on the table when out of their holes, but

always place them in the lid of the box. The box, when not in use, should be kept in a stout cardboard outer box, so as to preserve the ebonite from dust and the action of light, both of which spoil its insulating power.

The terminals of this plug resistance box are then connected to the terminals 1 and 2 of the bridge, and between the terminals 3 and 4, is connected the unknown resistance to be measured. A battery of two or three cells is then connected through a contact key with the terminals A, A′, and a galvanometer, preferably of the suspended coil type, is connected in between the terminal T, and the slider on the bridge wire P. Some plug or plugs are then removed from the box, and the slider is moved along the wire until the galvanometer shows no current. When this is the case, the resistances of the two sections of the bridge wire into which it is divided by the slider, and which are to one another in the ratio of the lengths of these sections, are in the same proportion as the ratio of the effective resistance of the plug box to the unknown resistance. The two sections of the bridge wire are the resistances P and Q in the formula above; the resistance R is the resistance of the plug box, and W is the unknown resistance. Hence, since

$$\frac{P}{Q} = \frac{R}{W},$$

when the bridge is balanced, we have

$$W = R\,\frac{Q}{P},$$

and the resistance W becomes known, being equal to the resistance R, taken out of the plug box, multiplied by the ratio $\frac{Q}{P}$, in which the slider divides the bridge wire.

In the Post Office pattern of Wheatstone's bridge (see Fig. 94), the slide wire is replaced by a set of coils joined in series through plugs and blocks, and these coils are generally a series of 1000, 100, 10, 10, 100, 1000 ohms. The centre terminal *c* is placed between the two 10 ohm coils, and the terminals *a* and *b* at the ends of the series. (See upper diagram, Fig. 92.)

By taking out plugs, the ratio of the resistance between *a* and *c* can be made to be to that between *c* and *b* in any decimal ratio such as 1000 : 10, 100 : 10,

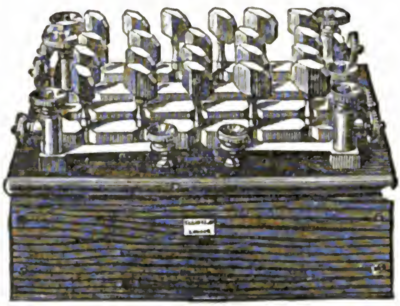

Fig. 94.—Post Office pattern Wheatstone's Bridge.

10 : 100, &c. These resistances constitute the *ratio arms* of the bridge, and are the P and Q in the above formula. The remaining arm of the bridge R is a series of coils as described. The bridge is generally completed by two contact keys, one in the galvanometer, and one in the battery circuit for closing the circuit. The battery circuit should always be closed a short time before the galvanometer circuit. With a bridge of the above kind it is very easy to measure any resistance between one ohm and 10,000 ohms very accurately.

§ 5. **Construction of Standards of Resistance.**—Every electrical laboratory must be provided with cer-

tain coils called *Standard Coils*, the resistances of which are exactly known. These resistances take various forms according to their mode of use.

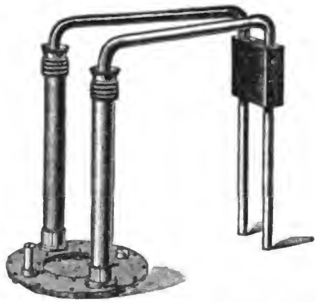

Fig. 95.—Fleming Pattern of Standard Resistance Coil.

A standard ohm coil (see Fig. 95) consists of a coil of double silk covered platinum-silver wire, which is wound up inside a flat brass box, or case of a ring-shape. The ends of the wire are attached to two thick copper rods or terminals, and these are put in connection with the bridge by means of mercury cups and thick wires. The approximate length of the wire to be used in making a resistance of this kind may be judged by the following table:—

For a Resistance Coil of	Take of Platinum-silver Wire double-silk covered—	
	Length	Gauge in B.W.G.
	feet	inch diam.
1 ohm . .	9	No. 22 = ·036
10 ohms . .	42·5	No. 24 = ·025
100 " . .	133	No. 30 = ·014
1000 " . .	675	No. 34 = ·010

Having cut these lengths of wire they are doubled on themselves, so as to make the circuit as little inductive as possible, and the ends soldered to the copper terminal rods, using rosin as a flux. The resistance of the loop is then measured on the bridge. If it exceed

the required amount, the covering is stripped off the wire at the loop, and the loop twisted up until the resistance required is nearly attained. The loop is then touched with solder. The exact adjustment of a wire to a certain resistance is a matter requiring time and patience. Having given the wire loop the requisite resistance, the covered wire is then coiled up in the brass case, and this latter made water-tight. In use the brass case is placed in a vessel of water, the temperature of which is taken with a thermometer, so that when the wire in time takes the temperature of the water, its own temperature becomes known.

To prevent the water short-circuiting the thick copper connectors, they are enclosed for a certain length in outer brass tubes, with an insulating layer between the rod and the tube. There are certain authorised standards of resistance, which are kept at the Cavendish Laboratory, Cambridge, by the British Association Committee on electrical standards, and it is possible to send a coil of the above mentioned kind to Cambridge and obtain a certificate stating its exact resistance at a certain temperature in terms of these standards. This certificate is called the "Cambridge Certificate." It is useful to have a *one-ohm standard* and a *ten-ohm* standard thus certified.

Small resistances, such as a tenth of an ohm, are best constructed by joining in parallel ten resistances, each of one ohm. Measure off on the Wheatstone's bridge ten platinoid wires of one ohm resistance and 0·02 inch diameter. Solder the extreme ends of these wires to two thick copper blocks, so that the wires are all bunched together like the strands of a rope, and terminated by the copper blocks. This compound resistance will then have a resistance of $\frac{1}{10}$th of an ohm. In the same way a $\frac{1}{100}$th of an ohm may be made by bunching 100 wires, each having a resistance of one ohm. If ten or eleven one-ohm resistances are joined up with mercury cups in the manner shown in Fig. 56, it is possible, by means

of two copper combs or connectors, to throw these resistances into parallel when the combs are placed in the cups, or into series when the combs are removed. If the ten resistances are all made exactly equal to begin with, we can then measure with great accuracy on a Wheatstone's bridge the resistance of the ten one-ohm coils in series, and know that the joint resistance of the ten coils arranged in parallel is *one-hundredth* part of their joint resistance when arranged in series. A resistance of $\frac{1}{10}$th of an ohm can in the above manner be constructed.

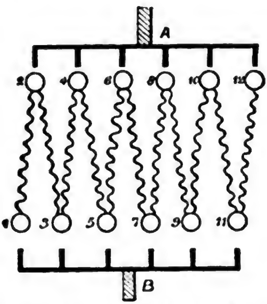

Fig. 96.—Resistances arranged in Parallel or Series.

In the construction of resistances of the above kind, we have to guard against the changes produced in the resistance by the rise of temperature caused by the current.

Referring to the Table of Resistivities of Metals and Alloys in Chapter V., it will be seen that many alloys combine small temperature coefficients with high resistivity. The attention of metallurgists has been directed to the production of such alloys, with the result that we now have many alloys of more or less constant composition, and known by trade names, which fulfil the above conditions. Examples of these are given in the following table, for the sake of comparison.

COMPOSITION AND TEMPERATURE COEFFICIENTS OF VARIOUS ALLOYS.

Alloy.	Composition.	Approximate Resistivity in Microhms per Cubic Centimetre.	a = Temperature Coefficient per Degree Centigrade.
German-silver.	Copper 4 parts, Nickel 2 parts, Zinc 1 part.	20 to 30	·0004
Platinoid	German-silver with a little tungsten.	40 to 45	·0003
Manganin	Copper 84 parts, Manganese 12 parts, Nickel 4 parts.	42 to 46	Zero at about 15° C.
Resista	..	76	·0011
Platinum-silver.	Platinum 33 parts, Silver 66 parts.	25 to 30	·00025
Manganese-copper.	Manganese 70 parts, Copper 30 parts.	100	·00004
Cast-iron	Iron with carbon and impurities.	80 to 100	·0008 or ·001
Steel	Iron with carbon and minute quantities of other elements.	15 to 50 according to composition and temper.	·002 to ·004

The resistance (R_t) of a wire at any temperature t° Cent. is given in terms of the resistance (R_0) at 0° Cent. by the formula

$$R_t = R_0 (1 + a t)$$

where a is the temperature coefficient in the above table. Roughly speaking, we may say that if the resistance of a copper wire of any size is taken as unity, the resistance of wires of the same size, but of the different materials stated, will be greater in the ratio following.

For	*Brass*,	multiply the *copper*	resistance	by	4·5
„	*Iron*	„	„	„	6
„	*German-silver*	„	„	„	12·5
„	*Platinum-silver*	„	„	„	15
„	*Platinoid*	„	„	„	24
„	*Manganin*	„	„	„	30

It is very useful to carry in the memory the following facts:—

Platinoid wire, No. 36 gauge S.W.G., has a resistance of about 4 ohms to the foot, and No. 18 a resistance of $\frac{1}{10}$th of an ohm per foot.

In the case of copper, a length of 400 feet of No. 16 wire has a resistance of one ohm; of No. 18, 230 feet, and of No. 36, about 6 feet will be required to make a resistance of one ohm.

§ 6. **Current Carrying Capacity of Wires.**—In all cases where resistances are used, we have to consider the current carrying capacity as determined by the temperature to which the resistance may safely rise. The rate at which heat is generated in a conductor is measured in watts by the product of the square of the current strength in amperes, and the resistance in ohms.

The heat so generated can only escape through the surface of the wire. Hence there must always be surface enough in contact with the surrounding medium. If bare wires traversed by a current are exposed to the air, they will get too hot to touch unless they have a surface at least of one square inch per watt wasted in them. In the case of round bare wires of different diameters, the currents which bring them to the same constant temperature are determined by the diameter of the wire, and by a certain numerical constant depending on the nature of the wire. The constant state is reached when the rate at which the heat is generated in the wire by the current is equal to the rate at which the heat gets out of the surface. The current producing any selected temperature varies as the diameter of the wire raised to the power of $1\frac{1}{2}$, or

$$C = k\,d^{\frac{3}{2}}$$

The coefficient k depends on the material of which the wire is made. The *current density* in the wire is measured by *the amperes per square centimetre* or per square inch of cross section of the wire.

In the case of insulated copper wires used for electric lighting conductors, the current density should never exceed the amount which will bring the wire to 130° Fahr. or 55° Cent., when the current is passed continuously through them. The following table shows the sizes of cables and wires, insulated with indiarubber and cotton twist in the manner usual for electric lighting work, which will carry currents of various stated strengths.

CURRENT CARRYING CAPACITY OF INSULATED COPPER WIRES AND CABLES.

Size of Cable, S.W.G.	Safe Working Current in Amperes.	Size of Cable, S.W.G.	Safe Working Current in Amperes.
No. 18	3·1	$\frac{7}{14}$	32·0
,, 17	4·0	$\frac{19}{18}$	31
,, 16	4·9	$\frac{19}{16}$	49
,, 15	5·9	$\frac{19}{14}$	70
,, 14	7·0	$\frac{19}{12}$	100
$\frac{7}{20}$	9·3	$\frac{61}{15}$	150
$\frac{7}{18}$	14·0	$\frac{37}{12}$	180
$\frac{7}{16}$	23·0		

A fraction such as $\frac{7}{18}$ indicates a cable made of seven strands of copper wire, each No. 18 S.W.G. in size.

In making what are called *rheostats*, which are variable resistances to be placed in the path of a current to regulate it, it is usual to employ spirals of galvanised iron or German-silver wire. These spirals are arranged in an iron frame or open box, having on some part of it a *rheostat handle*, by means of which one or more of the coils of wire can be inserted in circuit. In the construction of rheostats it is necessary to know the size of wire which can be used for different currents, and the fol-

lowing tables will supply that information. The spirals are assumed to be in a vertical position, and fairly well open to access of air. The temperature to which the different sizes of wire will be brought by the currents called the *safe working currents* is not more than can be easily borne by the hand placed upon them.

The table marked Hadfields' Resista Wire gives the safe working currents for various sizes of a special high resistance alloy called *Resista.*

SAFE CURRENT CARRYING CAPACITY OF BARE GALVANISED IRON WIRE.

Size S.W.G.	Safe Working Current in Amperes.	Approximate Number of Feet per Ohm.
10	18	200
12	13	135
14	8·5	80
16	6	50
18	4	28
20	3	16
22	2	9
24	1·25	6
26	1	4

CURRENT CARRYING CAPACITY OF BARE RESISTA WIRE.

Size S.W.G.	Safe Working Current in Amperes.	Approximate Number of Feet per Ohm.
10	15	32
12	11	22
14	7·5	13
16	5·7	8·4
18	4	4·8
20	3	2·6
22	2·25	1·6
24	1·6	1·0
26	1·3	·66

CURRENT CARRYING CAPACITY OF BARE GERMAN SILVER WIRE.

Size S.W.G.	Safe Working Current in Amperes.	Approximate Number of Feet per Ohm.
12	8	86
14	4·5	49
16	3·2	30
18	2	16·3
20	1·2	9·2
22	0·8	5·9
24	0·6	4·4
26	0·4	2·5

A series of experiments was made by the author some years ago to determine the currents which, when passed through bare wires coiled on wooden rods, would bring them ultimately to one steady temperature, say of 60° C.

A number of wires were prepared of copper, brass, iron, German silver, each 25 feet long, and of six sizes respectively, Nos. 10, 12, 14, 16, 18, 20 B.W.G., the diameters being as given in the table below. These wires were coiled into spirals round wooden rods about one inch diameter, and the turns of the wire well separated, so that each coil or spiral was about 18 inches long. Measured steady currents were sent through these for some hours, and so adjusted that after the temperature had become steady the wires were all at a temperature of 60° C. The currents respectively carried were as follows:—

Size of wire	No. 10 B.W.G.	12	14	16	18	20
	·134 inch diam.	·109	·083	·065	·049	·035

Safe Currents carried expressed in Amperes.

German silver .	18·75	13·5	8·25	6	4·12	3
Brass . .	30	18·75	15	9·75	7·5	5·25
Iron . .	18	13·25	10·5	8·25	5·25	3·75
Copper . .	49·5	38	26·25	20·25	15·0	9

These currents passed through the above-described naked spirals bring the respective wires to about a temperature of 60° C., when equilibrium is established.

In the construction of resistances for use in testing dynamo machines, it is necessary to adopt some form which is cheap, not too bulky, and affords a large surface for radiation of heat. Thin hoop iron bent into zig-zag shape answers fairly well. Narrow bands of iron wire gauze make an effective resistance for the purpose, as it affords an immense surface of cooling, and can therefore be made to take a very high current density.

One yard of hoop iron about half an inch wide and one-thirty-second of an inch thick measures about one-hundredth of an ohm. Hence 100 yards of it have a resistance of an ohm. Rods of electric light carbon are sometimes employed as resistances, but they are not suitable for use in cases where the resistance may be roughly used. One difficulty which attends the use of carbon is in getting effective contacts. The best plan to adopt is to dip the ends of the carbon rods in molten paraffin, and after they are well saturated, let the wax harden, then file the surface to free it from wax, and electrotype the ends with copper, and tin them by dipping in melted solder. To these tinned ends wire connections can then be soldered.

In electric lighting circuits it is always necessary to insert at some place *safety fuses*, which are wires intended to be melted or fused by the current if it exceeds a certain value, so that the circuit may be interrupted, and the insulated copper conductors preserved from over-heating.

Mr. W. H. Preece has determined the sizes of wires

made of various materials which will melt with stated ampere-currents, and it is exceedingly useful to have these values at hand. They are as follows :—

PREECE'S FUSE WIRE TABLE.

Melting Currents in Amperes.	Approximate Gauge of Wire in B.W.G. Fused by the stated Currents in the case of the Metals.			
	Tin.	Lead.	Copper.	Iron.
1	36	35	47	40
2	34	32	43	36
3	30	28	41	33
4	26	26	39	31
5	24	24	38	29
10	20	20	33	24
15	18	18	30	22
20	17	17	27	20
25	16	16	26	19
30	15	15	25	18·5
35	14·5	14	24	18
40	14	13	23	17·5
45	13	12·5	22·5	17
50	12	12	22	16
60	11·5	$\frac{2}{15}$	21	15
70	10·5	$\frac{2}{14}$	20	14
80	10	$\frac{2}{13}$	19	13·5
90	9·5	$\frac{3}{15}$	18·5	13
100	9	$\frac{4}{16}$	18	12
120	8	$\frac{3}{13}$	17·5	$\frac{2}{15}$
140	$\frac{2}{10}$	$\frac{4}{14}$	17	$\frac{2}{14}$
160	$\frac{4}{13}$	$\frac{4}{13}$	16·5	$\frac{2}{13}$
180	$\frac{3}{11}$	$\frac{4}{12}$	16	$\frac{2}{13}$
200	$\frac{4}{12}$	$\frac{5}{13}$	15	$\frac{2}{12}$
250	$\frac{5}{12}$	$\frac{5}{12}$	$13\frac{1}{2}$	$\frac{5}{16}$

§ 7. **Direct Reading Ammeters and Voltmeters.**—In all practical electrical engineering and laboratory

work it is very convenient to have instruments which show at once by an indicating needle on their scales the value of a current in amperes or a potential difference in volts. Such instruments are called *direct reading* instruments. The desirable qualities for a direct reading instrument are that it should be accurate, keep accurate or at least keep a constant error, be dead-beat, that is, the indicating needle should come to rest without many vibrations after it has been disturbed. It should also not be disturbed by neighbouring magnetic fields, and not sensibly affected by changes of external temperature. It

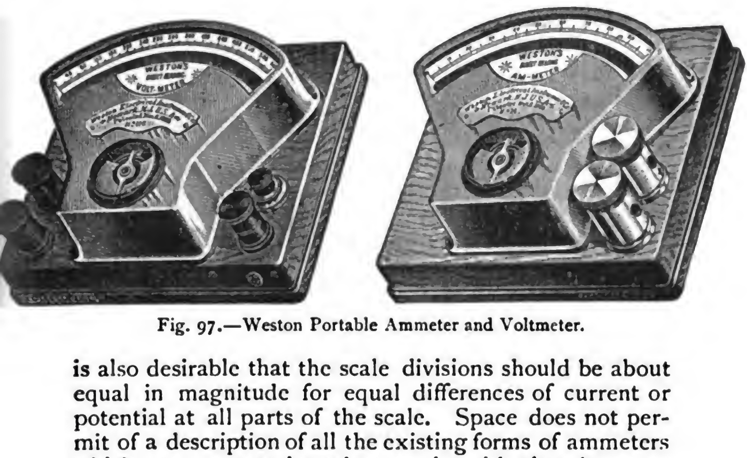

Fig. 97.—Weston Portable Ammeter and Voltmeter.

is also desirable that the scale divisions should be about equal in magnitude for equal differences of current or potential at all parts of the scale. Space does not permit of a description of all the existing forms of ammeters which even approximately comply with the above requirements. The Weston amperemeters and voltmeters (see Fig. 97) are, however, so convenient for most purposes for continuous current work that they should be mentioned. These instruments are practically movable coil galvanometers. In their construction a specially prepared steel magnet creates a constant magnetic field. In this field is placed a small coil of wire, the axis of

which is carried on jewelled pivots. The coil is controlled by a delicate spiral spring like the hair spring of a watch. The coil is fixed in the field of the magnet so that any current passing through the coil turns it round in the magnetic field. The coil carries a pointer moving over a divided scale.

The instrument is practically a very sensitive movable coil galvanometer which can be used in any position. To convert it into an amperemeter for measuring large currents, the instrument includes in the case a resistance suitable for carrying the largest currents to be used. The terminals of the movable coil are connected to the ends of this resistance. Hence, when a current is passed through the instrument it really measures the difference of potential between the ends of a fixed constant resistance traversed by the current. If the current is only a very small one, it is measured by passing it directly through the working coil of the instrument. If the instrument is to be used as a voltmeter, then the working coil has in series with it another non-inductive coil of 15,000 or 20,000 ohms resistance. It may here be pointed out that, given any principle on which a galvanometer or current measuring instrument can be constructed, the same class of instrument may be used as a voltmeter if it has high resistance, and as an ammeter if it has low resistance. When we use an instrument as a voltmeter, what we desire to do with it is to measure in some way the difference of potential of two points on a circuit. In order that this may be correctly done, the instrument used must not sensibly alter the potentials. Hence, if the instrument works by reason of a current flowing through it, that current must be small, and therefore the internal resistance of the instrument large. If, on the other hand, the instrument is to be used as an ammeter, and inserted *in* a circuit, then we must have a low resistance instrument, in order that it may alter as little as possible the currents flowing in that circuit.

A galvanometer-voltmeter, as it may be called, working by means of a current passing through it, and which is intended to measure potential differences up to 100 or 150 volts, should not have a resistance of less than 2000 to 30000 ohms.

An amperemeter, on the other hand, should have a resistance of only a fraction of an ohm.

An amperemeter intended for measuring very small currents is often called a *milamperemeter*.

Since every direct reading instrument is certain to have some scale errors, it must from time to time be checked. This is best done by means of the potentiometer. If the instrument to be checked is an ammeter and can be employed with continuous currents, then it is joined up in series with a suitable low resistance and a steady electric current sent through the two in series. The true value of the current is then obtained by the potentiometer as above described.* If the instrument is a voltmeter, then it is joined up in parallel with a volt-box resistance and a potential-difference applied to their common terminals. The value of this is determined by the potentiometer. A series of such observations are taken, recording the actual scale reading of the instrument for various currents or voltages, and the true value of the current or voltage. We then construct a *curve of errors* as follows :—

A straight line is taken (see Fig. 98), and marked with equi-spaced divisions to represent the scale divisions of the instrument. At the several divisions perpendicular lines are set up to represent to some scale *the difference* between the scale reading and the true value of the current or volts which produces that scale reading. This perpendicular is drawn *above* the horizontal line if the scale error is positive, that is if the instrument reads too high at that point of the scale, and *below*, if the scale error is negative. A curved line is then drawn joining

* For details of these processes the student is referred to the author's 'Electrical Laboratory Notes and Forms.'

all the tops of these perpendiculars. This curve shows the nature of the *error* at any point in the scale. Thus suppose that the instrument is an ammeter and indicates 20 amperes when the real current going through it is

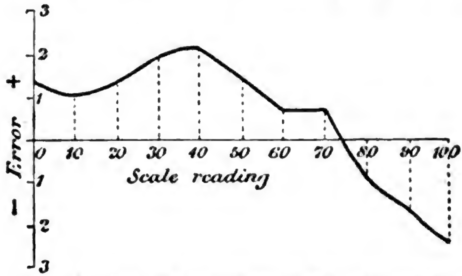

Fig. 98.—Error Curve of Direct-Reading Voltmeter.

18·5 amperes, the error is a *plus* or positive error, viz. +1·5 amperes, and this amount has to be subtracted from the scale reading to obtain the true value of the current. Every direct reading ammeter and voltmeter should in this way be checked by the potentiometer.

§ 8. **Alternating Current Instruments.**—All the instruments so far described in this chapter can only be employed for direct or continuous current measurement. Instruments of the electrodynamic type can be used for alternating as well as for continuous current work.

One of the most useful of these is the Siemens electro-dynamometer. In this instrument there is a fixed coil of wire (see Fig. 99) attached to a support, and another larger coil embraces this fixed coil without touching it. The outer coil is suspended by silk fibres, and is free to move. The plane of the outer coil in its normal position is at right angles to the plane of the fixed coil. The outer or movable coil is terminated in two wires, which dip into fixed mercury cups, and by means of which the current passes into and out of the movable coil. The circuits of the fixed and movable

coils are joined up in series with each other. When a current is sent through the coils a *couple*, or torque, is brought into existence, tending to twist round the movable coil so as to place its axis more or less in line with the axis of the fixed coil. The mechanical couple thus created is proportional to the square of the strength of the current through the coils. Hence, if the current is doubled in strength the couple is made *four* times, and if the current is trebled in strength the torque becomes *nine* times as great. A controlling torsion spring is provided with an indicating needle moving over a divided scale, and when the head of the spring is rotated, the twist given to the upper end of the spring, as read on the divided scale, can be made to counteract the action of the electrical forces and maintain the movable coil, to which the lower end of the spring is attached, in its zero or normal position. The angular twist given to the upper end of the spiral spring is a measure of the mechanical torque due to the spring, and this may be made to equilibrate or balance the torque due to the electrical forces. Hence the angular twist which must be given to the head of the spring to maintain the movable coil in its zero position is proportional to the square of the strength of the current. Accordingly, if in two separate

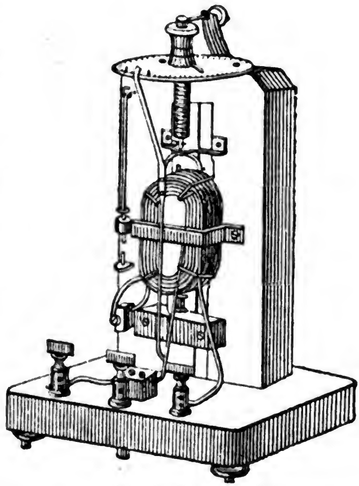

Fig. 99.—Siemens' Electrodynamometer for measuring Alternate Currents.

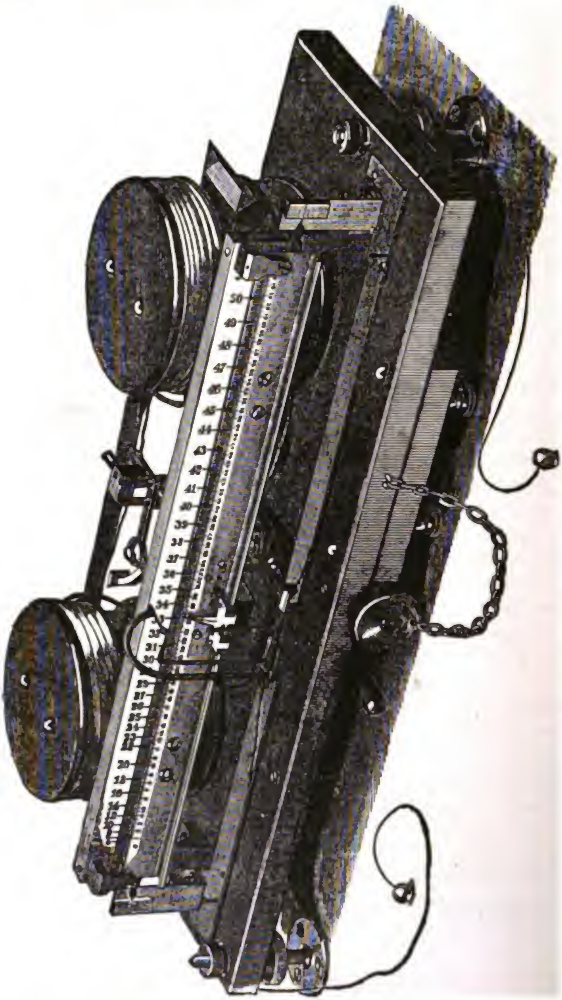

Fig. 100.—Lord Kelvin's Standard Centi-ampere Balance.

cases we find that twists of 25° and 49° are required to restore the movable coil to the normal position with regard to the fixed coil, we know that the square roots of these numbers, viz. 5 and 7, are proportional to the current strengths making those deflections. The *constant* of the electrodynamometer is the number by which the square root of the restoring twist of the spring head must be multiplied in order to obtain the ampere-value of the current through the coils. This constant can be obtained by calibrating the electrodynamometer by means of a potentiometer. The value of the constant is always furnished by the maker, together with a table of square roots.

Lord Kelvin has devised a complete series of instruments for measuring electric currents, called *Ampere Balances*, which are really electrodynamometers. In order to avoid the employment of mercury cups, he invented a very ingenious method of suspending a coil so that it was exceedingly free to move through a small range, and yet maintained in conducting connection with fixed terminals. The general construction of a Kelvin balance is seen in Fig. 100. In these instruments an arm, like the arm of a balance, carries at each end a circular coil of wire, with its plane horizontal. This arm is suspended from two trunnions, placed in the centre, by *ligaments* consisting of many parallel strands of fine copper wire. These strands are soldered to the trunnions of the arm and to fixed supports. The current enters and leaves the coils by these ligaments. The movable coils are placed between fixed coils, two fixed coils being situated at each end of the balance, and the six coils are so joined up in series (see Fig. 101) with each other that the movable coil at each end of the arm is repelled by one fixed coil and attracted by the other when the currents flow through them, in consequence of the fact that conductors parallel to each other repel or attract according as the electric currents in them are in the opposite or in the same direction. The forces are so

arranged that they tend to tip over the balance arm in one and the same direction. The balance is then restored by sliding a weight along a tray attached to the balance arm, and from the position of this weight, as read off on a scale fixed to the tray, the ampere-value of the current can be obtained. A whole series of these instruments have, been devised by Lord Kelvin, called kilo-ampere balances, deka-ampere balances, deci-ampere balances, &c., according to the range of current values for which each instrument is suitable. These instruments, like the Siemens electrodynamometer, are adapted for the measurement of both alternating and continuous

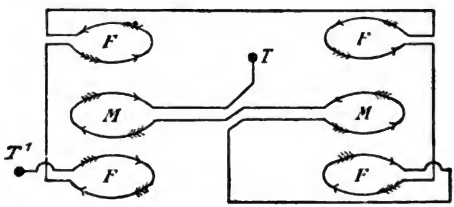

Fig. 101.—Connections of Kelvin Ampere-Balance.

currents. The reason for this is as follows. If two conductors are placed in contiguity to one another, and if the same current is sent through these conductors, a mechanical force is created which tends to attract the conductors together or to repel them apart. The mechanical force requisite to hold the conductors in their initial position is proportional to the square of the strength of the current. Hence, if the current is an alternating one, the mean mechanical force required to maintain the conductors in position is proportional to the *mean square value* of the current, and hence the square root of the restoring force is proportional to the root-mean-square value of the varying current. This last, however, is the value which we wish to determine. The only condition necessary to secure success is that the periodic time of

the current must be small compared with the time of free vibration of the movable coil. The readings of the dynamometer are then independent of the frequency of the current, and only determined by the R M.S. value of the current.

Alternating and continuous currents are said to have the same numerical value in amperes if they equally affect an electrodynamometer or Kelvin Balance. In the same way currents thus equal in dynamometer value will produce heat at an equal rate in the same wire.

When an electric current flows through a wire it heats it, and the rate of production of heat is at any instant proportional to the square of the current strength. Hence, if the current strength is changing from instant to instant, the total quantity of heat produced in the wire in any given time is proportional to the mean of the square of the current strength. Under fixed conditions, the temperature which the wire will assume will depend upon this mean square value of the current; for the wire attains its final temperature when there is a balance between the rate at which heat is generated in it and the rate at which heat is lost by it. The wire loses heat in three ways—bv convection, by radiation, and by conduction. If a wire is enclosed in a tube in such a way that convection is nearly prevented, then the wire attains a final state of temperature, when there is a balance between the rate at which the wire loses heat by radiation, and gains it by internal generation. Under these conditions, the final temperature, and therefore the length of the wire, are determined, not by the true average, but by the mean square value of the current strength. If, therefore, the current is an alternating current, the final temperature, and therefore the length of the wire, enables us to measure this mean square value of the current.

We have seen that a continuous current of one ampere has been defined as an unvarying current of electricity, which when passed through a solution of a silver salt, de-

posits ·001118 gramme of silver per second, or 4·0248 grammes per hour. This may be taken as the practical definition of what is meant by an electrical current having a strength of one ampere when that current is an unvarying current. An alternating current cannot, however, be estimated by this electrochemical method, but it is defined as follows:—An alternating current of one ampere is understood to be a periodic current, which, when passed through a conductor, brings that conductor to the same final steady temperature as would an unvarying current of one ampere if passed through it under the same conditions. It is therefore an alternating current whose root-mean-square value is unity, assuming the instantaneous values to be measured in fractions or multiples of an ampere.

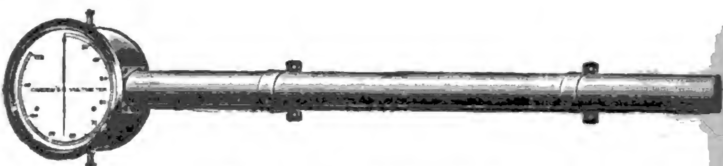

Fig. 102.—The Cardew Voltmeter.

We can accordingly make use of the above fact to construct instruments which shall measure the R.M.S. value of a periodic current, and which depend upon the thermal action of the current.

The well-known Cardew voltmeter is an instrument of this kind.

In this instrument a platinum-silver wire (see Fig. 102), of about 300 ohms resistance, is stretched in a tube, and, for the sake of compactness, the wire is folded backwards and forwards four times over small ivory pulleys. One end of this wire is fixed, and when a current, alternating or continuous, is sent through the wire it becomes heated, and it attains a final constant

temperature if the current passing through it is constant or regularly periodic. The wire therefore elongates, and the expansion of the wire is measured and detected by a multiplying gear of the following kind. The elongation of the wire is made to cause revolution of a mechanism consisting of an inter-gear of wheels and pinions, and to the last axis of the series an indicating needle is attached, moving over a divided dial. The wire has to be held in a tube or frame, and there are two types of this instrument, called respectively the *rod* and the *tube* type. In the rod type of instrument, which is the easiest to manufacture, the platinum-silver wire is kept extended by being fastened to two rods, each formed one third of iron and two-thirds of brass. The whole instrument is then enclosed in a brass case. When a current is passed through the wire it heats it, and the rods become heated also by radiation from the wire. It takes a certain time before the rods settle down into a final state of temperature, in which the heat received by them is equal to the heat radiated by them. Until this is the case, the instrument does not come to its final reading. In the other type of instrument, called the tube instrument, the wire is attached simply to a similarly compound brass and iron tube, which forms the case of the instrument. In this latter type the outside tube arrives very much more quickly at its final state of temperature, and hence the instruments of the tube type are preferable for accurate work on account of the fact that they thus come much more quickly to their final readings when put upon the circuits.

In either case the final state is reached when the wire has attained a constant temperature. When this is the case the recording mechanism measures the difference between the total expansion of the wire, and that of the rods or tube. This expansion is a measure of the excess of temperature of the wire, due to the current, and, therefore, of the rate of generation of heat in it. Hence it is also a measure of the root-mean-square value of the

current flowing through the wire, and therefore also of the R.M.S. value of the potential difference of the ends of the wire. The instrument may therefore be graduated as a voltmeter or as an ammeter.

A disadvantage which the rod instrument possesses is that there is generally a considerable negative variation of the needle, on taking off the current. The rods do not cool as quickly as the wire, and therefore when the current is taken off, the needle goes back beyond the zero of the scale. The instruments are generally made for reading voltages from 40 to 150 volts. In the manufacture of the instruments the wire has first to be carefully aged by putting current on and off for some time at intervals of one minute, so as to heat and cool the wire. In this way irregular variation in expansion is abolished, and the platinum-silver wire is brought into a condition in which it always is the same length at the same temperature. These Cardew voltmeters are really of course alternating current ammeters, and take a current of about one-third of an ampere at 100 volts. The instrument of this range therefore dissipates a power of 30 watts, and, as we shall point out later on, has the disadvantage of wasting a certain amount of energy if kept continuously upon the circuit, but, in spite of this fact, when carefully made the Cardew voltmeter is an instrument of great value for measuring alternating current pressure. A special form of this Cardew voltmeter is made by the Edison-Swan Company for engine-room purposes, the dial of which is very large, and which is graduated say from 80 to 110 volts. In this way such a graduation is given to the instrument that a variation of one volt can be easily seen at a considerable distance.

In the employment of thermal voltmeters it is necessary to avoid convection currents or movements of air in the enclosing tube or case. In using a Cardew volt meter it is generally found best to place the instrument with the tube in a horizontal position. The variable cooling effects of the air currents in the tube are then, to

some extent, prevented. If a Cardew voltmeter is used with its tube vertical, the needle will be observed to make small movements to and fro, even if the current is perfectly steady. These movements of the needle may, if they occur, prevent the pressure from being accurately read within less than one volt.

We must next pass on to consider instruments which depend on electromagnetic action, and in order to understand the principles on which such instruments act, we must first explain one or two elementary facts with regard to the behaviour of iron in a magnetic field. A piece of soft iron, when placed in a varying magnetic field, tends to move from weak to strong regions of magnetic flux density, and other things being equal, the mechanical force so displacing the iron at any point is proportional to the product of the strength of the field, and the rate of change of the field at that point. This principle has been employed by many inventors in the design of ammeters and voltmeters.

In these instruments a small plate of soft iron is attached to a lever pivoted eccentrically in between two flat coils of insulated wire in such a manner, that if the iron is displaced from the centre to a point near the interior surface of the bobbin, that movement is indicated by a needle. Hence, when any current is passed through the coil, a moving force is brought to bear on the iron, tending to displace it from the centre to the edge of the coil. The reason for this movement is that the field of a shallow circular coil is weakest at the centre or axis of the coil. A weight on the lever is so arranged as to resist this movement, and the instrument may be calibrated for different current strengths, and constitutes what is called a gravity instrument, because no springs are used in its construction.

Fig. 103 shows the appearance of a Thomson ammeter for alternating currents made on the above principle.

Another very similar instrument has been devised by

Dolivo-Dobrowolsky. In this instrument, which is intended for the measurement of alternating currents, there is a bobbin of wire, and in the aperture of this bobbin is suspended a very slender fragment of iron wire. When an alternating current is passed through the coil, the iron is drawn down into the coil, owing to the tendency it has to move from weak to strong places in the field. This movement of the iron is resisted by the weight of a small mass placed on the lever which carries the iron. A needle, attached to the axis which

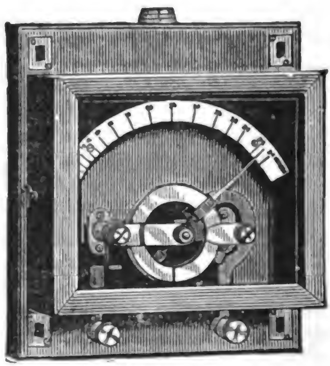

Fig. 103.—Thomson Alternating Current Ammeter.

carries the fragment of iron wire, moves over a graduated scale, and the instrument can be calibrated as an alternating current ammeter.

Another instrument largely in use, which resembles in general construction the ammeter of Elihu Thomson, has in it a flat fixed coil through which the current to be measured passes; and on an eccentric axis, passing through this bobbin, there is fixed a small plate of iron. The field of the coil is made weaker in one part than in others by placing a fixed plate of iron in the opening

of the bobbin. When a current is passed through the coil, the movable plate of iron is repelled from the fixed plate, and in so doing turns round the axis to which it is attached. An indicating needle fastened to the axis moves over a divided scale.

A fourth instrument depending on the electromagnetic principle is that of Evershed. In Evershed's alternating current ammeter there is a fixed coil of wire through which an axis passes. This axis carries a small piece of soft iron fixed on a shaft like the head of a hammer. and an indicating needle is also fixed to the axis. Within the coil there are two soft iron cheeks, and between these is formed a strong magnetic field when a current passes through the coils. On passing the current the movable piece of iron is drawn down between the two fixed cheeks, and this movement is resisted by a weight carried on the axis. The piece of soft iron therefore takes a definite position under any given current which depends on the relative forces acting upon the piece of soft iron. In these electromagnetic instruments it is necessary that the calibration should be made for the particular frequency at which they are to be used. We must not take it for granted in using an instrument of this class that its readings for the same R.M.S. value of the current will be identical for alternating currents of different frequencies, as the inductance of the coil will in general tend to reduce the readings as the frequency increases. In the Evershed ammeter, a compensation is provided to meet this difficulty, and to make the instruments give identical indications, although the frequency of the alternating current may be varied. The working coil is shunted by an inductive shunt ; this shunt takes about 6 per cent. of the total current in the case of continuous currents, but only about 2 per cent. or less, in the case of alternating currents. The result is that the working coil takes more current with alternating currents as the frequency increases, and a compensation is thus provided for the inductance of the coil.

All the instruments so far described, the Siemens dynamometer, the alternating current ammeters made by Nalder, Evershed, Thomson, and Von Dolivo-Dobrowolsky, can be converted into alternating current voltmeters by winding them with very high resistance wires, or putting part of that resistance in the form of a non-inductive resistance associated with the measuring part. In Evershed's voltmeter there is a compensation for frequency which is worth noting. The voltmeter coil has placed in series with it a coil, the terminals of which are shunted by a condenser. This shunted condenser has the property of neutralising the inductance of the voltmeter coil, and if properly adjusted, the instrument may be made to give identical scale indications for alternating pressures of widely different frequencies, and be practically compensated for frequency.

We must pass on to notice instruments for the measurement of alternating electromotive forces which depend upon electrostatic attractions. These electrostatic instruments have for many purposes great advantages, the most notable of which is that they do not consume power, and that therefore they may be left on the circuits indefinitely without expense. Lord Kelvin's electrostatic voltmeter for high pressures is a well-known instrument of this kind. It consists (see Fig. 104) of four quadrant-shaped plates which are connected to one terminal of the instrument. Suspended between these plates, but insulated from them, is a paddle-shaped aluminium needle, which swings on very delicate pivots. The needle is connected to the other terminal of the instrument. The instrument, in fact, forms a condenser, of which one plate is fixed and the other is movable. When a difference of potential, varying from 1000 to 5000 volts, is produced between the terminals of the instrument, the movable plate is attracted in between the fixed plates. This movement is resisted by weights, which are hung on the bottom of the needle.

When two plates, in fixed positions, have produced

between them a difference of potential, the force required to hold them in any given position is proportional to the square of the difference of potential between them. To the end of the aluminium needle is attached

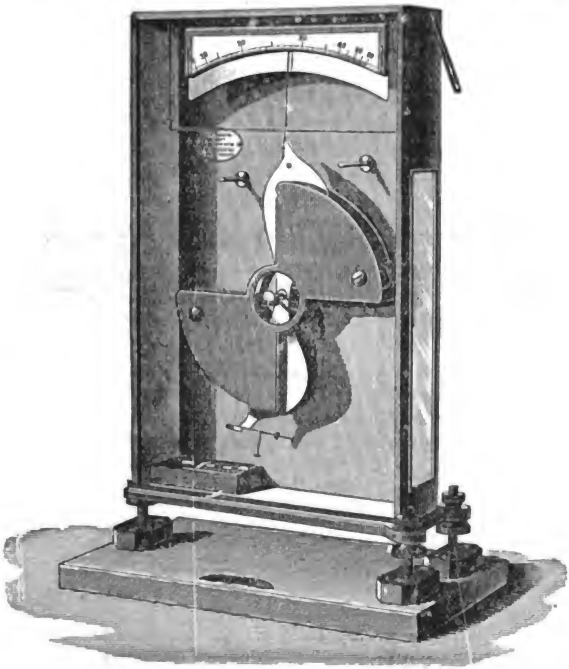

Fig. 104.—Lord Kelvin's Vertical Electrostatic Voltmeter.

a long pointer, moving over a divided scale, and the instrument is graduated in such a manner as to read directly in volts. Since the attraction between the plates depends upon the square of the potential difference, it

is independent of its direction, and therefore the instrument works equally well with direct or alternating pressures ; and, in this latter case, it gives us the root-mean-square value of the potential difference between the terminals of the instrument.

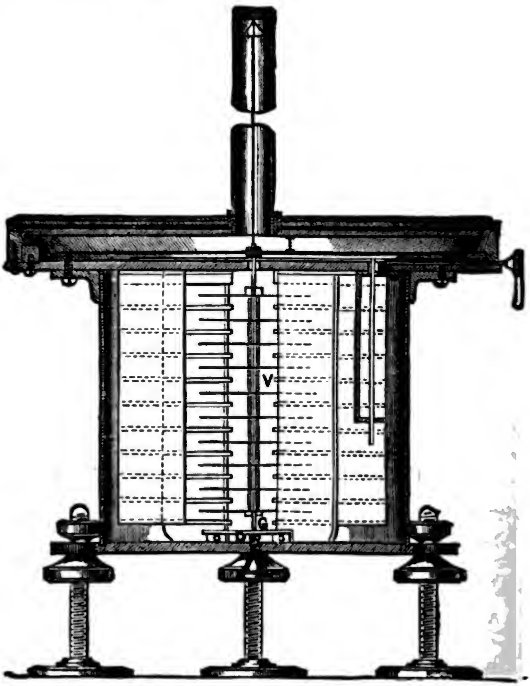

Fig. 105.—Lord Kelvin's Multicellular Electrostatic Voltmeter.

Another instrument, also invented by Lord Kelvin, but adapted for measuring lower pressures, is the multicellular electrostatic voltmeter. In this instrument there

are a series of quadrant-shaped plates (see Fig. 105), placed one above the other, which are called cells. There are then a series of paddle-shaped needles, all attached to a common axis, which is hung up by a platinum silver wire. The normal position of the needles is just outside the quadrants, but if a difference of potential is created between the needles and the cells, the needle is drawn or attracted into the cells. This movement is resisted by the torsion of the suspending wire. An indicating needle, attached to the axis, moves over a divided scale; and the instrument, which can be arranged to measure from 40 volts upwards, gives us, therefore, the root-mean-square value of the potential difference between the cells and the needle.

In these instruments there may be a very small error in the reading, which is dependent upon the existence of a small electromotive force of contact between different metals. If the cells are made of brass and the needle of aluminium, there is a small contact difference of potential which is due to these different metals, and which may amount to something less than half a volt. Accordingly, it will be found that in such an instrument reading, say 100 volts, the reading given by the instrument will depend, to a slight extent, upon whether the cells are positive or negative, and a small correction has to be applied depending on the manner in which the instrument has been joined up in calibrating it.

Lord Kelvin has also devised a form of multicellular voltmeter which is useful for many purposes, as it has a vertical scale, and the instrument (see Fig. 106) is intended to be attached to a switch-board. The advantage of these electrostatic instruments in that they take up no power, is considerable in electrical engineering practice. Take for instance a thermal voltmeter absorbing 30 watts, and assume that this instrument is kept connected to the circuits in the dynamo room of a central station all the year round. Since there are, roughly, 8000 hours in a year, this instrument, absorbing

30 watts, would in one year dissipate 240 Board of Trade units of electric energy. If we reckon this energy as costing 1*d.* per unit, it is evident that this instrument will cost 240*d.*, or 1*l.* per annum, to keep it going. In reading alternate current pressures higher than 100 volts, a transformer has to be interposed between the circuits and the voltmeter, to reduce the pressure. This transformer will also use up energy, and if it takes no more than the voltmeter it will also waste 1*l.*'s worth of electric energy in the course of a year. We see, therefore, that an electrostatic instrument which wastes no energy at all has, in cost of upkeep, a great advantage over the electrothermal or the electromagnetic type of instrument, and in fact we can afford to spend a larger sum of money on an electrostatic instrument, and yet effect a total saving in the cost of the electric measurements in that station. For suppose we capitalise our annual energy loss at 10 per cent., we see that an instrument of the electrostatic type, compared with one of the electrothermal or the electromagnetic type, effects a very considerable economy.

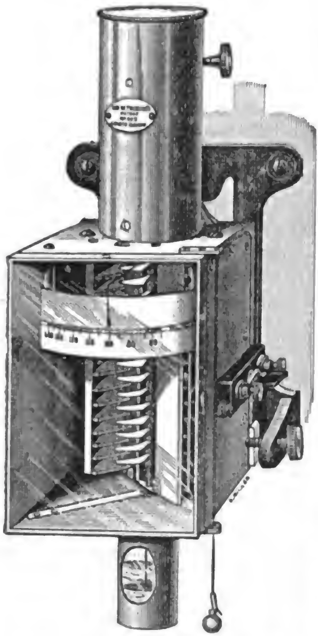

Fig. 106.—Lord Kelvin's Vertical Electrostatic Voltmeter.

We may employ these electrostatic voltmeters for the measurement of alternating currents, and convert them into alternating current ammeters in the following way. Let a non-inductive resistance, say of platinoid wire, be constructed, which is capable, without sensibly heating, of passing the current to be measured, and let the resistance of this wire be accurately known. Then to the ends of this circuit attach an electrostatic voltmeter. Knowing the difference of pressure between the ends of the wire and its resistance, we know the current flowing through it. In many cases, as in measuring the primary current of a transformer at no load, it is much more convenient to employ such an electrostatic voltmeter and non-inductive resistance than any other method. A suitable non-inductive resistance can always be made with coils of platinoid wire, joined in parallel; and these must be so adjusted that the final temperature they attain is not more than a few degrees above the normal temperature.

§ 9. **Measurement of Electrical Power.**—When any electrical device or apparatus is being supplied with electric current at a certain pressure it is taking up electrical power. Thus, for instance, if a current of 0·5 ampere is passing through an incandescent lamp, and if the terminal voltage or potential difference at the terminals of the lamp is 100 volts, then the lamp is taking up electrical power represented by $\cdot 5 \times 100 = 50$ watts. In all cases where the current is supplied as a continuous current, whether the device be a lamp, motor, electrolytic cell or anything else, the electrical power taken up reckoned in watts is obtained by multiplying together the ampere value of the current and the number representing the terminal volts, or volts between the terminals of the apparatus.

When a current is flowing through a conductor it dissipates energy, and the rate at which this energy is dissipated is spoken of as *the power* taken up in that circuit. When a constant current is flowing through a

conductor, if we measure the current in amperes and the difference of potential between the ends of the circuit in volts, the product of these two numbers gives us the power taken up in the circuit measured in watts. In this case two simple measurements give the required rate of dissipation of energy in the conductor. If, however, we have to deal with an alternating current circuit, in which the current strength is varying from instant to instant, according to a periodic law, and if likewise, the difference of potential between the ends of the circuit is varying in the same periodic manner, we cannot always obtain the measurement of the mean power taken up in the circuit, by taking the product of the root-mean-square value of the amperes and root-mean-square value of the volts. What we really require in this case is the *mean value of the power* taken up in the circuit. We can obtain the measurement of the mean power if we can measure at every instant the true value of the current strength and the difference of potential. Suppose these instantaneous values of the current and pressure known at equidistant intervals taken throughout one complete period. If we then multiply the instantaneous value of the current by the corresponding value of the pressure, or difference of potential, we obtain a number representing the instantaneous value of the power, and if we imagine the period divided into a large number of equidistant intervals of time, and those products taken at every such instant, then the mean value of these products taken throughout the period will give us a close approximation to the mean value of the power being absorbed by that circuit.

We have seen in a previous chapter that it is possible to determine and describe curves representing the instantaneous values of the current and electromotive force in the case of an alternating current circuit, but this in general is not a simple matter to do, and we have therefore to resort to other methods of obtaining the required power measurement.

In dealing with the power taken up in alternating current circuits, there are two cases to be considered.

The first case is that in which the circuit is non-inductive. In that case, as before explained, the impedance of the circuit is the same as its resistance, numerically speaking. For such circuits the alternating current flowing in the circuit is in step, as regards phase, with the alternating potential difference between its extremities. When this is the case the power taken up in that circuit can very easily be measured. If we measure the root-mean-square value of the alternating current by means of any of the balances or dynamometers already described, and if by means of any of the electrostatic or thermal voltmeters we measure the root-mean-square difference of potential between the ends of the circuit, and multiply these two mean-square values together, we obtain the mean value of the power taken up in the circuit, and we arrive at the same result as if we had been able to measure separately the instantaneous values of the current and potential difference at numerous equidistant intervals throughout the phase, and taken the mean value of their products.

As an instance of this, it may be pointed out that an incandescent lamp may be treated as a practical non-inductive circuit. If an incandescent lamp is traversed by an alternating current, and we measure the current flowing through the lamp by means of, say, a Siemens dynamometer, and the potential difference between the terminals of the lamp by means of an electrostatic voltmeter, and if we multiply the two scale readings of these instruments together, we obtain the mean value of the power measured in watts taken up in the lamp.

So far then all is quite simple, and in dealing with any circuit which we know, or can prove to be, practically non-inductive, we have no difficulty, by means of two instruments of the proper kind, in determining the real mean power taken up in the circuit. Our difficulties come in when we have to deal with circuits such as

transformers, which, when not fully loaded, we know to be inductive. If, in this case, we can determine numerous instantaneous equidistant values of the current, and difference of potential between the ends of the circuit, then proceeding as above described we can graphically find the mean value of the power taken up in the circuit. If, however, it is not convenient to do this, we cannot

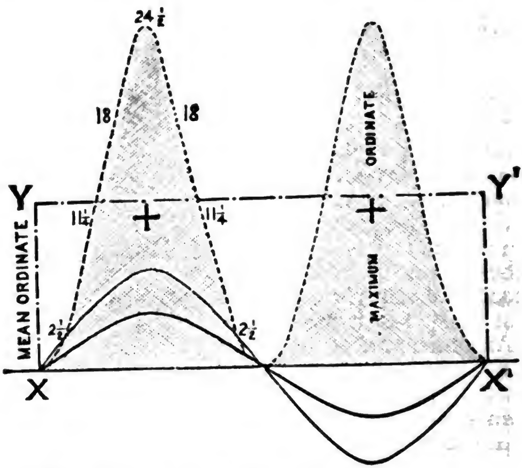

Fig. 107.—Power Curves for a Non-inductive Circuit.

proceed to measure the root-mean-square values of the current and electromotive force, and then multiply them together. Such a proceeding would lead to a considerable over-estimate of the real power taken up in the circuit. Without going into elaborate proof of this, it may be simply sufficient to present the following figures.

In Fig. 107 are shown two simple harmonic curves

in step with one another. Let the semi-period of each curve be divided into eight equidistant parts, and ordinates be erected at each point. The values of these ordinates for the two curves represent equidistant instantaneous values of these periodic current and electromotive force curves. By squaring each of the values of the ordinates, and taking the square root of the mean of the squares, we obtain for each curve a number which would represent the instrumental value obtained by an alternating current dynamometer or voltmeter. If we multiply together the simultaneous values of current and electromotive force, we obtain a number, given by the dotted curve, which represents the instantaneous value of the *power* taken up in the circuit, and if we take the mean value of all these separate instantaneous values of the power, we obtain the same number as we do if we take the *products of the square roots of the mean of the squares of the instantaneous values of the current and electromotive force.* Hence, we see that when the two simple harmonic curves are in step with one another, the product of the square roots of the mean of the squares of the separate ordinates is equal to the mean value of the products of the corresponding ordinates.

In Fig. 108 are shown two periodic curves, which may be taken to represent a current and an electromotive force curve, but one of which is displaced backwards relatively to the other. This is what happens in an inductive circuit, where the periodic current always lags in phase behind the periodic electromotive force.

If we perform the same operations on the ordinates of these curves, we find that the product of the root-mean-square values for the two separate curves is in excess of the mean of the product of the instantaneous equidistant values for the two curves. In other words, if in such a circuit we measure the current by means of a dynamometer, and the potential difference between the ends by means of an alternating current voltmeter, the product of these two numbers gives us a number which is in

excess of the true value of the mean power taken up in the circuit.

We can at once measure the power-absorption of an energy transforming apparatus, when using continuous currents, by the employment of an ordinary direct cur-

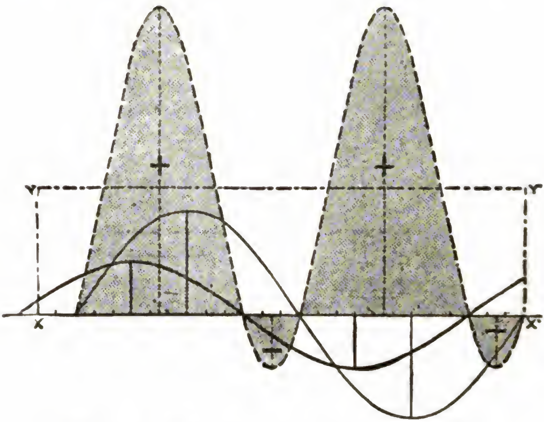

Fig. 108.—Power Curves for an Inductive Circuit.

rent ammeter to measure the current going into it, and a voltmeter to measure the terminal volts, the instrument being arranged as shown in Fig. 109.

In this use of the instruments, the voltmeter must either be an electrostatic voltmeter or else of very high resistance if a current-taking instrument. Otherwise, if a rather low resistance voltmeter is used, the current which it takes up is reckoned into the reading of the ammeter; and a correction must be made for it before we can obtain the true value of the current taken by the device being tested.

The potentiometer can, of course, be used to make

both measurements of volts and currents. In the testing of incandescent electric lamps, where we wish to know the power taken up by the lamp, it is usual to employ either a potentiometer or a good direct-reading ammeter and voltmeter.

The absorption of power being measured in watts, we can immediately obtain the *horse-power* absorbed by dividing the *watt-power* by 746. Thus, if a house takes a total current of 50 amperes at 100 volts to electrically light it, the *power* taken up is $50 \times 100 = 5000$ watts; the horse-power being used is $\frac{5000}{746} = 6{\cdot}7$ H.P.

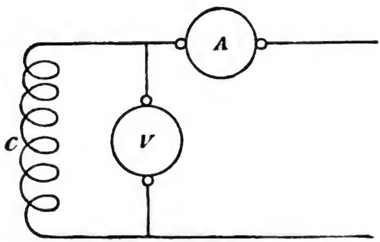

Fig. 109.—Measurement of Continuous Current Power. A, ammeter; V, voltmeter; C, coil in which electrical power is being taken up.

If, however, the current is supplied in the form of an alternating current, we have just shown that we cannot obtain the value of the power absorbed by separate measurements with an ammeter and a voltmeter, *unless the circuit to which the power is being supplied is non-inductive.* It has already been explained that when an alternating current flows through an ammeter suitable for measuring it, the instrument tells us the root-mean-square value of the current. In the same way an alternating voltmeter reads for us the R.M.S. value of the voltage.

Suppose, then, that an alternating current is being

supplied to a transformer, and that we desire to know the power or mean power being taken up in it.

The current is not continuous but is varying through a cycle of values, and, as we have seen, the alternating potential difference at the terminals of the transformer is not in general *in step* with the alternating current. The current lags behind the voltage in phase. At any instant, however, the power being given to the transformer is measured by the product of the instantaneous value of the voltage and current; and this product gives us what may be called the instantaneous value of the wattage or power being taken up in watts. The power absorbed is therefore not constant, but fluctuating. We can, however, discover the *true mean power* being taken up. The value of this true mean power cannot be obtained merely by multiplying the R.M.S. value of the current by the R.M.S. value of the voltage, *unless* the circuit tested is a non-inductive circuit, and therefore the current in step with the impressed voltage. If the current is not in step with the voltage, then the true mean power being taken up is obtained by multiplying together the R.M.S. value of the current, the R.M.S. value of the voltage, and a number called the *power factor* of the circuit.

If the current and voltage curves are sine curves, then the mean power can be obtained by multiplying together the R.M.S. value of the current and volts, and the *cosine of the angle of lag* of the current behind the potential difference or voltage.*

We can, however, employ an electrodynamometer such as that of Siemens (already described), in such a way as to enable us to measure the mean value of the power being given by an alternating current to any circuit, inductive or not, and whatever may be the form of the current wave.

* The proof of these propositions cannot be given without a mathematical discussion. The student who is desirous of further information on these points is referred to the author's treatise on 'The Alternate Current Transformer in Theory and Practice,' vol. i.

In this use of the instrument it is called a *Wattmeter*, and is arranged as follows:—

Let the fixed coil of the electrodynamometer, similar to that shown in Fig. 99, be placed in series with the circuit in which we desire to measure the power being taken up. Let the movable circuit of the dynamometer consist only of a few turns of wire, and let this movable circuit have joined in series with it a non-inductive resistance, which may be formed either of coils of wire or incandescent lamps. Let this movable circuit with its added resistance be placed as a shunt across the ends of the circuit in which it is required to measure the power, being joined up as shown in Fig. 110. Then, when the alternating electromotive force is applied to the circuit, the fixed coil of the dynamometer (now called a wattmeter) will be traversed by a periodic current identical with that passing through the inductive resistance. The movable coil of the wattmeter will be traversed by a current which will be in step as regards phase with the potential difference between the ends of the inductive circuit. When the dynamometer thus has its two circuits traversed by two currents, the force required to hold the movable circuit in its normal position against the electrodynamic forces is at any instant proportional to the product of these currents. If, then, the currents vary from instant to instant, and if the time of vibration of the movable coil is very long compared with the

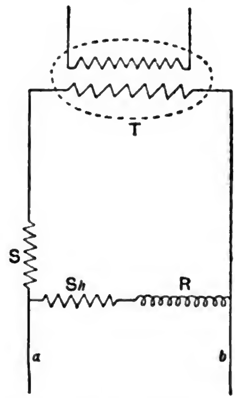

Fig. 110.—Diagram of Connections for Measuring Alternating Current Power with the Wattmeter. S, fixed or series coil of Wattmeter; Sh, shunt or movable coil of Wattmeter; R, added non-inductive resistance; T, circuit in which power is being measured.

periodic time of the current, the mean value of the torque required to hold the movable coil in its normal position —with its axis at right angles to that of the fixed coil—will be proportional to the mean value of the products of the currents in the fixed and movable coils respectively; that is to say, will be proportional to the mean power being taken up in the inductive circuit. The torque required to hold the movable coil in its normal position may be furnished by the torsion of a spring, and hence we can with such an instrument read off the mean power being taken up by the inductive circuit, provided that the wattmeter is already standardised. The best way to standardise the wattmeter is to apply the wattmeter to measure the power taken up in a known standard non-inductive circuit, and at the same time to measure —with an alternating current ammeter and voltmeter—the root-mean-square value of the current flowing through this circuit and root-mean-square value of the potential difference between its ends. In this way we apply the wattmeter to measure the known power being taken up in a non-inductive circuit (obtained by taking the product of the readings of the aforesaid instruments), and we then obtain the *constant* of the instrument. The constant of the instrument is the number by which we must multiply the necessary twist of the head of the torsion spring of the wattmeter reckoned in scale divisions, which gives the required torque, to obtain the mean power in watts passing through the instrument.

The conditions of success in the use of the wattmeter are as follows:—

1st. The current through the series coil of the instrument must have the same value as the current through the circuit to be measured, and the current through the shunt coil of the wattmeter must be exactly in step with the difference of potential between the ends of that shunt circuit; in other words, the shunt circuit must be strictly non-inductive. This can only be secured by winding the movable coil of the wattmeter with no very large number of turns.

It is convenient to call the product of the root-mean-square value of the amperes and the root-mean-square value of the volts the *apparent watts* taken up by the circuit, and to call the true mean value of the power as read by the wattmeter the *true watts* taken up by the circuit.

The ratio between the *true watts* and the *apparent watts* is called the *power-factor* of the circuit. Thus, for instance, in the case of a transformer on open circuit—the transformer being of the closed magnetic circuit type—the power-factor is about 0·7; in other words, the real power is only $\frac{7}{10}$ of the apparent power. In the case of a transformer of the open magnetic circuit type, the power-factor may be as small as 0·1. Hence we see that a great error might be committed by taking the product of the instrumental readings as simply representing the true mean power taken up in the circuit.

The power-factor (P.F.) is related to the apparent power or product of the amperes (A) and volts (V) (R.M.S. values), and to the true power or true watts (W) (mean value) in the manner represented by the formula—

$$\text{P.F.} = \frac{\text{W}}{\text{AV}} = \frac{\text{true watts}}{\text{apparent watts}}.$$

§ 10. **Watt-hour Meters and Ampere-hour Meters**—In the supply of electric energy for commercial purposes it is essential to record the total *energy* given to any circuit in any time, and the total electric *quantity* irrespective of variations in the power or the current. Consider, for instance, the supply of electric current to a house, and for the sake of simplicity let it be a continuous current. Let the current vary from time to time. Suppose a horizontal line taken to represent the 24 hours of the day, and let it be divided into 24 parts, and at each interval a vertical line drawn representing to some scale the current then flowing into the house. The curve drawn to touch the tops of all these ordinates includes

an area which represents the total quantity of electricity in *ampere-hours* which has passed into the house. An instrument which will record automatically this quantity is called an *ampere-hour meter.*

Again, let the vertical lines drawn in a similar diagram represent the *power* in watts delivered at each hour to the house; the area of the curve then delineated represents the total electric energy in *watt-hours* which has passed into the house. Any instrument which will automatically record this energy is called a *watt-hour meter.*

Instruments for the measurement of electric energy or quantity are generally called simply *meters,* and they are classified into ampere-hour meters and watt-hour meters. A more detailed classification of all the different forms of meter already invented would be a rather difficult thing to make on a perfectly correct basis. An approximate classification of meters for the measurement of alternating currents may be made as follows:—

1. Graphic recording ampere-hour meters.
2. Graphic recording watt-hour meters.
3. Continuously recording ampere-hour meters.
4. Intermittent recording watt-hour meters.
5. Continuously recording watt-hour meters.

Of the first two classes the Holden ampere-hour and Mengarini watt-hour meter are good examples. In these instruments an arm carrying a pen is displaced over a paper-covered drum, which is revolved uniformly in 24 hours by a clock. The motion by which the pen is displaced is regulated by a part of the instrument which is simply an ammeter or a wattmeter, and the displacement of the pen is proportional to the current or the power passing through this measuring part. When, therefore, the diagram is cut off and unrolled, we find on the paper a curve which represents, by its ordinates, either the power or the current at any instant; and, if the whole area of the curve is integrated, then such area represents the whole energy or quantity which has passed through the meter in 24 hours. These instruments have the

advantage, therefore, that we practically record two quantities at once; and they serve two purposes, of indicating the instantaneous current or power, and the total current quantity or energy, but they have the disadvantage that they are not self-integrating.

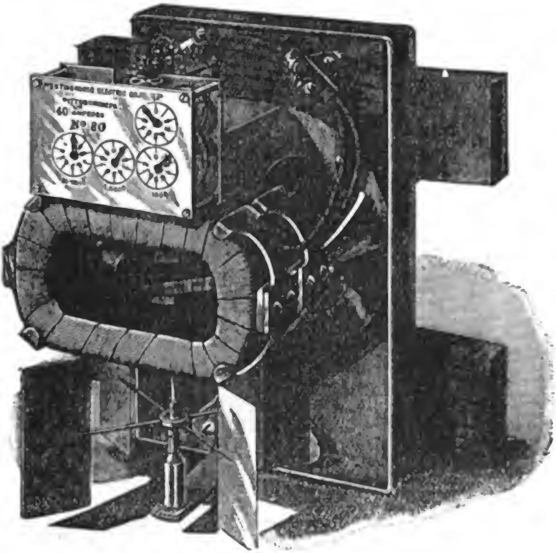

Fig. 111.—Shallerberger Meter.

Of the continuously recording ampere-hour meters there are two well-known types in use, respectively for recording alternating current quantity and continuous current quantity. These were invented by Shallenberger and Ferranti. Taking first the Shallenberger alternating ampere-hour meter, it is constructed as follows.

It consists (see Fig. 111) of a small transformer, one

coil of which we may call the primary, and which is in series with the circuit in which the current to be measured is flowing. The core of this transformer is a little soft iron disc, which is capable of revolving on an axis. This axis is geared at the top with a counting mechanism which records the number of revolutions of the disc, and at the bottom there is a vane or fan of thin aluminium which serves to retard the rotation of the disc.

The secondary circuit of this transformer (see Fig. 112) consists of a small coil of copper, which is

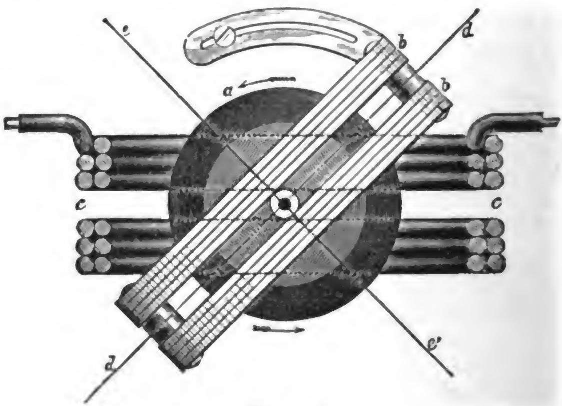

Fig. 112.

closed upon itself, and which is placed with its axis inclined at 45° to the axis of the primary coil. When the primary current flows through the primary coil it does two things—it magnetises the core, and it induces a secondary current in the closed secondary circuit. The phase of this secondary current is about 90° behind the phase of the primary current, and thus the magnetism of the iron core, which is in a direction at right angles to the plane of the primary coils, also lags in phase

behind the primary current by about 90°. The magnetism of the core and the induced secondary current are, therefore, in step, and are in such directions that the axis of the disc is always being pulled round by the induced field of the secondary coil. If, then, there were no friction of any kind, the iron disc would be therefore continually accelerated in speed, but since the air opposes a resistance which varies approximately as the square of the velocity, and since the mean driving force is proportional to the mean square of the current strength, it follows that the total number of revolutions which the disc makes in any given time is proportional to the total mean quantity or ampere-hours which have passed the primary circuit. The meters can therefore be calibrated by a constant in such a way that they read directly on counting dials ampere-hours, and if the pressure between the mains is kept constant, they may be graduated to read in Board of Trade units.

These meters are very simple to construct and very fairly accurate in performance, and they have therefore come into extensive use. The velocity of the disc being at any time proportional to the mean current passing through the meter, we can, if the current is kept tolerably constant, employ the instrument as an ammeter. By moving the position of the secondary coil a little adjustment can be made in the meter for change of frequency, and the meter can be calibrated for the particular frequency for which it is intended to be used.

The Ferranti continuous current ampere-hour meter consists of an electromagnet, the coil of which is of mild steel with a certain retentivity for magnetism. The core is worked at such a low flux density that the magnetisation of the core is always nearly proportional to the strength of the magnetising current. This electromagnet has a disc-shaped cavity in the core (see Fig. 113) lined with an insulating material, and which is filled with mercury. The main current flows through the electromagnet coils, and then entering the mercury at the

periphery of the disc-shaped cavity, which is not insulated, flows radially inwards in all directions to the centre of the mercury, whence it goes to the terminal of the meter. Under these conditions the mass of mercury is set in rotation, and its rotation is retarded by radial grooves which are formed on the sides of the chamber. The force driving the mercury is proportional to the square of the

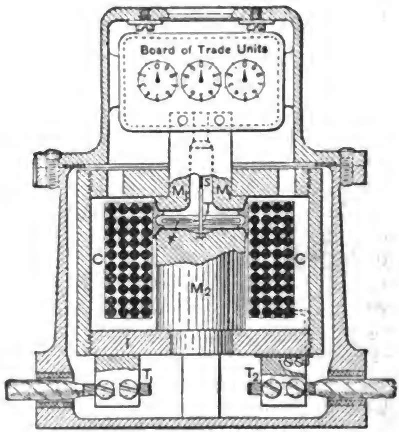

Fig. 113.—Ferranti Ampere-hour House Meter.

strength of the current, and the force retarding the rotation of the mercury to the square of the speed. Hence it follows that the number of rotations in any given time is proportional to the total quantity of electricity which has passed. The rotation of the mercury is communicated to a counting mechanism by means of a little vane F immersed in it. The counting mechanism is so devised that the dials read electric energy in Board of Trade units on the assumption that the voltage of the circuit remains constant. The two meters last described, viz. the

Shallenberger meter for alternating current quantity, and the Ferranti mercury meter for continuous current quantity, are both only ampere-hour meters, and record the total quantity of electricity which has passed through them. If, however, that current is being supplied at a constant voltage, then the readings of the dials may be made to show the volt-ampere-hours or watt-hours which have passed into the circuits through the meter. Since 1000 watt-hours are one Board of Trade unit, those meters are sometimes so arranged as to read on the dials directly Board of Trade units. They are not, however, true watt-hour meters.

Coming next to the continuously recording true watt-hour meters, we reach that class of meter which may be said to be the best adapted for general wants, and one of the most efficient of these continuously recording watt-hour meters is the one invented by Professor Elihu Thomson. The Thomson recording watt-meter (see Fig. 114) is a watt-meter in which one coil C, called a series coil, carries the current to be measured. These series coils really form the field magnet of a small electromotor. The armature A of this motor is one without any iron in it, having a small commutator and brushes of the usual kind, and this armature circuit, together with an external added resistance, constitutes the shunt circuit of the wattmeter. When the meter is attached to a circuit, the main current passes through the series coils, and the shunt coil being attached to the two mains of the circuit is traversed by a current proportional to the voltage of the circuit, and the armature then begins to revolve. The shaft which carries the armature carries also a copper disc D, which is embraced by three horse-shoe magnets M. When the disc revolves, eddy currents are set up in the disc which retard its motion. The number of revolutions of the disc in a given time is recorded by the counting mechanism P attached to the shaft. Then since the force driving the armature round is at any instant proportional to the

power passing through the instrument, and since the retarding force is proportional to the velocity, it follows that the number of revolutions in a given time represents the watt-hours that have passed through the meter. In order to overcome the constant friction of the train,

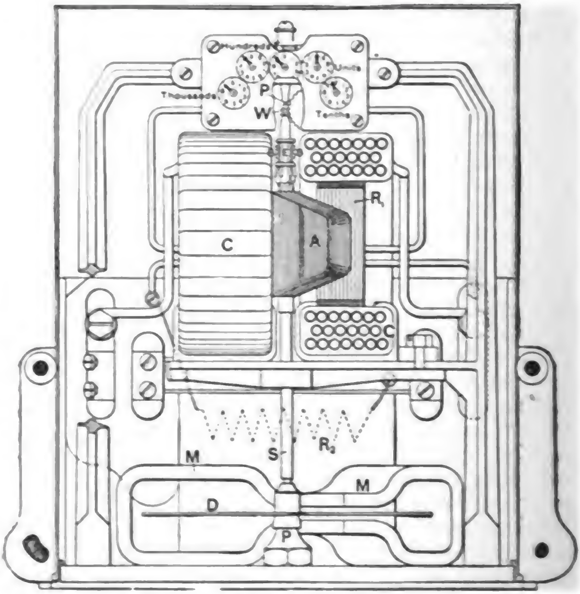

Fig. 114.—Thomson Recording Wattmeter.

there is a compound winding on the field magnet, consisting of a few turns of the shunt coil which is arranged in such a direction that the driving force due to the fixed and movable shunt-winding, tends to overcome the permanent friction of the armature shaft. By pro-

perly treating the permanent magnets, it is found that they retain a constant magnetism for long periods of time. The meter is so arranged that, when working on a 100 volt circuit, the shunt coil has a resistance of 1000 ohms altogether, and takes, therefore, ·1 of an ampere. The loss in the meter, therefore, is only 10 watts, which is the power taken up in driving the meter. By properly arranging the shunt coil, it is possible to make the constant of this meter perfectly constant for a very large range of its action, and one great advantage which this meter has is that it can be employed with both alternating and continuous currents. When properly adjusted, this watt-hour meter is capable of very great accuracy in the measurement of electric energy.

The force driving round the armature of the motor is proportional to the product of the strengths of two currents, one of which is the current going into the circuit which is being measured, and the other of which is proportional to the terminal voltage of the circuit being metered. Hence, the mean driving force is proportional to the mean product of these two quantities, the current and the terminal voltage. Hence it is proportional to the mean power in watts given to the circuit.

The instrument, therefore, measures the energy in watt-hours, which have passed through it quite independently of the frequency, or whether the current is alternating or continuous.

We may ask ourselves at this stage what ought to be the characteristics and requirements in a good commercial meter for the measurement of electric energy supply to houses, for lighting and other purposes. Most persons without experience would probably say that the first requirement in a meter is accuracy; but as a practical matter, accuracy is not of so much importance as that the meter should have a uniform percentage error. A type of meter which is capable of measuring,

under some conditions, to ·1 per cent. of accuracy, and at other times is liable to make errors of 150 per cent. in actual practice, is not nearly so useful as a meter which will not read closer than 1 per cent., and yet which, in actual practice, never proves to go more than 3 per cent. wrong.

The next condition which a meter must comply with is that of the consumption of small power. Since the meter is connected to the circuit continually, if it absorbs power, the total energy dissipated by it may amount to a serious item. We have pointed out, in speaking of voltmeters, that a continuous power absorption, say of 20 watts, during the whole year, amounts to a yearly consumption of 160 units of electric energy, and that, therefore, small power consumption is certainly an important item in appraising a meter.

The third great requisite is, that the meter ought not to require elaborate care and delicacy in fixing, and that it ought to be hardy enough to endure carriage. House meters have to be fixed in places which are sometimes subject to vibration and dampness, &c., and a meter for house purposes ought, therefore, to be hardy enough to stand these conditions.

The fourth condition is, that the meter must be so constructed in principle as not to be capable of being easily tampered with, or its indications made to vary, even when enclosed in a lock-up case.

To the above requirements ought, of course, to be added the broad and general conditions of simplicity of structure and cheapness in price, as far as consistent with good workmanship.

Space will not permit of further reference to the exceedingly numerous forms of electric house meters. For additional information on them, as well as for a fuller discussion of electric measuring instruments generally, special treatises must be consulted.

CHAPTER X.

THE GENERATION OF ELECTRIC CURRENTS.

§ 1. **Electric Current Energy.**—We have already pointed out that when an electric current is created by the application of an electromotive force in a circuit, energy has to be expended in the first place to bring the current into existence, and energy has also to be expended to maintain this current at a constant value, even if the electromotive force remains unchanged.

The initial stage involves *work done* against the inductance of the circuit, and it can be shown that the amount of work so spent in creating a current is measured in joules by the product of the *coefficient of self-induction*, or the inductance of the circuit reckoned in henrys, and half the square of the current strength in amperes. When the current is then created, work has to be spent to maintain it against the resistance of the circuit, and the amount so spent per second is measured by the product of the resistance of the circuit, reckoned in ohms, and the square of the current strength in amperes. There is a close analogy between these energy expenditures, and that involved in setting in motion a heavy body immersed in a fluid. To set and keep such a body in motion with a constant velocity involves the expenditure of energy, in the first place, to get the body up to the required speed, and this energy is measured by the products of the mass of the body, and half the square of its final steady velocity. When once this speed is attained, energy has still further to be expended to keep it in motion against the resistance of the fluid, and the energy so spent per second is nearly

measured by the product of a certain constant (depending on the nature of the body and of the fluid), and the square of the velocity. A current in a conductor, therefore, always represents an expenditure of energy in some form involved in its manufacture, and if it is a constantly maintained current, even though cyclic or periodic in value, it represents a continual expenditure of energy to keep it going. To produce and maintain a current, some other form of energy must, therefore, be continually transformed, and this may be done by the transformation of many forms of energy, but chiefly by the expenditure of mechanical energy, chemical energy or thermal energy.

It is only, however, under certain conditions that this transformation can take place. The condition for transforming mechanical energy or energy of motion into electric current energy is that the energy must be spent in making a conductor forming a closed circuit, move in a magnetic field in a particular way. In the same manner, the conversion of heat energy into electric current energy can only take place at a junction of two dissimilar bodies, called a thermo-electric junction, or in an unequally heated conductor. The making of an electric current involves, therefore, the possession of a certain apparatus or instrument which may be called the *instrument of transformation*, and if energy in one form is continually supplied to this machine, a part, at least, of that supplied energy is converted into its equivalent in electric current energy. The remaining portion of the supplied energy is always frittered away into useless heat, which gets radiated and lost as available energy. This dissipated energy is, at it were, the price we pay for the convenience of the remaining transformation. It is like a money-changer's commission charged for changing a draft or cheque, or foreign money to English.

The progress of electrical invention has, therefore, consisted in the discovery of the way to make these

instruments of transformation. The most remarkable and useful steps of which have been the invention of *the dynamo* and *voltaic battery*, and of lesser importance the *thermopile* and *electrical machine*.

The reverse transformation is also important, and transforming devices have been evolved for executing the reversal of the process and transforming back part, at least, of the energy of an electric current into *light* by means of *electric lamps*, into chemical potential energy by means of the *electrolytic cell*, into *heat* by means of the *electric furnace*, and into *kinetic energy* by means of the *electric motor*.

We shall limit our attention in this chapter to a consideration of some points in connection with the structure of the dynamo-electric machine, and primary and secondary batteries, as instruments of transformation.

§ 2. **The Dynamo.**—Referring the student to special treatises for the history of the development of the dynamo machine from the fundamental discoveries of Faraday, we shall consider here merely the essential mode of operation of some of the more modern machines.

We have already explained the chief facts concerning the production of a current in a circuit, by the insertion into or withdrawal from it of a stream of magnetic flux.

Consider the arrangement shown in Fig. 115. Let F be a fixed semi-rectangular bar of iron wound over with insulated wire so as to form an electromagnet, and let it be excited by a current of electricity through its coils. Let A be another similar semi-rectangular bar also wound over with insulated wire to form another electromagnet. Let A be capable of being set in revolution round an axis *a b* so that its poles *n* and *s* are alternately presented to the poles N and S of the fixed electromagnet. Suppose the two electromagnets are in the position shown in the figure, and let the circuit of F be traversed by a separately generated exciting current, and let the circuit of A be closed on itself. Then, if A is set in revolution, the stream of magnetic flux

from F will be directed through the iron core of the magnet A, first in one direction and then in the other. This reversal of the direction of the flux creates an electromotive force in the coil of the magnet A. This electromotive force is a periodic and alternating electromotive force, because its value depends upon the rate at which the flux is being changed through the coil of the A magnet. If we follow in imagination the magnet A through one complete revolution, we shall see that the

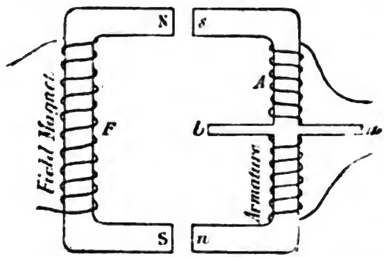

Fig. 115.—Principle of the separately excited Dynamo.

electromotive force in the A coil has a zero value when the poles *n s* of A are just opposite the poles N S of F, and has a maximum value when the position of the magnet A is at right angles to that of F. The instantaneous values of the electromotive force induced in the coil of the A magnet may be roughly represented as regards direction and magnitude by the ordinates of the curve shown in Fig. 116, where the round circles represent the end-on view of the poles of the magnet F, and the figures on the base line the angular displacement of the electromagnet A from the plane of the electromagnet F. The current, therefore, in the closed circuit of the A magnet is an *alternating current*, and reverses its direction twice in every complete revolution of the electromagnet A.

It is possible to add to the axis of revolution of the A magnet a device called a *Commutator* for obtaining from this internal alternating induced electromotive

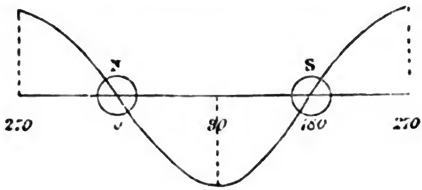

Fig. 116.

force a continuous current in an external circuit attached to the coil of the A magnet. The original simple arrangement for effecting this change is called a *Split-tube Commutator.* Let a wood or insulating drum be fixed on the shaft *a b*, carrying on its surface two semi-cylindrical plates *x* and *y*, nearly touching, and let one end of the coil of the armature A be attached to the plate *x* and the other end to the other plate *y*. Then let two light springs X and Y be fixed so as to touch these commutator plates, and let the external circuit E be terminated at X and Y (see Fig. 117). The drum must be so attached to the shaft *a b* as to revolve with it, and so that the spring X passes from contact with the plate *x* to contact with the plate *y*, just at the instant when the electromotive force generated in the circuit of A changes its direction.

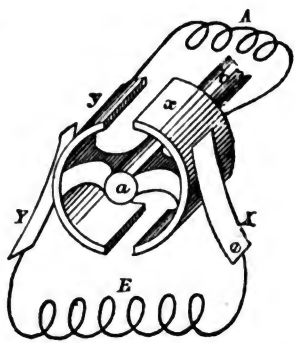

Fig. 117.—Split-tube Commutator.

The line joining the contact-points of the springs X and Y is called the axis of commutation, and this axis of commutation must be at right angles to the line joining the poles N and S of the magnet F.

It will then be found that on rotating the armature A, a fluctuating, but not alternating, current will be produced in the external circuit E connecting the springs X and Y.

The student or teacher accustomed to the use of tools will have no difficulty in making a working model of the apparatus just described. The electromagnets can be made of bars of soft iron about 1 inch wide, $\frac{1}{4}$ inch thick, and 18 inches long. Any blacksmith will bend these bars up into a semi-rectangular shape. One of them can be mounted on a board and wound over with insulated wire, say with six layers of No. 18 wire on each leg. The other must be supported on a steel shaft, and have a simple split-tube commutator attached to it. The windings on both magnets may be of the same sized wire. If a galvanometer is then attached to the spring contact slips, it will then be possible to show the cyclic variation of electromotive force in the A magnet circuit, as it is turned round over the poles of the F magnet, which last must be separately excited by a current from a battery. The split-tube commutator is made by forcing a short brass tube on to a cylinder of wood fixed on the shaft. This tube is secured by brass pins, and then slit lengthways into two half cylinders.

In the simple arrangement described above we have all the essential organs present in the modern dynamo-electric machine. The magnet F is called the *Field Magnet*, and the function of this is to create the magnetic flux, the variation of which causes the induced electromotive force. The electromagnet A, in the coils of which the electromotive force is induced is called the *Armature* of the machine. The device (if present) for *rectifying* or making continuous the current in the external circuit is called the *Commutator*. The springs X and Y are called the *Brushes*.

The essential principle of action of the machine is

that the magnetic flux due to the field magnet is caused to change its direction through the circuit of the armature. This may be done by causing the armature to revolve whilst the field magnet is stationary, or the armature may be stationary and the field magnet may revolve. In some cases both field magnets and armature may revolve. In some cases both field magnets and armature are stationary. In order, then, to create the change of flux through the armature, an intermediate piece of iron has to revolve which directs the flux one way or the other through the armature core. Machines of this class are called *Inductor Machines.* A comprehension of their mode of working may be obtained from

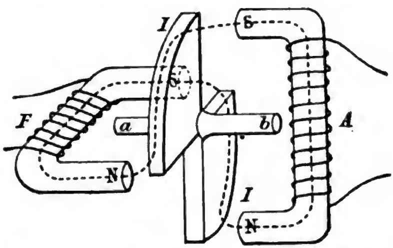

Fig. 118.—Principle of the Inductor Dynamo.

a consideration of the simple model shown in Fig. 118. In this case two semicircular electromagnets are fixed with the line joining their poles at right angles to each other. One of these is the fixed field magnet F, and is excited by a current. The other of these is the fixed armature A. To a revolving shaft *a b* are fixed two quadrantal-shaped segments of iron I, I, which revolve in between the poles of the two magnets, as close as possible but just not touching the poles. When these pieces of iron are in the position shown in the figure they conduct the magnetic flux of the field magnet through the armature in the direction as represented by the

dotted line, but when they have moved round through a quarter of a turn they conduct the flux round through the armature in the opposite direction. Thus the necessary reversal of flux is made through the coils of the armature without any movement of the iron core of the armature itself.

It will then be observed that we may classify dynamo-electric machines as follows—

(i) Continuous current machines;

(ii) Alternating current machines, or alternators; according to whether they furnish in the external circuit a continuous or an alternating current.

Both classes may be either—

(*a*) Fixed field and revolving armature machines,

(*b*) Revolving field and fixed armature machines,

(*c*) Fixed field and armature, or inductor machines.

Alternators may be—

(1) Single-phase alternators, giving one single alternating current.

(2) Two-phase alternators, giving two alternating currents with a fixed difference of phase between them.

(3) Polyphase alternators, giving several alternating currents having different phases.

The output of a machine is always reckoned in kilowatts. Hence a 30-kilowatt (K.W.) machine is one which can produce an electrical power in the external circuit of 30,000 watts.

The *efficiency* of a dynamo is the ratio, expressed as a percentage, between the power given out by it in the external circuit and the power required to drive it round, both powers being measured in the same units. In the case of continuous current machines, or of single phase alternators working on a non-inductive current, the output of the machine in watts can be obtained by measuring the potential difference of the brushes in volts, and the outgoing current in amperes, and then multiplying these two values together.

§ 3. **Excitation.**—In the model machine described

in the last section, we considered that the magnetic flux was created by an electromagnet energised by a separate electric current. There are, however, three ways in which this field flux can be created. In the first place, the field magnet may be a permanent steel magnet once for all magnetised. Next, the field magnet may be an electromagnet, and the current required to create it may be obtained from a separate or external source of current, such as a battery. Thirdly, the principle of self-excitation may be employed. In this latter case either a part or the whole of the current produced in the armature may be led through the circuit of the field magnets. If the field magnets are slightly magnetised initially in the right direction, then beyond a certain speed of revolution, the electromotive force set up in the armature will be sufficient to generate, and increase up to a limit, an exciting current which, when led through the coils of the field magnet, will fully magnetise the field magnet cores. This is called *self-excitation*. The self-exciting machine is the one to which the name of *dynamo* was originally given. Permanent field magnets and separately excited field magnet machines were previously called magneto-electric machines.* Hence we have the following classification of machines depending on the method of the production of the field flux—

(1) Magneto-machines. Permanent magnet fields.

(2) Separately excited machines. Electromagnet fields.

(3) Self-excited machines.

(*a*) Series wound fields.
(*b*) Shunt wound fields.
(*c*) Compound wound fields.

The last class (3) can obviously only be continuous current machines.

The sub-classification of self-excited machines de-

* The method of self-excitation was devised by S. A. Varley in 1866, and soon after independently by Siemens and Wheatstone.

pends upon the mode of winding of the field magnets. If the whole current from the armature goes through the field magnet coils, the machine is called a *Series* machine.

If the ends of the field magnet circuit, and also the ends of the external circuit, are both joined to the brushes of the machine, that is, to the terminals of the armature circuit, the machine is called a *Shunt* machine.

If a shunt machine has in addition field magnet coils which carry the whole external current, it is called a *Compound wound* machine. (See Fig. 119.)

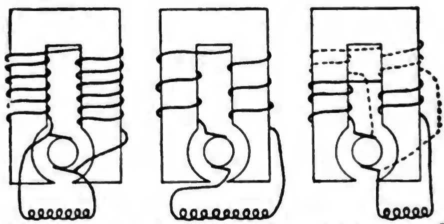

Fig. 119.—Types of Field Magnet Winding.

The power reckoned in watts taken up in the exciting coils of the field magnets is called the *exciting power* of the machine.

§ 4. **General Design of Continuous Current Dynamos.**—From the remarks in the previous section it will have been made clear to the student that a continuous current dynamo essentially consists of two electromagnets, one of which carries a commutator, and revolves in front of, or between, the poles of the other. Experience has shown that these electromagnets should have very different forms, and with some slight modifications most continuous current machines now in use consist of a field magnet having massive round iron or

mild steel legs united by a yoke, and furnished at the opposite ends with curved pole pieces nearly embracing the armature. (See Fig. 120.)

The other electromagnet or armature takes the form of a *drum*, or *ring*, or cylinder of iron, wound in a particular way with coils of insulated wire.

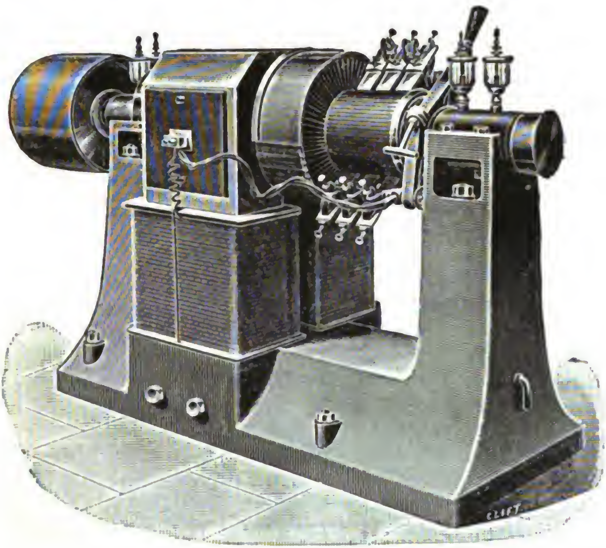

Fig. 120.—Modern Two-pole Dynamo.

The field magnet cores are constructed of cast steel of the highest permeability, or else of wrought iron, and the wire windings on them consist of double cotton-covered copper wire, insulated with paper and shellac varnish. The core of the armature is always built up of stampings of thin sheet iron or sheet steel. The proper

shaped circular discs or rings are stamped out of sheet iron, and piled one on the other with thin paper between them. They are then held in some suitable manner on the shaft. The reason for thus making the armature *laminated* is, that if it were not so constructed the revolution of a mass of solid iron in between the poles of the field magnet would create in it eddy electric currents, which would heat it and cause a waste of energy.

The armature core is fixed to the shaft by gun-metal carriers, which communicate to the iron core the driving power of the shaft. The laminated iron core is then wound over with the armature coils, and this winding takes a different form according as the iron core is a drum or a cylinder. To the driving shaft is also affixed the *Commutator*. This consists of a number of copper segments insulated with mica between and beneath them, and which fit together like the stones of an arch. The commutator segments are drawn or cast with a section like the stones of an arch, so that when fitted together they form a smooth cylindrical surface composed of separate insulated bars of copper. To each copper segment is affixed a radial rod, by means of which the connection is made to the coils of the armature. The armature winding for a *ring* armature is then effected in the following manner.

Starting from one commutator segment an insulated wire is taken several times round the iron core and back to the next commutator segment. Starting again from the same segment a second wire is taken the same number of times round the core, and back to the third commutator segment. This process is followed until the last end of the last wire is brought back to be joined to the first commutator segment. The windings on the ring or cylinder thus form an endless spiral, and have the arrangement shown in Fig. 121.

The armature when wound presents on the outside a smooth cylindrical surface of cotton-covered wires, all placed parallel to the shaft. To keep them in their

places they are bound round with cross windings of steel wire, kept from touching the armature wires by mica. The whole armature is then well varnished and baked. The armature is next placed in its position between the poles of the field magnets, and in a well constructed machine there is just clearance enough to allow the bobbin to rotate without any risk of the armature winding touching the pole pieces.

If the armature core is a *drum* core, and not a cylinder or ring core, the winding is conducted on similar lines. It will, however, in this case, be clear on consider-

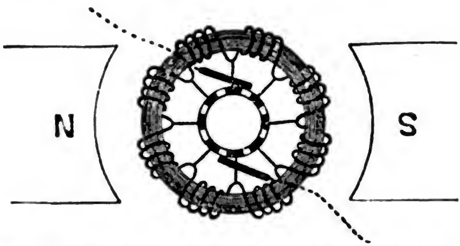

Fig. 121.—Diagram of Gramme Ring Armature Winding.

ation, that after half the armature winding has been put on, and half the commutator segments attached to coil ends, the drum is then uniformly covered with longitudinal layers of wire. The second half of the winding has therefore to be laid over the first layer, or else the coils have to be sandwiched in between those of the first half of the winding in order to complete the winding of the armature.

It will be best at this stage to endeavour to make clear to the reader, by the help of some diagrams, the manner in which the currents are generated in the armature coils in each of these two kinds of armature, when it is rotated in the field of the field magnet. Consider, in the first place, the cylinder or ring armature, usually

called after its inventor a Gramme ring. Let the circular ring I, in Fig. 122, stand for the armature core, and let it be wound over in the manner described with single loops of wire, the ends of which are brought out to the segments of an *external commutator*, the segments of which are represented by the black curved lines. Let *brushes* touch this commutator along an axis of commutation at right angles to the line joining the poles of the fixed field magnets, N and S. Then suppose the ring to revolve in *clock-wise* direction. The magnetic flux

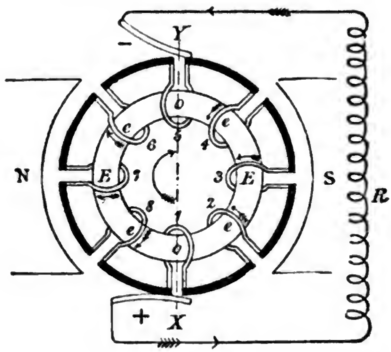

Fig. 122.

from the pole N passes across the air gap, and separates into two streams of flux round the opposite sides of the ring, and enters again at pole S. This flux threads through, or is linked with, the coils wound on the ring. A little reflection will make it clear that as the ring revolves, the flux keeps its place, but each armature coil as it passes the line joining the poles will experience a change in the direction of the flux passing through it. Hence it has created in it an induced electromotive force. The coils which are at any moment situated along the axis of commutation, will have no change of flux produced

through them by a small movement at that place, and hence no electromotive force is generated in them. The coils in the quadrantal or intermediate positions will have small electromotive forces created in them.

The relative magnitude of these electromotive forces is in some sense indicated by the size of the letter E attached to each coil. If the student will then suppose the ring in Fig. 122 to make a small portion of a revolution in a clock-wise direction, and consider how the flux changes through each of the eight coils, and the direction of the induced electromotive force in each coil, he will see that this last is represented as to direction by the arrows marked on each of the coils respectively. It must be remembered that when the flux *coming out* from a *North* pole is *inserted into* a conducting loop, it generates a *counter clock-wise* electromotive force in the loop, as seen from that side at which the flux is put in. If we consider the four coils on the half of the ring Y N X, we see that the electromotive forces in them all conspire to force a current *out* at the brush X. Also the electromotive forces in the four coils in the half of the ring Y S X, all conspire to do the same thing. If the brush X is not externally connected with the brush Y, these equal resultant electromotive forces round the two halves of the ring-winding simply oppose each other, and no current results. If, however, the brush X is connected with the brush Y by an external circuit, then these two resultant electromotive forces conspire to generate a current in the direction X R Y, through the external circuit R. If the armature winding were straightened out it would be found (see Fig. 123) to consist of a pair of wires which are joined in parallel between the two points of contact of the brushes. In each of these branches there is a series of unequal electromotive forces, all helping one another. The sum of these graded electromotive forces in each side of the armature is, however, the same. When the brushes are connected by an external circuit, these two series of electromotive forces

both act together in parallel to urge a current through the external circuit R. The resistance of the armature as a whole, from brush to brush, is exactly half that of the whole resistance of one half of the armature, or a quarter of the total all-round resistance of the armature. It will be seen that as each pair of commutator bars or sections passes under a brush, the coil connected to these segments is short-circuited for an instant. As soon as this pair of segments passes away from under the brush, the corresponding armature coil is opened again. As these coils possess very considerable inductances, it is necessary that this opening of the short-circuit should

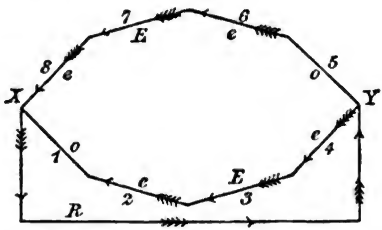

Fig. 123.—Gramme Ring Winding unwound.

take place when there is no electromotive force in that coil, or else there will be a *spark* under the brush. This sparking at the commutator is injurious to it, and must be avoided by giving the brushes a proper position.

The action of the drum winding is a little more difficult to understand. It can however best be made plain by considering the drum as extended into a strip by a sort of projection. In Fig. 124 the long rectangle *a b c d* represents the drum armature core thus extended. The rectangles represent the coils wound upon it. Let the magnet pole N be supposed to move sideways down the row of coils. The stream of flux from the pole will pass as shown in the dotted lines, and it is easy to see

that as the magnet passes over each coil, the direction of flux through it will be reversed, and this will create in it an electromotive force. Also, it will create lesser electromotive forces in the neighbouring coils. Hence, on tracing out the connections and the direction of the electromotive force in each loop it will be found that all the loops connected to one side of the commutator have a series of electromotive forces in them, acting in them, say

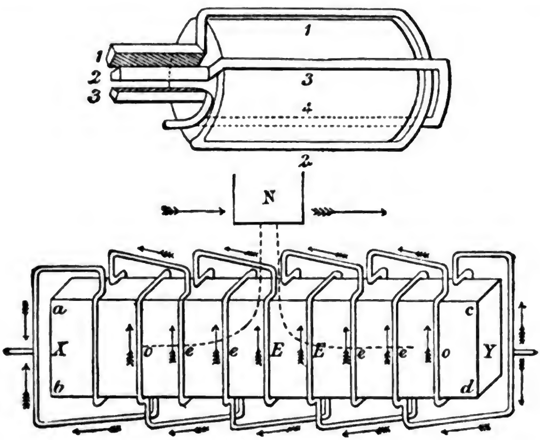

Fig. 124.

from Y to X, and all the other set of loops have likewise an equal total electromotive force in their circuit, acting from Y to X. Hence if X and Y are joined by an external circuit, a current will be urged through it from X to Y. The reader will notice that in both cases, viz. in the drum, and in the cylinder or ring winding, certain coils are *active* at each instant, and have generated in them the maximum electromotive force, and certain coils are *idle*, and have no electromotive force in them, and others are in interme-

diate states. The effective or resultant electromotive force of the machine is the sum of all the individual electromotive forces in the several coils on one half of the armature, or attached to one half of the commutator. Supposing all the wire taken off the drum and laid open, it will, as above stated, be found to form an endless wire. The points of contact of the brushes are at the ends of a diameter of this loop. If we imagine the armature wire marked off into the portions which form each loop when it is wound on the ring or drum armature, and an arrow drawn on each, representing by its size and direction the electromotive force in those loops at any instant when the armature is turning in the field, and if we imagine this wire taken off the core and opened out, we should have an endless loop, as shown in Fig. 123, which shows the series of electromotive forces in each loop of the wire on each side of the armature for the Gramme ring winding delineated in Fig. 122. Even although a coil may be inactive as regards the production of electromotive force in it, it is traversed by a current, and hence if we consider a drum armature in section and as seen when looked at *end-on*, in one half of the conductors on the armature the current would appear to be moving towards the observer, and in the other half current to be moving away from the observer. For some purposes it is very convenient thus to represent the armature *in section.* In order to show in which direction the current is moving in a wire when seen end-on, we represent that wire by a *black dot* or circle if the current is moving in it *away from* the eye, and by a *white circle* if the current in it is moving *towards* the observer.

§ 5. **Armature Reaction.**—Up to the present moment we have said nothing about the reaction which the armature exerts upon the field magnets. The flux from the field magnets passing through the moving armature coils in their varying positions generates in them electric currents. These electric currents magnetise the armature core, and thus create an opposed magnetomotive force

which reacts upon the field magnets and changes the disposition and strength of its magnetic flux.

In order to see what happens, let us examine the effect of the armature current in the case of a two-pole dynamo with drum winding. Let the cores and circuits be represented in section as shown in Fig. 125. The windings on the armature may be divided into two

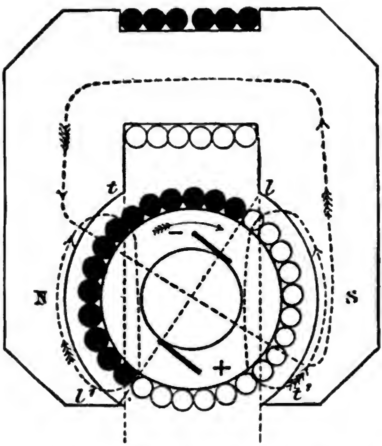

Fig. 125.—Diagram illustrating the cause of Armature Reaction in a Two-pole Dynamo.

groups, viz. one which is contained between the two vertical dotted lines which just touch the curved horns of the pole-pieces, and which are called the *back turns* on the armature; and secondly, the remaining portion of the winding which lies under the curved horns right and left. This last part is called the *cross turns* on the armature. The direction of the magnetic flux due to the field-magnet windings is as shown by the dotted line drawn round inside them.

Regarding now the armature core and field core as forming one magnetic circuit with two air-gaps in it, consider the magnetic effect of the *back turns* on the armature on the magnetisation of this circuit. It is clear that they oppose the field-magnet turns in their magnetising effect. Consider, in the next place, the magnetising effect of the *cross turns* on the armature. It is evident that they create a magnetic flux in the armature-core and pole-pieces, the direction of which is as shown by the looped dotted lines. If the direction of this flux is traced out, it will be seen that the flux in the pole-pieces due to the cross turns tends to help the flux due to the field magnet at two horns or corners *t t* of the pole-pieces, and to weaken it at two other horns *l l*. These horns are called respectively the *trailing horns* and the *leading horns*. Hence the joint effect is to displace the interpolar magnetic field and twist it round, so that the lines of the flux from pole to pole are no longer straight across, but obliquely directed, as shown in Fig. 126. The effect, therefore, of the *cross turns* is to displace the interpolar field, and the effect of the *back turns* is to partly counteract the magnetic effect of the field coils. The back turns have the effect of driving out of the armature core some of the flux that would otherwise traverse it.

There is another important practical point which must be considered, and that is the *magnetic leakage*. The whole of the flux created by the magnetising effect of the field-magnet turns or windings does not pass through the armature core. A good deal of it leaks across from leg to leg and pole-piece to pole-piece without traversing the core. This waste field is called the leakage field, and in even well-designed dynamos as much as 30 to 35 per cent. of the total flux traversing the median section of the field magnets never gets through the armature core at all. As our object is to magnetise the armature, and not merely to magnetise the field, all this leakage means so much increased cost

in exciting power. The *leakage coefficient* varies somewhat in dynamos of different types, but is approximately represented by the above figures. The student will see therefore that a number of ampere-turns have to be put upon the field magnets over and above those necessary merely to magnetise the iron, to do the following things :—

In the first place, the magnetic flux has to be forced

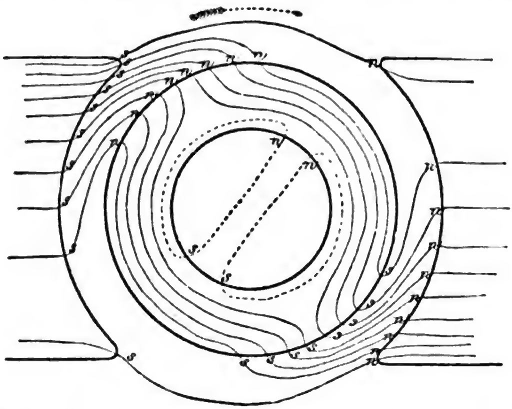

Fig. 126.—Magnetic Flux Lines through the Core of a Gramme Armature when generating a current.

across the air gap. As a very high flux density is necessary in the gap, this involves a considerable additional expenditure of magnetomotive force. In the next place, the reverse magnetising effect of the back turns on the armature has to be overcome or neutralised ; and in the third place we have to provide for the leakage of flux.

The student must be referred to more advanced treatises for information as to the mode in which the

necessary total exciting power in ampere-turns on the field magnets is calculated in any given case. Our object here is more to indicate general principles than enter into details.

With regard to the distortion of the field produced by the cross-turns on the armature, one result of it is to shift the axis of commutation (or line joining the point of contact) of the brushes round through an angle called the *angle of lead*, in the direction of the rotation of the armature. If there were no armature reaction at all, the proper position of the brushes for sparkless collection of current would be on a line perpendicular to the line joining the centres of the pole-pieces. We have already explained that the proper position of the brushes to prevent sparking on the commutator is such a position that the ends of any armature loop pass under a brush at the instant when there is no electromotive force in that loop. If the direction of the field is displaced or twisted forward, then it follows that the position of the brushes at the non-sparking point will be also twisted forward, or have a *lead*, as it is called, beyond the line at right angles to the polar line. The proper position for the brushes is generally found to be just under the leading horn of the pole-piece, the brushes being shifted forward from the median line in the direction of rotation. Since the distortion of the interpolar field depends upon the magnitude of the current going out of the armature, it follows that the non-sparking point shifts with the load. The greater the load on the dynamo, the further round must the brushes be set. The invention of methods for preventing sparking at the brushes and avoiding the necessity of shifting the position of the brushes with every change of load has considerably occupied the attention of dynamo builders. The student must be referred to larger treatises for information as to the special remedies suggested by Edison, Sayers, Mordey and others. Meanwhile, it is sufficient to state that one remedy (although a costly one) is to make the field

magnet so strong that the cross magnetising turns of the armature have not a great effect upon it. With a sufficiently strong field magnet, the effect of changes in the load is less marked as regards sparking at the brushes than with a weak field. The matter is one largely of first cost; and dynamos may be designed without regard to cost, so that they do not spark at all under changes of load. The brushes for collection are always held on a rocking arm, and can be shifted round so as to touch the commutator at the non-sparking points. Brushes are now most usually made of copper gauze folded up into a rectangular shape. They should never press so hard on the commutator as to score or mark it. The commutator must be kept smooth and true, and the brushes slightly shifted laterally from time to time to avoid always working in one place.

§ 6. **Alternators.**—Alternating current dynamos are designed to give an alternating current, and have therefore no commutator. Hence they are simpler in design than continuous current machines. The field magnets must be excited by a continuous current from some external source, and this is now generally done by means of a small continuous current machine fixed to the shaft of the alternator. The field magnets of an alternator may take many different forms. In the alternators of Siemens, Kapp and Ferranti the magnets consist of a pair of iron rings or frames, from which project iron teeth or poles, on which last are placed magnetising coils. The rings or backs are carried on a bedplate, so that the poles project inwards. In the Mordey alternator the field magnet consists of a stout shaft wound with a magnetising coil, and on this shaft are carried curved shield-shaped iron discs, to which are fixed pole-pieces projecting inwards.

In other cases the field magnets consist of an iron ring, which carries radial teeth of iron projecting inwards, on which are placed magnetising coils. The Westinghouse and Thomson alternators are of this form.

The armature may be wound on an iron core, in which case it is called an *iron-cored armature.* Armatures of this kind are found in the Siemens, Kapp, Westinghouse, Thomson and Ganz alternator. The armature may, on the other hand, consist simply of coils of copper wire or band with no iron in it, in which case it is called a *coreless armature.* Of this type are the armatures in the Ferranti and Mordey alternators. A convenient method of diagrammatically representing the relation of the armature windings to the field poles is to suppose the field ring or rings cut open and laid out straight, and then to represent the movement of the armature as a procession of coils

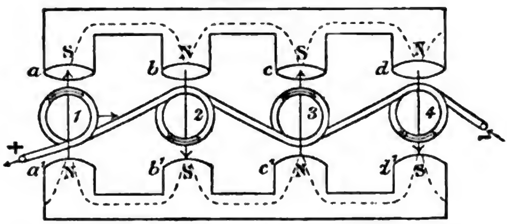

Fig. 127.—Diagram illustrating an Alternator Construction.

passing between or beneath these teeth. We can thus, as in the diagram in Fig. 127, represent the construction of a Ferranti or Siemens alternator.

In Fig. 127 the armature coils 1, 2, 3, 4 are supposed to be moving from left to right between field poles *a a'*, *b b'*, &c. It will be seen that as coil 1 passes from between *a a'* to between *b b'* the flux through it is reversed, and hence an electromotive force is generated in it. The ends of the armature circuit are brought to the collector rings, against which press collecting brushes, and as the coils move through a space equal to the distance between the magnet poles the electromotive force in them changes its direction, and thus creates an *alternating* current.

Alternators at the present time are generally constructed to give an alternating current having a frequency of 50 ∽ to 150 ∽, most usually 50 ∽ on the Continent of Europe, 100 ∽ in England, and 130 ∽ in the United States. If the machine is intended to produce a two-phase current, then there are two independent sets of armature windings, which are so arranged that the electromotive force due to one lags 90° behind that due to the other. An alternator which produces an electromotive force (R.M.S. value) of 250 volts and upwards is called a high tension alternator. One which produces an electromotive force of less than 250 volts or so is called a low tension alternator. If an alternator is supplying current to a non-inductive circuit, such as a load of incandescent lamps, the current coming out of the machine is in step with the electromotive force, and under those conditions if we multiply together the R.M.S. value of the current as read on an alternating current ammeter and the R.M.S. value of the terminal voltage as read on a voltmeter, we obtain the output of the machine in watts.

This, however, is not the case when the alternator is working on an inductive circuit, such as a bank of transformers lightly loaded. In this last case there is a considerable difference of phase between the current and the electromotive force. The output can then only be measured by a properly arranged wattmeter.

When alternators are set to work to send electric currents through conductors between which there is *electrical capacity*, many remarkable effects are produced, which are due to the change in the armature reaction which then takes place. For information on these matters, however, and other details of alternator working, the student must consult special text-books.

§ 7. **Efficiency and Efficiency Measurement.**—If we consider some particular form of dynamo, say a shunt-wound continuous current machine, when in action we may note the following distribution of energy losses. In the first place power is applied to the shaft of the

dynamo to turn it round. Part of this power is wasted in *friction* at the bearings of the machine. A second portion of the power is wasted in the *eddy electric currents* set up in the core of the armature, and in the *hysteresis* loss in reversing the magnetic flux in the core at every revolution. A third part is wasted in heating the magnetising coils of the field magnet and in heating the coils of the armature. Some portion is wasted in continually reversing in direction the currents in the armature core and in sparking (if any) at the brushes. In an alternator some not inconsiderable portion of power is spent in churning up the air round the armature. Finally, a portion of the power supplied is converted into electric power which appears in the external circuit. This last portion is the only one really useful. The efficiency of the machine is the ratio, expressed as a percentage, between the power which is taken out electrically and the power put into the machine mechanically. In very good modern continuous current dynamos this efficiency may reach 94 per cent.; most commonly about 90 per cent. when the machine is at full load. As the load is decreased the efficiency falls off considerably.

An accurate and simple mode of testing a continuous current dynamo for efficiency is that due to Dr. Hopkinson. It is based on the fact that a dynamo is a *reversible engine*, that is to say, it can not only be used to convert mechanical power to electrical power, but can also be used as a *motor* to convert electrical power put into it in the form of a current back into mechanical power. In other words, if a current is put into a dynamo from some external source the dynamo begins to revolve, and is then said to act as a motor.

If a pair of identical dynamo machines have their shafts coupled together in one line (see Fig. 128) and their brushes joined together so that their armature circuits are connected, we may so arrange the connections that when the coupled machines turn round together, one machine, which we will call the dynamo D, sends out an

electric current passing into the armature of the other machine which we will call the motor M, in the right

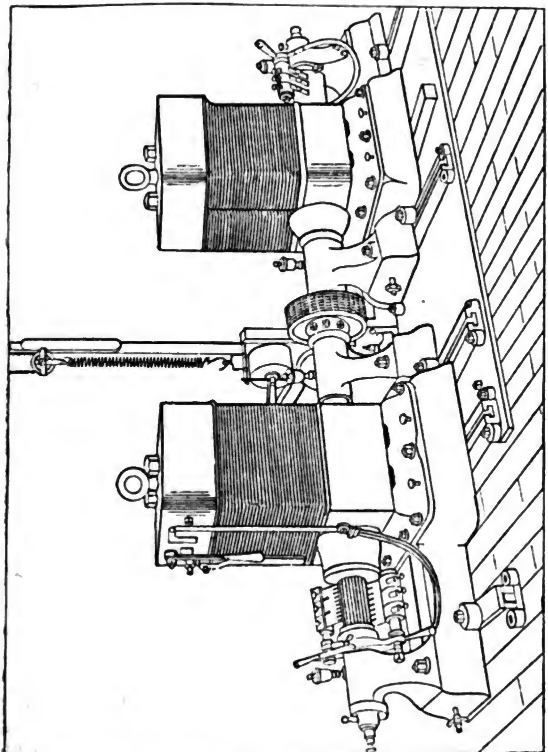

Fig. 128.—Dr. J. Hopkinson's Method of Measuring the Efficiency of a Dynamo and Moto

direction to preserve the requisite rotation of the shaft. Since, however, the power given out by the motor is not sufficient to create through the dynamo all the power

required to revolve the motor, we have to supply an additional amount of power from outside sufficient to make up all the energy losses in the two machines. This is done by inserting a secondary battery or third dynamo in the circuit of the armature to supply the necessary power. The combination of the two dynamos will then start revolving, and can be kept at the standard speed. The motor drives the dynamo, and the dynamo and assistant battery together make up the necessary voltage to supply the motor with the required current.

The power coming out of the battery can be measured by measuring the current A circulating through it, and

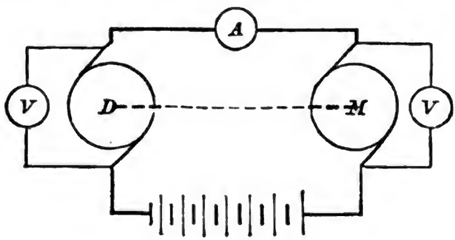

Fig. 129.

multiplying this by the voltage drop V over the battery. We then assume that this supplied power, equal to AV watts, is divided equally between the motor and the dynamo to make up their internal losses.

Hence the efficiency of the dynamo is equal to the ratio of the power in watts coming out of the dynamo to the same power *increased* by half the product AV. The efficiency of the motor is equal to the ratio of the watts coming out of the motor *diminished* by half the product AV to the power in watts put into the motor. The power coming out of the dynamo is measured by the product of the current and the terminal voltage (V_D)

of the dynamo. and the power put into the motor is measured by the product of the current in amperes and the terminal voltage (V_M) of the motor.

Hence, if we measure with an ammeter the current (A) circulating round the system (see Fig. 129), and measure with a voltmeter the voltages V_D, V_M, V_B, at the terminals of the dynamo, motor, and battery respectively, we have the following rules for calculating the efficiency E_D of the dynamo, and that E_M of the motor.

$$E_D = \frac{A\,V_D}{A\,V_D + \frac{1}{2}A\,V_B}.$$

Also

$$E_M = \frac{A\,V_M - \frac{1}{2}A\,V_B}{A\,V_M}.$$

These electrical measurements can all be made with great accuracy. It will be seen that we really only require to know the current A in order to be sure that we have the full load current coming out of the dynamo, and that we are testing it under normal conditions as to load and speed.*

§ 8. **Primary and Secondary Batteries.** — The transformation of chemical potential energy into electrical energy, or the reverse process, is effected by transforming devices, which are called *Primary* or *Secondary Cells.* A collection of cells is called a *Battery.* The elementary theory of cells is best approached by studying the facts of Electrolysis. We have already seen that if two plates of a chemically inactive metal, say platinum, are placed in a solution of an electrolyte (for the sake of simplicity let us consider it to be a solution of hydrochloric acid (HCl) in water); and, if an electric current is passed through this electrolytic cell or voltameter, *Electrolysis,* or electro-chemical decomposition, takes place. The products of this electrolysis appear at the

* For practical details of the test see the author's 'Electrical Laboratory Notes and Forms,' Advanced Series, No. 40.

plates, or electrodes, and are called the *Ions*. The modern view of this process is as follows :—A chemical molecule like hydrochloric acid (HCl) consists of an atom of hydrogen (H) united to an atom of chlorine (Cl) by a chemical bond. Each of these atoms is called a monovalent atom. When hydrochloric acid exists in solution in water, it is considered that the same atom of hydrogen is not always in contact or in union with the same atom of chlorine, but the molecules are continually being broken up and reformed just like couples dancing in a ball-room who are constantly changing partners. The forces holding the atoms together to form a molecule are considered to be the electrical attraction of charges of electricity of opposite sign carried by the atoms. The causes of a rupture of a molecule may be considered to be the collisions it receives in moving rapidly about. Moreover, in a molecule of hydrochloric acid, we must picture to ourselves the atoms of hydrogen as positively electrified, and the atoms of chlorine as negatively electrified ; these charges being exactly equal in amount but opposite in sign. The molecules of hydrochloric acid are thus moving rapidly about and exchanging partners. Hence at any instant there is a certain number of complete molecules of hydrochloric acid (HCl) and a certain number in a state of dissociation as free atoms of hydrogen (H) and chlorine (Cl). These disunited or free atoms are called *Ions*, and the charges of electricity they carry are called the *Ionic Charges*. If, then, we introduce into a vessel containing such an electrolyte a pair of platinum plates connected with a source of electromotive force so that one plate is at a higher potential than the other, we immediately disturb the distribution of the free ions. Before the introduction of these electrodes, we may picture to ourselves the free ions with their electrical charges moving rapidly about in different directions, one moment being in combination and the next moment free again. The instant, however, that the charged electrodes are introduced these last will exercise

an electrical attraction upon the ions, when in the free state, in virtue of the ionic charges. The positive ions will begin to make their way towards the negative electrode, and the negative ions towards the positive electrode. Each ion will make its way as through a crowd, free one moment and for a short time in combination the next, but always struggling on towards the oppositely electrified electrode during its moments of liberty. The result will be that the electrodes will be soon covered over with a layer of ions, the positive electrode with a layer of negatively charged ions, and the negative electrode with a layer of positively charged ions. These electrodes are then said to be *polarised*. If, then, the source of electromotive force is removed and the platinum plates connected with each other by a wire, we find that for a short time there is a current of electricity in this connecting wire. This is called the *Discharge Current* of the electrolytic cell. The polarisation is then found to disappear as this discharge current proceeds. The electrified ions move back into the liquid, and this is associated with an electric current, viz. the movement of an equal amount of positive electricity in one direction and of negative in the opposite. If instead of discharging the cell we had continued to apply a sufficient electromotive force, bubbles of hydrogen would have made their appearance at the negative pole, and of chlorine at the positive pole,* and energy has continually to be supplied to the cell to effect this free decomposition. In order to cause the ions to appear in the form of free neutral elements, energy has to be given to the cell.

As soon as the electrodes are polarised, any attempt to send more current through the cell meets with an *opposing electromotive force*, due to the tendency of the ions to re-combine. Hence, to force a current through

* As a matter of fact in decomposing hydrochloric acid it is necessary to use a carbon rod for the positive pole or electrode, because the chlorine ions attack the platinum. This, however, does not affect the general explanation.

the cell and liberate freely the ions as neutral elements or compounds at the electrodes, an electromotive force has to be applied greater than the back electromotive force due to the polarisation of the plates or to the liberated ions. This necessitates *work* being done, and accordingly, to make a continual liberation of ions at the surface of the electrodes requires two things: firstly, the passage of a certain quantity of electricity through the voltameter; and secondly, the presence of a certain electromotive force creating a certain minimum potential difference between the electrodes of the cell.

It will be noted, therefore, that on this view the electric current, when causing electrolysis, is not supposed to tear the molecules of the electrolyte asunder. The electric forces simply guide the ions when they *are* free, one half of them migrating in one direction, and the others in the opposite.

In order to produce what is called free decomposition, work has, however, to be done in the cell to overcome the back electromotive force, due to the ions liberated against the electrodes. This *work* may be looked upon as the equivalent of the potential energy represented by the products of electrolysis.

The amount of electricity which must pass through an electrolytic cell to liberate one gramme of free hydrogen gas from an electrolyte containing it, is 96,340 coulombs, or 9,634 absolute C.G.S. units.

Faraday discovered that if we pass the same current through a number of electrolytic cells arranged in series, or one after the other, each containing different electrolytes, the current will liberate in each cell weights of free ions which are *chemically equivalent*.

Thus if we have three electrolytic cells joined in series, one containing dilute hydrochloric acid, the next melted silver chloride, and the next melted zinc chloride, all of which substances are electrolytes, the weights of hydrogen, silver and zinc, liberated against the negative electrode of each cell by the passage of 96,340 coulombs of electricity, will be respectively 1 gramme of hydrogen

107·7 grammes of silver, and 32·45 grammes of zinc. These weights are in the ratio of the chemical equivalents of these bodies, or proportions in which they combine with one and the same element such as chlorine.

If a number of grammes weight of a body is taken equal numerically to its molecular weight, this mass is called a *gram-molecule* of the body. If a number of grammes equal to the chemical equivalent is taken, this mass is called a *gram-equivalent* of the body. Thus the atomic weight of hydrogen is 1, and the valency of hydrogen is unity. The atomic weight of oxygen is 16, and the valency is two. The molecular weight of water (H_2O) is 18, and the chemical equivalent is said to be 9. Accordingly, 1 gramme of hydrogen, 8 grammes of oxygen, and 9 grammes of water are *gram-equivalents* of these bodies.

Faraday's law of electrolysis is that *one gram-equivalent of any electrolyte is decomposed by sending through it* 96,340 *coulombs of electricity*. This quantity of electricity accordingly will decompose 9 grammes of water, and liberate from it 1 gramme of hydrogen. It will also decompose 36½ grammes of hydrochloric acid, and liberate from it 1 gramme of hydrogen. This shows that there is a certain definite and unalterable electric charge associated with an atom or ion of a monovalent element, such as hydrogen, chlorine, silver, &c., and exactly double that quantity of electricity associated with a divalent atom or ion such as copper, zinc, or calcium. Some elements, such as iron, can exist in two states, in which they are either divalent or trivalent.

In order to force this quantity of electricity (96,340 coulombs) through the electrolyte, a certain minimum electromotive force is necessary. Thus dilute sulphuric or hydrochloric acid cannot be freely electrolysed with an electromotive force of much less than 1·8 volts.

Since the mass of an electrolyte decomposed is proportional to the quantity of electricity which has passed through it, an electrolytic cell becomes an exact means

of measuring a quantity of electricity. By weighing the amount of copper or silver deposited in a copper or silver voltameter we have seen that a uniform electric current can be measured. The student should carefully examine for himself the effects taking place when an electrolyte, say dilute sulphuric acid, is electrolysed.

In a small glass beaker (called a voltameter) fix two platinum plates (see Fig. 130) and provide two or three voltaic cells of the Daniell type. These cells are made by placing in a gallipot a cylinder of copper, and in the interior of the cylinder a porous pot of unglazed clay. A rod of amalgamated zinc is placed in the interior of the porous pot. A strong solution of zinc

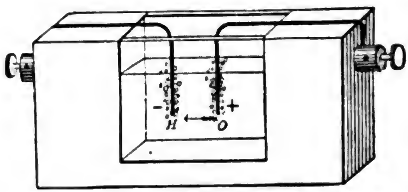

Fig. 130.—Voltameter for Electrolytic Decomposition, adapted for use in the Projection Lantern.

sulphate is then placed in the interior of the porous pot, and a strong solution of copper sulphate is placed in the space outside the porous pot. These solutions are made by dissolving the white crystals of zinc sulphate and the blue crystals of copper sulphate respectively in boiling water. Prepare three cells of this kind, and provide terminals to the zincs and coppers so as to connect up the cells in series.

Join up the voltameter, one Daniell cell and a galvanometer with wires, as shown in the diagram in Fig. 131, and arrange the wire from the voltameter so as to connect it quickly with the cell or the galvanometer. Then note the following facts. If the voltameter is connected for a few minutes with the Daniell cell, and then removed and switched on to the galvanometer, the galvanometer will show a feeble current for a moment or two. This is the polarisation current of the volta-

meter. If the Daniell cell is kept in steady connection with the voltameter no bubbles of gas will be seen coming off from the plates. The electromotive force of the Daniell cell being only about 1 volt is insufficient to overcome the back electromotive force of polarisation and send a continuous current through the voltameter. If, however, two or three Daniell's cells are joined in series, and employed as the battery, then a current is continuously forced through the voltameter, and free decomposition of the dilute acid occurs. Hydrogen is liberated in bubbles at the negative electrode and oxygen at the positive electrode. During, and after the decomposition, however, the electrodes are covered or saturated with hydrogen ions and oxygen ions, and if the voltameter is disconnected from the battery and put on to the galvanometer a brief reversed current will be found lasting as long as the platinum plates remain electrochemically different by the ions deposited on them. The cell therefore becomes when polarised a reservoir of energy.

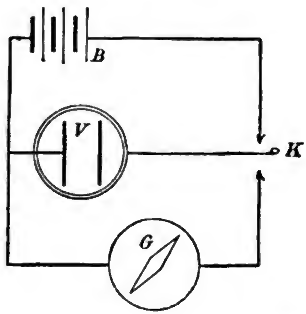

Fig. 131.—B, Battery; V, Voltameter; G, Galvanometer; K, Key.

The polarisation of platinum plates, when placed in dilute acid and made the electrodes in the electrolysis of dilute acid, was discovered soon after the invention of the voltaic cell, and such a polarisation cell was called a *Secondary cell.* It was not until 1859 that Planté took up the subject, and showed by exhaustive experiments that *lead* was the best metal to employ for the electrodes in the construction of a secondary cell; and that secondary cells, made of lead plates placed in dilute sulphuric acid, might be made a very efficient means of storing electric energy.

Planté discovered a process by which the amount of

the electric charge accumulated in a lead secondary cell could be greatly increased.

His process, which is called *forming* the plates, briefly is as follows:—He placed two lead plates in dilute sulphuric acid containing 10 per cent. of the strong acid. He then passed a current through the electrolytic cell so made, and after a short time one of the plates, viz. the positive electrode, was found to be covered with a brown layer of peroxide of lead (PbO_2). He then allowed the cell to stand disconnected, and after a period of rest discharged the cell by connecting the plates by a wire. In the next place he charged the cell in a reverse direction, and then after a second period of rest discharged it again. After these operations had been many times repeated, it was found that the capacity of the cell to take up current energy and give it out again was very much increased. One plate had become superficially covered with peroxide of lead, and the other with reduced or spongy lead. Planté also found that a preliminary treatment of the lead plates with nitric acid had the effect of greatly increasing their electrical capacity when *formed* as above, and used as the electrodes in a secondary cell.

Since that date numerous improvements have been made in the process of "forming" the plates, and also in the mode of constructing them.

We may take as a typical instance of a modern Planté type of cell the secondary cell known in practice as the D.P. cell. The lead plates of this cell are constructed of narrow slips of lead plate, which are all autogenously soldered or *burnt* together into a square lead frame. The construction gives a plate somewhat like the gill of a fish in structure, and having a very large surface of lead exposed to the electrolyte. This plate is then *formed* by treatment in a special electrolyte, and finally a series of such plates are placed in a glass box containing dilute sulphuric acid. The alternate plates are connected to connector bars of lead, and insulating pieces or separators placed so as to prevent the plates

from touching. (See Fig. 132.) This construction then gives us two lead plates of very extended surface, placed opposite and in proximity to each other in dilute sulphuric acid. One set of connected plates is converted on the surface into peroxide of lead, and the other set of connected plates into spongy lead on the surface. These plates are charged by passing a current through the cell in one direction until the electrolyte begins to give off bubbles of hydrogen and oxygen. When this process is finished, it is found that the cell possesses a store of electric energy, and will give out a reverse current for some considerable time.

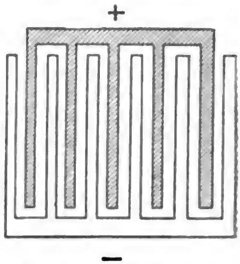

Fig. 132.—Arrangement of Plates in a Secondary Cell.

We have then briefly to consider the electrical and chemical facts connected with this process of energy storage. In order that a secondary cell may be charged, it is found that an electromotive force, gradually rising from 2 volts to 2½ volts, must be applied to its terminals. Hence to charge 54 cells in series an electromotive force must be available rising gradually from 108 to 135 volts.

Whilst the cell is charging, a certain quantity of electricity is passed through it. This quantity, reckoned in *ampere-hours*, is called *the charge*. When the cell is discharged, a certain lesser quantity is given up by it, and the ratio between the ampere-hours of discharge which can be got out of the cell to the ampere-hours put into the cell is called the *ampere-hour efficiency*. The ampere-hour efficiency varies with the rate of charge and discharge, and on the time the cell has been standing in between charge and discharge, and is generally expressed as an efficiency for such and such a rate of discharge in amperes.

The most appropriate rate of charge in amperes is

determined by the area of the opposed surface of the plates, and this surface also imposes a limit to the normal or safe rate of discharge. In order to keep the cell in good condition, the cell must be charged at a certain current density, or at a certain number of amperes per square foot of plate surface, and the rate of discharge must not exceed a stated maximum current density.

In charging the cell, a current is put into it under a certain electromotive force. The product of the in-going current, reckoned in amperes, and the terminal potential difference in volts, is the *charging power* in watts. The total amount of energy put into the cell can be reckoned out in *watt-hours*. The ratio expressed as a percentage between the energy which can be taken out of the cell on its discharge to the energy which is put into it in charging is called the *watt-hour efficiency* of the cell. This can never be a number greater than about 80, because the average charging voltage is about 2½ volts per cell, and the average discharge voltage is not more than two volts.

These two efficiencies are, in practice, obtained as follows.

A cell has a measured current sent into it to charge it, and the potential difference between the cell terminals is measured in volts at regular intervals. A horizontal line is then taken (see Fig. 133) on which to mark off time in hours. Vertical lines are then drawn to some suitable scale to represent the ampere-current going into the cell, and also the power in watts, viz. the numerical product of the current in amperes and terminal volts, supplied to the cell, the readings being taken at frequent intervals. If lines are drawn joining the tops of these perpendiculars, we shall obtain curves showing the variation of charging current and charging power during the charge.

If the area lying between the horizontal line, the two extreme perpendiculars and the curve is taken, and

reckoned out in units, each equal to the area of a rectangle, one side of which is the length taken to represent an hour, and the other, the length taken to represent an ampere or a watt, then these areas give us respectively the *ampere-hours* and *watt-hours* put into the cell.

Let the cell be then discharged and measured in the same manner, and let the discharge be considered as complete, when the potential difference between the terminals of the cell has fallen to 1·9 volts. If then we construct another pair of diagrams drawn as above, to represent the ampere-hours and watt-hours of the

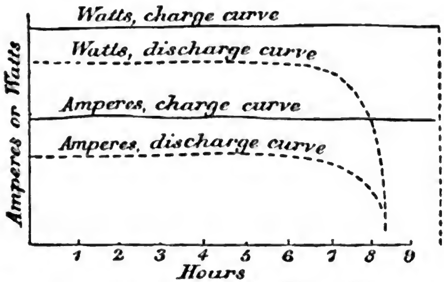

Fig. 133.—Ampere-hour and Watt-hour Areas for Charge and Discharge Diagram of Secondary Cell.

discharge, we are finally able to find the ratio of the area representing the ampere-hour discharge to that of the ampere-hour charge, and the area representing the watt-hour discharge to that representing the watt-hour charge. This gives us the *ampere-hour efficiency* and the *watt-hour efficiency.* The ampere-hour efficiency will be smaller the greater the rate of discharge, and so will the watt-hour efficiency.

To get the greatest ampere-hour efficiency, the cell must be discharged with a small out-going current, and the charge must be stopped the moment bubbles of gas are seen coming off from the plates.

In conducting a test of a secondary cell, these electrical measurements are best made by the potentiometer. The internal resistance of the cell can be calculated for any out-going current by measuring the voltage (V) of the cell on open circuit, and by measuring the terminal potential difference (v) when the circuit of the cell is closed through a known resistance of (R) ohms.

Then, since the out-going current is equal by Ohm's law to $\frac{v}{R}$, and also since by the same law

$$\frac{v}{R} = \frac{V}{r + R},$$

where r is the internal resistance of the cell, we see that r can be calculated by the formula

$$r = R\left(\frac{V - v}{v}\right).$$

The chemical changes which go on in the lead secondary cell during charge and discharge have been the subject of much investigation, and even now can hardly be said to be fully understood. When a lead secondary cell of the Planté type has been properly *formed* and *charged*, on looking at the plates we see that one plate, called the positive plate, which has been in connection with the positive pole of the charging dynamo, is converted on the surface, and to some depth below the surface into a dense puce-coloured or chocolate-coloured deposit, which is closely adherent to the lead-plate. This is composed chiefly of *peroxide of lead* (PbO_2). The other plate, called the negative plate, has a clean grey colour, and the surface of this lead plate has been converted into lead in a highly porous condition, in which it is termed *spongy lead*.

Faure made the discovery, that the process of forming the plates could be accelerated by mechanically putting on to the surface of the lead, red oxide of

lead (minium), Pb_3O_4, which was held there, and was reduced and peroxidised, instead of forming the peroxide and spongy lead entirely out of the material of the plate itself, as did Planté. Subsequent improvements by Mr. J. W. Swan and others, finally led to the perfection of a type of plate for secondary batteries, called the *pasted plate* or *packed grid* type. This plate is made by casting a lead grid with rectangular apertures (see Fig. 134), and having a conducting lug attached to it. This plate is then packed with a cement made by mixing up litharge (PbO) or red lead (Pb_3O_4) with dilute sulphuric acid, according as it is intended to make a negative or positive plate. This cement is packed and pressed into the apertures of the grid, and after a time sets hard. The plate so prepared is placed in dilute acid and *formed* by means of a current, and finally we obtain, as a result of the process, a series of positive plates mostly converted into hard peroxide of lead on the outside and outer portions, and negative plates in the same way reduced to spongy lead. The lead grid serves as a backing to give mechanical strength.

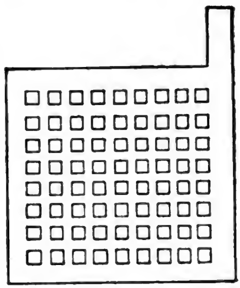

Fig. 134.—Lead Grid for Secondary Cell Plate.

When these *peroxide* and *reduced* plates, as they are termed, are placed in dilute sulphuric acid, they are electrochemically very different. The reduced plate acts to the peroxide plate as the zinc to the carbon in a primary cell, and, if connected by a conductor, the plates will produce a current in the connecting circuit. The electromotive force of the cell is 2·1 or 2·2 volts, but falls gradually as the discharge proceeds. The nature of the complicated chemical processes which go on in the cell cannot yet be said to be perfectly ascertained, but

opinion inclines on the whole to the view that during this discharge the plates take up sulphuric acid out of the dilute sulphuric acid, and both have white sulphate of lead produced in them and on them, which exists intimately mixed up with the remaining peroxide of lead and reduced lead. The process of charging the plates consists in breaking up this sulphate of lead electrolytically and reforming peroxide of lead and spongy lead on the positive and negative plates respectively. The chemical equation for the reaction is then as follows.

Before the discharge we have the active materials

$$(+)PbO_2 \mid H_2SO_4 + H_2SO_4 \mid Pb\,(-).$$

During the discharge we have formed out of them

$$(+)PbSO_4 \mid H_2O + H_2O \mid PbSO_4\,(-),$$

where the signs $+$ and $-$ denote the positive and negative plates.

Hence the peroxide and reduced lead disappear, and lead sulphate is formed. Sulphuric acid at the same time is taken up out of the dilute acid, thus reducing the specific gravity of the electrolyte. On charging again, the sulphate of lead is broken up and the density of the electolyte rises. The variation of the specific gravity of the electrolyte is therefore found to be an indication of the state of the cell. The dilute acid should have a specific gravity of 1·235 to 1·240 when the cell is fully charged, and should not be allowed to fall below 1·205 or 1·210 when the cell is discharged.

Cells should never be allowed to stand long uncharged or partly discharged. Chemical actions, called local actions, then come into play, which result in the formation of a dense layer of badly conducting sulphate of lead on the surface of the plates, which it is difficult to decompose. The peroxide of lead and the spongy lead are good conductors, and hence as long as the sulphate of lead is only formed in intimate mixture with these in the body of the plate it is easily got at and reduced by the current.

In charging a series of cells the charge should be continued until every cell just *boils*, as it is called, or gives off gas freely from each plate. The plates should then after full charge have a clean chocolate-red and blue-grey colour respectively, and the cell should never be discharged beyond the point at which the electromotive force of each cell on open circuit falls below 1·9 volts.

To obtain a steady voltage of 100 volts for the electric lighting of buildings it is customary to put in 54 cells, and to provide a dynamo capable of giving an electromotive force of 135 volts. The cells can then be charged fully by giving each its necessary 2 to 2·5 volts. At the beginning of the discharge only 49 or 50 cells are employed to give the voltage, and they provide 100 volts. As the discharge continues more cells are taken into use, and finally near the end of the discharge the whole 54 are necessary to obtain about 100 volts.

In estimating the value of any type of secondary cell for various purposes we have to take into account—

1. The capacity of the plates in ampere-hours per pound weight of plates (positive and negative reckoned together) and per pound weight of the complete cell.

2. The maximum rate of discharge per square foot of plate (surface of one set of plates alone reckoned) and per pound weight of the whole cell.

3. The safe maximum rate of charge and discharge per square foot of plate.

4. The durability of the plates.

5. The weight of the whole cell per horse-power at the maximum rate of discharge.

The capacity of the plates per pound of plates may generally be taken at about 4 ampere-hours per pound of plates, positives and negatives both taken together. In some types of cell it may rise as high as 9 ampere-hours per pound of plate. This weight-capacity is after all very much conditioned by durability. In estimating the value of a cell we have to consider not only its electrical

capacity but its duration or useful life and capability to stand some rough usage.

Plates can be made to have a large capacity in ampere-hours per pound, but this must not be gained at the expense of durability and by making a fragile plate. Generally speaking, a cell of the stationary type, consisting of a glass box containing the acid and lead plates, will weigh altogether from one-half to one-third as many pounds as it has ampere-hour capacity at a nine-hour discharge. In cells intended for traction work, capacities as high as 5 to 8 ampere-hours per pound of complete battery cell, can be obtained. Since the cell discharges at 2 volts, if we multiply the *ampere-hour capacity* by 2 we have the *watt-hour capacity* of the cell. This number divided by 746 gives us the *horse-power-hour capacity* of the cell. Hence the ordinary weight of complete secondary batteries may be taken at about one-hundred weight (112 lbs.) per horse-power-hour of contained energy.

The ampere-hour capacity of the cell depends upon the rate of discharge. Hence the discharge rate in amperes must be stated, or else the capacity figure has no useful meaning. It is generally stated by the makers for various durations of discharge. Thus the ampere-hour capacity is said to be so much for a 9-hour, 6-hour or 3-hour discharge.

The capacity of a cell is therefore intimately connected with the current-density during discharge, and the more rapidly we take the charge out of a cell the less quantity in ampere-hours do we find in it. The reason for this is that a rapid rate of discharge converts the superficial layers of the plate into lead sulphate, and hinders the process of sulphating the deeper layers of the plate. It is not merely the surface but the interior body of the plate which should take part in the chemical processes, and the rate of discharge affects the depth to which the action proceeds.

Bearing in mind that a watt-hour is 3600 joules, and

that a joule is nearly three-quarters of a foot-pound, we can find the work stored up in a cell in foot-pounds by multiplying the capacity in ampere-hours by $2 \times 3600 \times \cdot 75 = 5400$. Hence, generally from 20,000 to 40,000 foot-pounds of energy can be stored up in two secondary plates, one positive and one negative, weighing together one pound.

For electric traction purposes makers have aimed at constructing a cell which will be as light as possible per foot-pound of capacity, and yet stand sudden brief high discharges without damage.

For stationary purposes, especially for central station purposes, the aim is to secure durability combined with the power to give occasional large discharge currents.

As regards the current density during charge and discharge, most modern cells are so used that the normal charge or discharge current is at the rate of about 3 to 6 amperes per square foot of plate surface, reckoning only one set of plates, say the positives.

If the normal rate of charge or discharge is exceeded and we attempt to put the energy into the cell or take it out at a much greater rate than corresponds to a current density of the above stated value, there is risk of damaging the plate. For central station purposes, a tolerably uniform rate of treatment can be obtained, but cells intended to be used for *traction* (as in electrical cabs, omnibuses and trams) must be made to stand without injury large sudden discharges at a high current density for short periods during the starting of the motor.

A few words must then be said in conclusion on the subject of *primary batteries*. In nearly all the widely used primary batteries the negative electrode is a plate or rod of amalgamated zinc. This is associated with a plate or rod of some substance not capable of being oxidised at a low temperature, and which is at the same time a good conductor. The materials most usually employed for the positive electrode are copper, silver, platinum, carbon, carbon in association with pyrolusite

or peroxide of manganese, or else compressed peroxide of lead. If a plate of zinc and one of any of the above bodies (which are electrochemically very different from zinc) are placed together in a vessel in dilute acid or alkali, we then have in this voltaic couple a source of electromotive force. If the plates are metallically connected by a wire outside the cell, a current is produced in the wire which flows in the wire from the positive electrode or pole to the negative pole, and back through the electrolyte of the cell.

Immense discussion has taken place on the question of the place and origin of this electromotive force of the cell. Volta proved that two metals placed in contact are at different potentials; and he placed the seat of electromotive force of the copper-zinc cell, joined by a copper wire, at the point of contact of the copper and zinc. The electrical energy exhibited in the cell circuit undoubtedly is the result of a transformation of a part, at least, of the energy represented by the chemical actions going on in the cell; and it seems most reasonable, therefore, to place the seat of the cell electromotive force at those places where chemical transformations are proceeding, viz. at the place of contact of the metals and the active liquids. We shall not attempt, however, any discussion of the difficult matter of the theory of the voltaic cell, but confine ourselves to the simple facts.

If a pure zinc and copper plate are placed in dilute sulphuric acid and connected, it is seen that in a few moments the copper plate is covered over with gas-bubbles, which are hydrogen gas. At the same time the zinc dissolves in the dilute acid and forms sulphate of zinc. This layer of hydrogen liberated on the copper plate reduces the electrochemical difference of the two metals; and in fact a copper plate thus coated (or *polarised*) with hydrogen *ions* becomes electrically more similar to a zinc plate. As soon as this happens, therefore, the current nearly ceases. To keep up the current, some means must be provided to destroy or remove this

hydrogen, or *depolarise* the positive plate. Even if this polarisation does not stop the current altogether, it slowly reduces the current given by the cell by introducing a back-electromotive force and an additional resistance.

The most obvious means of getting rid of this hydrogen layer is to put into the cell some body which is rich in oxygen, and which can give it up easily to combine with the hydrogen and form water. For this purpose, chromic acid, bichromate of potash, permanganate of potash, chlorochromic acid, bleaching powder and other oxidising substances are placed in the cell as *depolarisers* An endless number of *secret liquids* have been vaunted and introduced as specially effective for this purpose. Taking the simple and well-known *bichromate cell* we find it is usual in this cell to employ a plate of hard, highly conducting graphite or carbon as the positive pole. To this a lead head is fixed or clamped, and a terminal. The creeping of acid up the carbon can be prevented by first soaking the top of the carbon in melted paraffin wax before putting on the clamp. The zinc and carbon plate are immersed in a solution formed as follows:—In 1 pint of water dissolve 3 ounces of crystallised bichromate of potash, and add 2 ounces of strong sulphuric acid. This solution costs about 1*s.* 6*d.* per gallon to make. Owing to the fact that the solution deposits chrome alum crystals on the carbon and zinc, and creates somewhat insoluble salts in working, it is better to use hydrochloric acid instead of sulphuric acid. The oxidising action of the chromic acid in the potassic bichromate continually destroys the hydrogen which would otherwise appear on the carbon plate when the cell is working. The electromotive force of the cell on open circuit is about 1·9 or 2 volts. The objection to the use of chromic acid or other similar oxidising bodies placed in the acid electrolyte is that they chemically attack the zinc directly, and by producing useless consumption of the zinc, reduce the *zinc efficiency* of the battery. Moreover, as the depolariser becomes exhausted round the

carbon plate, the electromotive force of the cell falls off. Hence it is very difficult with any single fluid cell of this kind to keep a constant current for long together through a resistance which is not very large. The zinc-efficiency of the battery may be measured as follows. Since the electrochemical equivalent of zinc is 1·21330 grammes per ampere-hour, we see that for every ampere-hour of electric quantity that comes out of the cell, 1·2133 grammes of metallic zinc must at least be dissolved. If we allow the battery to send a constant current of x amperes for say 5 hours, we know then that $5x$ is the number of ampere-hours which have come out of the cell. Hence $5x \times 1{\cdot}2133$ grammes of zinc at least have been dissolved off the zinc plate to make this current. If the zinc plate is weighed before and after the experiment, then any weight lost over and above the amount so reckoned is due to waste, or local chemical action on the zinc. The ratio between the zinc which should have been theoretically used, and that which has been actually used up is called the *zinc efficiency* of the cell. It will vary with the duration of the experiment, being greater the larger the current taken from the cell.

In spite of its disadvantages the bichromate cell is a very favourite one in laboratory and for telegraphic purposes.

The amalgamated zinc-plates used in it cost about *6d.* per lb., and we may say that approximately a pound of zinc will be used up for every gallon of bichromate solution. This amount of zinc theoretically should produce about 400 ampere-hours of electric quantity, or say 700 watt-hours of electric energy at 1·8 volts. Hence, 1000 watt-hours would cost about *3s.* to produce by means of a bichromate cell. This amount of electric energy is called one Board of Trade unit.

A Board of Trade unit of electric energy can be supplied by means of engines and dynamos for *3d.* a unit, or less, and is usually supplied for house-lighting at *6d.* Hence it will be seen that the generation of electric

currents by zinc primary batteries is much more costly than by dynamo-electric machines. Nevertheless, for the production of small electric currents, and for portable batteries, such as miners' lamps, there is a useful field for primary batteries.

For telegraphic and telephonic work, and for electric bells and all purposes where a small electric current is required intermittently, the most convenient cell for the purpose is the Leclanché, or some modification of it in the form of the *Dry Cell*, now much used.

The Leclanché cell, in one form, consists of an amalgamated zinc rod and a hard carbon plate, this last placed in a porous pot and packed round with granulated carbon and peroxide of manganese in a crystalline form. The exciting liquid is either sal ammoniac solution or a mixture of chloride of zinc and chloride of ammonium dissolved in water. The peroxide of manganese is a highly oxygenated body, and it acts as the depolariser. The peroxide is sometimes mixed with the carbon and compressed into the form of a hard block, called an *Agglomerate Block*, or the compressed peroxide of manganese may be placed in the form of separate compressed blocks on either side of the carbon plate.

In the case of the so-called dry cells the exciting solution is mixed with plaster of Paris or some absorbent material, so that there is no free fluid in the cell, and it can be sealed up and easily transported. There must, however, be moisture sufficient present to permit the chemical action to take place and preserve the electrolytic character of the exciting material.

The above cells are all called single-fluid cells, with depolarisers either solid, like the manganese peroxide, or liquid, like the bichromate of potash solution.

In two-fluid cells the depolariser is placed around the positive pole and preserved from coming in contact with the exciting solution in which is placed the zinc plate, either by a porous partition or by a difference of specific gravity, as in the ordinary and gravity Daniell cell,

In this case the ionic hydrogen set free from the dilute sulphuric acid in which the zinc is placed never reaches the copper plate at all. It enters the sulphate of copper solution in which the copper plate is placed and liberates from it an equivalent of ionic copper, which is deposited on the copper plate. The student may with advantage compare the chemical actions supposed to take place in the lead secondary battery with those taking place in a simple copper-zinc-acid cell and in a Daniell two-fluid non-polarisable cell. These are exhibited in chemical symbols below. The thick block letters stand for the metallic part of the electrodes, taking no share in the action.

I. Chemical Actions in the Lead Secondary Cell.

Before discharge—

– Plate	Electrolyte	+ Plate
| **Pb** Pb	$\left.\begin{matrix} SO_4 \\ H_2 \end{matrix}\right\}, \left.\begin{matrix} SO_4 \\ H_2 \end{matrix}\right\}$ with additional dilute acid	$\left.\begin{matrix} Pb \\ O_2 \end{matrix}\right\}$ **Pb** |

After discharge—

– Plate	Electrolyte	+ Plate
| **Pb** $PbSO_4$	H_2O, H_2O with additional dilute acid	$PbSO_4$ **Pb** |

II. Chemical Actions in the Zinc Primary Cell.

Before discharge—

– Plate	Electrolyte	+ Plate
| **Zn** Zn	$\left.\begin{matrix} SO_4 \\ H_2 \end{matrix}\right\}, \left.\begin{matrix} SO_4 \\ H_2 \end{matrix}\right\}$ with additional water	Cu **Cu** |

After discharge—

− Plate	Electrolyte	+ Plate
$\mid \mathbf{Zn}$	$ZnSO_4$, H_2SO_4 with additional water	H_2Cu **Cu** $\mid$

In the last equation H_2Cu stands for the polarised surface of the copper plate, the hydrogen being deposited on it and adherent to it.

III. Chemical Action in the Daniell Non-Polarisable Cell.

Before discharge—

− Plate	Electrolytes separated by porous division	+ Plate
$\mid \mathbf{Zn}\ \mathrm{Zn}$	$\left.\begin{matrix} SO_4 \\ H_2 \end{matrix}\right\} \left.\begin{matrix} SO_4 \\ H_2 \end{matrix}\right\} \Vert \left.\begin{matrix} SO_4 \\ Cu \end{matrix}\right\} \left.\begin{matrix} SO_4 \\ Cu \end{matrix}\right\}$ with additional water	Cu **Cu** $\mid$

After discharge—

− Plate	Electrolyte	+ Plate
$\mid \mathbf{Zn}$	$ZnSO_4$, H_2SO_4 $\mid$ H_2SO_4, $CuSO_4$ with additional water	Cu Cu **Cu** $\mid$

In the last equation, Cu Cu **Cu** stands for the original copper plate *thickened* by a deposit of copper made upon it from the sulphate of copper solutions.

Lord Kelvin and Von Helmholtz showed that the electromotive force of a chemical cell could be calculated from a knowledge of the heat equivalents of all the chemical and physical actions going on in the cell, *and* of the *rate* at which the cell changes its electromotive force with temperature when heated or cooled. In some cases external heating increases the electromotive force of a cell. In other cases, such as the Clark cell, heating the cell diminishes the electromotive force. In the case of the

Daniell cell there is practically no change. The resultant mechanical equivalent of all the heat actions taking place in the cell can be calculated from thermo-chemical data.

Suppose H to stand for the total amount of heat, *reckoned in mechanical units*, which would be liberated by the chemical actions taking place in the cell if all the energy represented by one C.G.S. unit of electricity passing round the cell circuit assumed a thermal form.

The unit of heat, be it remembered, is the heat required to raise one gramme of water one degree Centigrade at or near 10° C., and this is called one water-gram-degree, or *one calorie.* The gram-degree represents a mechanical work of 42 million ergs, or 4·2 joules; and heat is therefore measured in C.G.S. mechanical units when its calorie-value is multiplied by 42 million.

If the electromotive force of the cell is E volts, or $E \times 10^8$ C.G.S. units, then since the work done by one C.G.S. unit of electricity in falling through a potential of E volts is $E \times 10^8$ ergs, we must have the equation

$$E \times 10^8 = H$$

expressing the fact that the work done by the unit of electric quantity in going round the circuit under the action of the electromotive force, E volts is, by the law of conservation of energy, numerically equal to the total heat H represented by the resultant chemical action in the cell measured in mechanical units.

Suppose then that h represents the resultant thermal action *in calories*, we must have

$$H = h \times 0{\cdot}42 \times 10^8,$$

or

$$E = h \times 0{\cdot}42.$$

This was Lord Kelvin's original equation for the electromotive force of a voltaic cell. Von Helmholtz, however, in addition showed that if the cell changes its electromotive force with temperature, then, at any temperature

t° Centigrade, the electromotive force of the cell is represented more properly by the equation

$$E = 0{\cdot}42\,h - (273 + t^{\circ})\left\{\begin{array}{c}\text{The rate of change of the electro-}\\ \text{motive force with temperature.}\end{array}\right\}$$

In the case of the Daniell cell this last term is zero. In the Daniell cell the passage of one C.G.S. unit of quantity of electricity (10 coulombs) round the circuit involves the necessity for chemical changes in the cell, of which the resultant or *nett* thermal equivalent (h) is $2{\cdot}592$ calories. Hence the electromotive force of the Daniell cell by the above equation is $2{\cdot}592 \times 0{\cdot}42 = 1{\cdot}09$ volts, which closely agrees with observation.

The passage of 10 coulombs of electricity round the circuit necessitates the solution of ·003370 of a gramme of zinc, and this amount is called the absolute electro-chemical equivalent of zinc.

APPENDIX.

I.

The Measurement of the Earth's Horizontal Magnetic Field Strength.

The measurement of the Earth's Horizontal Magnetic Field Strength or Force in absolute or C.G.S. units is an important and fundamental physical measurement. It is based on the following principles.

If a point P (see Fig. 1) revolves in a circle round a fixed point O, with a uniform angular velocity w, and in a periodic time T, then $w\,T = 2\,\pi$. If the position of the point P is at every instant projected on to a fixed straight line X Y, the projection of O P, viz. O p, increases and diminishes with a simple harmonic motion (S.H.M.). The point p moves backwards and forwards along X Y. The distance O p is called the *Displacement* of p at any instant. Let the displacement be denoted by x, and let the maximum displacement of p, viz. O X, be denoted by X. Then O P = O X = X. Let the velocity of the point p, moving in the straight line X Y, at any instant be denoted by v, and its maximum velocity by V. Similarly let the acceleration of p, that is to say *the velocity of the velocity* of p be denoted by a, and its maximum value by A.

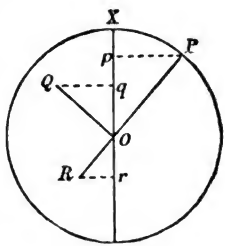

Fig. 1.

We have seen in Chap. VIII. that the maximum velocity of p is equal to w times O P. Hence V = w X. The velocity of p is represented as regards magnitude at every instant by the

projection on X Y of a line O Q drawn at right angles to O P, and of such length that $OQ = w\,OP$. Hence it follows that the maximum value of the acceleration of the point p will be represented by $w^2 X$, and the acceleration at any instant will be represented as regards magnitude by the projection on X Y of a line O R, drawn at right angles to O Q, or in the prolongation of O P, and equal in length to $w^2 X$. We have therefore

$$A = w^2 X,$$

and

$$V = w X;$$

also

$$w T = 2\pi;$$

therefore

$$V = \sqrt{A X} = \frac{2\pi}{T} X,$$

and hence

$$T = 2\pi\sqrt{\frac{X}{A}} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

This last equation shows us that the periodic time of a particle moving with a simple harmonic motion backwards and forwards along a straight line X Y is equal in magnitude to 2π times the square root of the quotient of the maximum displacement by the maximum acceleration. Moreover, the diagram shows us that the acceleration *at any instant* in the case of such a motion is proportional to the displacement, since Or always varies as Op, as O P moves round. The moving force on a body is by the definitions of force and acceleration (see Chap. II.) equal to the product of its mass and its acceleration. Hence if f stands for the *force* acting at any instant to restore a particle moving with a S.H. motion to its zero position, and if m is its mass we have always

$$f = m\,a,$$

and

$$F = m\,A \text{ for the maximum values.}$$

Substituting this last value for A in equation (i) we have for the periodic time T of a body of mass m, executing a S.H.

motion under a maximum force F, the force varying as the displacement, the equation

$$T = 2\pi\sqrt{\frac{mX}{F}} \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

Consider then the case of a simple pendulum. Let a small particle P (Fig. 2) oscillate in a *small arc* by a weightless cord O P. Let the particle P have a mass m. The force acting on P to restore O P to its vertical position if it is displaced, is equal to $m g \sin \theta$, where $\theta =$ the angle Y O P and g is the acceleration of gravity. If the angle Y O P is small, we may write θ for $\sin \theta$, and if we call the length of the arc Y P x, and the length of the pendulum O P l, we have then $x = l\theta$.

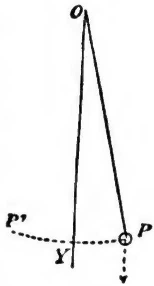

Fig. 2.

Hence the force acting to restore the particle P to its position if displaced is equal to $\frac{mgx}{l}$, and is therefore proportional to the displacement for any given length of pendulum.

The bob P moves therefore backwards and forwards with a simple harmonic motion. By equation (ii) the periodic time T of the motion must be such that

$$T = 2\pi\sqrt{\frac{mX}{F}},$$

but

$$F = \frac{mg}{l} X.$$

Hence

$$T = 2\pi\sqrt{\frac{l}{g}} \quad . \quad . \quad . \quad . \quad . \quad \text{(iii)}$$

This last equation is the well-known formula for the time of vibration of a simple pendulum. Suppose the pendulum is *not* a simple pendulum, consisting of a small particle placed at the end of a weightless cord, but is a mass of any shape vibrating

round the axis. We can always find an equivalent simple pendulum; that is to say, a pendulum consisting of a mass m equal to the whole mass of the vibrating body concentrated in a small particle and placed at the end of a weightless cord of length l, such that it makes a complete oscillation in a periodic time equal to that of the vibrating body. Then the couple or torque c acting to restore the simple pendulum to verticality at any instant is equal to $f\,l$, where f is the restoring force acting on the small particle in a direction at right angles to the suspending cord. Hence the maximum value of the restoring couple C is F l. The maximum value of the displacement X is equal to $l\,\theta$, where θ is the angular displacement of the equivalent simple pendulum at the extremity of its motion. Hence substituting in equation (ii) the values for F and X given by the equation

$$\mathrm{C} = \mathrm{F}\,l$$
$$\mathrm{X} = l\,\theta$$

we have for the periodic time T of the simple equivalent pendulum the expression

$$\mathrm{T} = 2\,\pi\sqrt{\frac{m\,\mathrm{X}}{\mathrm{F}}} = 2\,\pi\sqrt{\frac{m\,l^2}{\frac{\mathrm{C}}{\theta}}}.$$

The quantity $m\,l^2$, viz. the product of the mass of the particle and the square of its distance from the centre of suspension, is called the *Moment of Inertia* of the simple pendulum. The quantlty $\frac{\mathrm{C}}{\theta}$ is the *restoring couple per unit angle* of displacement. If we write I for $m\,l^2$, and K for $\frac{\mathrm{C}}{\theta}$ we can write the expression for the periodic time T of the equivalent simple pendulum as follows:—

$$\mathrm{T} = 2\,\pi\sqrt{\frac{\mathrm{I}}{\mathrm{K}}} \quad . \quad . \quad . \quad . \quad . \quad \text{(iv)}$$

In other words, the periodic time is equal to $2\,\pi$ times the square root of the moment of inertia divided by the restoring couple per unit angle of displacement for small displacements. This last is obtained by making a small displacement, and

taking the quotient of the numerical value of the restoring couple by the numerical value of the angular displacement. This equation (iv) is perfectly general, and applies to bodies of any form vibrating on *small* arcs round an axis of rotation under the action of a restoring couple which varies in magnitude proportionately to the angle of displacement.

Apply the foregoing facts to the consideration of a magnet of moment M vibrating in the earth's horizontal field round a vertical axis. If I is the moment of inertia of the magnet, then $M H \sin\theta$ is the couple restoring the magnet if displaced by an angle θ. If θ is *small*, then the couple C restoring the magnet when displaced through an angle θ is $M H \theta$. Hence $\frac{C}{\theta} = M H$, and therefore

$$T = 2\pi\sqrt{\frac{I}{M H}} \quad . \quad . \quad . \quad . \quad \text{(v)}$$

Hence if we set such a magnet vibrating in a small arc, and note the time T of a complete vibration, and know the moment of inertia I of the magnet, we can calculate the value of the product M H.

If the magnet has the form of a right-circular cylinder of mass W, and has a length $2l$ and a diameter d, its moment of inertia I is equal to

$$W\left(\frac{l^2}{3} + \frac{d^2}{16}\right).$$

Hence we have for the value of M H

$$M H = \frac{4\pi^2}{T^2} I,$$

$$= \frac{4\pi^2}{T^2} W\left(\frac{l^2}{3} + \frac{d^2}{16}\right) \quad . \quad . \quad . \quad \text{(vi)}$$

This enables us to calculate the product of the magnetic moment M, and the Earth's horizontal force H, when we know the time of vibration T, the mass W, the length $2l$, and the diameter d, of a cylindrical magnet which is set vibrating round

a vertical axis through its centre, and normal to its longest dimension.

In the next place, let the cylindrical magnet be placed with its magnetic axis at right angles to the magnetic meridian, and allowed to deflect a very small compass needle placed in the axial line of the magnet, and with its centre at a distance x from the centre of the deflecting magnet. Let the magnetic moment of this small compass needle be M′. Let $2y$ be the *magnetic length* of the deflecting magnet, that is to say, the distance between its poles. Let also $2y'$ be the magnetic length of the deflected magnet. Then it is easy to see that the force due to the poles of the deflecting magnet, which acts on *one* pole of the deflected magnet, must be equal to

$$\frac{\frac{M}{2y} \cdot \frac{M'}{2y'}}{(x-y)^2} - \frac{\frac{M}{2y} \cdot \frac{M'}{2y'}}{(x+y)^2}$$

or to

$$\frac{M}{2y} \cdot \frac{M'}{2y'} \left\{ \frac{1}{(x-y)^2} - \frac{1}{(x+y)^2} \right\},$$

or to

$$\frac{M\,M'}{y'} \; \frac{x}{(x^2-y^2)^2},$$

and similarly an equal force due to the deflecting magnet acts on the other pole of the compass needle.

Hence the couple due to the deflecting magnet which acts on the compass needle when deflected through an angle θ must be equal to

$$M\,M' \frac{2x}{(x^2-y^2)^2} \cos\theta,$$

and the opposing couple, due to the Earth's horizontal force, must be equal to

$$M'\,H \sin\theta.$$

These couples balance one another when the compass needle is at rest, and then we have

$$M' H \sin\theta = M M' \frac{2x}{(x^2 - y^2)^2} \cos$$

or

$$\frac{M}{H} = \frac{(x^2 - y^2)^2}{2x} \tan\theta \quad . \quad . \quad . \quad . \quad \text{(vii)}$$

From this equation we can determine the value of $\frac{M}{H}$ by a measurement of the distance x, the angle θ, and the magnetic length y of the deflecting magnet. For most purposes it is sufficient to take y as equal to ·82 of the length of the cylindrical magnet. If y is not known, we can take two observations of θ at different distances x and so eliminate the unknown quantity y from Equation (vii).

We have thus determined, by Equations vi and vii, the value of the *product* M H, and the quotient $\frac{M}{H}$, and the value of M and H separately is then immediately found.

For if $MH = a$, and $\frac{M}{H} = b$,

$$H = \sqrt{\frac{a}{b}}.$$

If the reader makes a determination in this manner of the value of H in a room free from large masses of iron, he ought to obtain a value not far from ·18 in the South of England, and about ·15 in the North of England or in Scotland.

II.

Table of Natural Sines, Cosines and Tangents.

Angle in Degrees.	Sine.	Cosine.	Tangent	Angle in Degrees	Sine.	Cosine.	Tangent.
0	·000	1·000	·000	22	·375	·927	·404
1	·017	·999	·017	23	·391	·920	·424
2	·035	·999	·035	24	·407	·913	·445
3	·052	·998	·052	25	·422	·906	·466
4	·070	·997	·070	26	·438	·899	·488
5	·087	·996	·087	27	·454	·891	·509
6	·105	·995	·105	28	·469	·883	·532
7	·122	·992	·123	29	·484	·875	·554
8	·140	·990	·141	30	·500	·866	·577
9	·156	·988	·159	31	·515	·857	·601
10	·174	·985	·176	32	·530	·848	·625
11	·191	·982	·194	33	·544	·838	·649
12	·208	·978	·212	34	·559	·829	·674
13	·225	·974	·231	35	·574	·819	·700
14	·242	·970	·256	36	·588	·809	·726
15	·259	·966	·268	37	·602	·798	·754
16	·276	·961	·287	38	·616	·788	·781
17	·292	·956	·305	39	·629	·777	·809
18	·309	·951	·325	40	·643	·766	·839
19	·326	·945	·344	41	·656	·754	·869
20	·342	·940	·364	42	·669	·743	·900
21	·359	·934	·384	43	·682	·731	·932

TABLE OF NATURAL SINES, ETC.—*continued.*

Angle in Degrees.	Sine.	Cosine.	Tangent.	Angle in Degrees.	Sine.	Cosine.	Tangent.
44	·695	·719	·966	68	·927	·374	2·475
45	·707	·707	1·000	69	·934	·358	2·605
46	·719	·695	1·035	70	·939	·342	2·747
47	·731	·682	1·072	71	·945	·326	2·904
48	·743	·669	1·111	72	·951	·309	3·077
49	·755	·656	1·150	73	·956	·292	3·271
50	·766	·643	1·192	74	·961	·276	3·487
51	·777	·629	1·235	75	·966	·259	3·732
52	·788	·616	1·280	76	·970	·242	4·011
53	·799	·602	1·327	77	·974	·225	4·332
54	·809	·588	1·376	78	·978	·208	4·705
55	·819	·574	1·428	79	·981	·191	5·145
56	·829	·559	1·482	80	·985	·174	5·671
57	·839	·545	1·540	81	·988	·156	6·314
58	·848	·530	1·600	82	·990	·139	7·115
59	·857	·515	1·664	83	·992	·122	8·144
60	·866	·500	1·732	84	·994	·105	9·514
61	·875	·484	1·804	85	·996	·087	11·43
62	·883	·469	1·880	86	·997	·070	14·30
63	·891	·454	1·963	87	·998	·052	19·08
64	·899	·438	2·050	88	·999	·035	28·64
65	·906	·422	2·144	89	·999	·017	57·29
66	·913	·407	2·246	90	1·000	·000	Infinite
67	·920	·391	2·356				

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